A TREATISE

ON

LAND

AND

LEVELLING,

ILLUSTRATED BY

COPIOUS FIELD NOTES, PLANS, AND DIAGRAMS.

IN FOUR PARTS,

BY THE

CHAIN, THEODOLITE, CIRCUMFERENTOR, AND SPIRIT LEVEL,

With Drawings,

EXPLAINING THEIR USE, AND EXHIBITING THEIR ADJUSTMENTS,
TOGETHER WITH INTRODUCTORY CHAPTERS
ON GEOMETRY, LOGARITHMS, MENSURATION, AND TRIGONOMETRY,

AND AN APPENDIX OF
TABLES OF LOGARITHMS, SINES, COSINES, TANGENTS, &c., TO SIX PLACES,

AND

A TRAVERSE TABLE

TO ANY DISTANCE, AND TO THREE MINUTES OF BEARING.

BY

HENRY JAMES CASTLE, D.P.S.,

SURVEYOR AND CIVIL ENGINEER.

LATE BOUNDARY COMMISSIONER TO THE HOME DISTRICT, UPPER CANADA,
AND LECTURER ON PRACTICAL SURVEYING AND LEVELLING TO
KING'S COLLEGE, LONDON.

LONDON:
SIMPKIN, MARSHALL, & CO., STATIONERS' HALL COURT.

TO

LIEUT. COL. SIR R. BONNYCASTLE, R.E.,

COMMANDING OFFICER OF ENGINEERS,
UPPER CANADA,

AS AN HUMBLE TOKEN OF RESPECT FOR HIS VIRTUES,

OF ADMIRATION FOR HIS TALENTS,

AND

AS A SINCERE, THOUGH INADEQUATE, OFFERING OF ACKNOWLEDGMENT

OF THE MANY

PROFESSIONAL SERVICES RECEIVED AT HIS HANDS,

WHILE EMPLOYED UNDER HIM IN THE FIELD,

THIS WORK

IS, WITH EVERY WISH FOR HIS HEALTH AND HAPPINESS,

RESPECTFULLY INSCRIBED BY HIS SINCERE FRIEND,

AND OBLIGED SERVANT,

THE AUTHOR.

It was from the want of any sufficient treatise, that I could put into my pupils' hands, on the subject of Land Surveying and Levelling, and the inconvenience I experienced in consequence, while engaged in my professional duties, as Lecturer at King's College, London, that I was induced to compile the following pages.

I found many parts of the subject ably discussed, scattered indifferently among several authors, but none sufficiently consecutive, or in detail, to suit my purpose.

Most of them were too elementary—confined to Chain Surveying—and sometimes not referring, even in that, to the modern system of "tyeing in," as it is called, or triangulating.

Others, on the contrary, were either purely military, or soared so highly into the depths of analytical calculations, that, though of invaluable assistance to the professional man of science, were, from the omission altogether of the more humble details of operation, both of the Chain and Theodolite, unfitted for the use of the civil surveyor;

and not one of them contained any information on the subject of the Circumferentor, or seemed sufficiently illustrated with plans and field notes.

From the direction also, that education had now taken towards the arts and sciences of life, *in most public and private schools,* I was induced to think that the present would be found a desirable *school book*, among the higher forms, as a full and corrected treatise of the theory and practice of Surveying. At the same time I trusted, that, by supplying the previous deficiencies, I should be furnishing the Surveyor with a complete *vade mecum* of reference.

To fulfill both these objects, I have introduced preliminary chapters on the most useful Problems and Theorems—on the Nature and Use of Logarithms—on Mensuration of Planes—and sufficient of Trigonometry as to enable the reader to understand fully any of the subsequent Trigonometrical Problems.

That portion, which refers to Surveying, has been, for facility of reference, divided into three Parts—by the Chain, Theodolite, and Circumferentor—containing copious field notes, plans, and diagrams, of each kind, together with drawings of the Theodolite and Circumferentor, and ample descriptions of their uses and adjustments.

In the Third Part, on Surveying, a special treatise has been introduced on the uses of the Circumferentor, in its application to the surveying of new countries. And the various methods, in actual operation in America—in the survey, by the needle, of extensive tracts of country—with the nature and application of the Traverse Table, to that purpose, are fully entered into and explained. The method of using the needle in this country, holding, as it does, but a subordinate part in surveying here, bears about an equal proportion to the same instrument in its perfection and accuracy in

new Countries, as the bubble of the Theodolite does to the same principle, when it is perfected in the Spirit Level.

In that part of the book that treats upon Levelling, in addition to the mere description of the nature and object of LEVELLING generally, and the use and adjustments of the *Instrument* and the reading of the Staff, have been introduced Field Notes of three miles of levels, taken on a line of railway, with the several Cross Sections. The method of computing the lowering and raising of the approaches have been given; and Drawings of the sections and cross sections, conformably to the regulations required for plans, that are intended for parliamentary deposit, have been added for illustration.

The several methods, whether correct or incorrect, adopted in practice, for calculating the CUTTINGS AND EMBANKMENTS, have been carefully investigated and examined, and corrections given for those that are wrong; and practical examples have been annexed, fully worked out, by the *Prismoidal Formula*—by Bidder's tables—and checked by a new Formula, which is specially adapted for cuttings of *any* length and height, for which I am indebted to Professor Moseley, of King's College.

And, lastly, there is an Appendix of tables of Logarithms, Sines, and Cosines; and a Traverse Table of Latitudes and Departures to any distance, and to three minutes of bearing.

I have now to acknowledge my obligation to the works of several authors, to whom I am indebted for much valuable assistance: among others, to Hall, Bridges, Keith, Hutton, Gummere, &c., &c.

Having intended these pages, as much for those who may have no opportunity of obtaining the assistance of a master, as for those who have, I have, in every case, annexed examples for practice;—to some of them have been given answers; to others, when the same result could be obtained by two or three methods, as additional practice for the Student, the answers have been omitted.

Before concluding, I have only to appeal to the kind indulgence of my readers for any mistakes that may have crept into the work, assuring them, that, as I shall have full opportunity of checking every example among my own Pupils, those mistakes shall be duly corrected in the next Edition.

30, *Queen Square, Bloomsbury,*
March 2, 1842.

P.S.—On further consideration, I have given, at the end of the work, the *corrected* and *omitted Answers* to *all* the Examples.

CONTENTS.

INTRODUCTION.

Plane Trigonometry.

CHAPTER I.

CHAPTER II.

Right-angled Triangles.

CHAPTER III.

Oblique-angled Triangles.

LAND SURVEYING.

Part the First.

CHAPTER I.

By the Chain.

CHAPTER II.

CHAPTER III.

CHAPTER IV.

CHAPTER V.

Methods of Surveying.

CHAPTER VI.

CHAPTER VII.

Of several Fields together.

CHAPTER V.

CHAPTER VI.

Division of Land.

CHAPTER VII.

CHAPTER VIII.

CHAPTER IX.

CHAPTER X.

CHAPTER XI.

LEVELLING.

CHAPTER I.

Its Nature and Object.

CHAPTER II.

CHAPTER III.

CHAPTER IV.

CHAPTER V.

CHAPTER VI.

CHAPTER VII.

CHAPTER VIII.

CHAPTER IX.

CHAPTER X.

CHAPTER XI.

ILLUSTRATIONS.

AND

ONE HUNDRED AND FORTY

WOOD CUTS.

INTRODUCTION.

GEOMETRY.

GEOMETRICAL DEFINITIONS.

A point has neither length, breadth, nor thickness.

A line has length.

A plane, or superficies, has length and breadth.

A solid has all three—length, breadth, and thickness.

An angle is the inclination of two straight lines meeting in a point.

A right angle is the inclination of one straight line to another, when, if either be produced, the second angle will be equal to the former.

An obtuse angle is greater: an acute angle, less: than a right angle.

Parallel lines never meet.

A parallelogram is a four parallel sided figure.

A rectangle is a right angle parallelogram.

A square is an equal sided rectangle.

A rhombus is equilateral, but not equiangular.

A rhomboid is neither equiangular nor equilateral.

A circle is a plane figure bounded by its circumference, every part of which is equidistant from the centre. The radius is this distance from the centre.

An arc is a portion of this circumference.

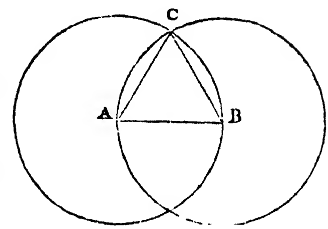
The chord of an arc is the straight line, joining the extremities of the arc.

A segment is the space, included between the chord and the arc.

The sector of a circle is the space, included between the two sides, subtending the angle, and the arc; therefore the sector of a right angle is a quadrant.

GEOMETRICAL PROBLEMS.

1. *To describe an equilateral triangle upon a given line.*

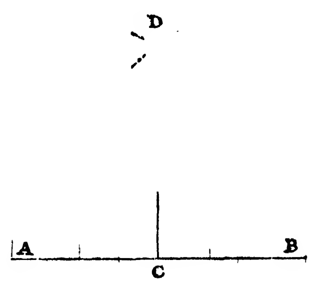


Let AB be the given line.

From A and B, with the radius AB, describe two circles intersecting in C; join AC and BC.

ABC is the equilateral triangle required.

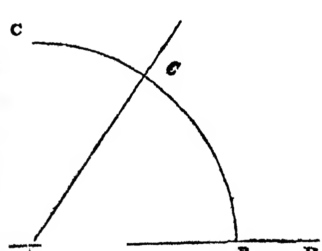
2. *From a point, within a given line, to erect a perpendicular.*



Let AB be the given line, and C be the given point.

From C, as a centre, take any distance, CA, and make CB=CA. Then from A, and B, as centres, at the distance AB, describe arcs intersecting at D; join the point of intersection at D with the point C. CD will be perpendicular to AB.

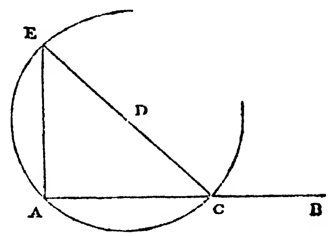
At a given point, in a given line, to construct a right angle, or an angle of any number of degrees, by means of a scale of chords; let AD be the given line, and A the given point.



Take off with your compasses, on any scale of chords, AB, the chord of 60 degrees (*radius*); and from the given point A, with that distance, describe a circle, intersecting the given line at B; then, from the point of intersection, with the distance of the chord of 90 degrees, BC, or of the chord of any other angle that may be required, BC, on the same scale, describe another

circle, intersecting the former. Join the points of intersection with the given point, and the lines will be perpendicular to, or making the required angle, with the given line. A protractor, which is a semicircle graduated into 180 degrees, and numbering, both from right to left, and from left to right, may be used, but not so accurately, for the same purpose.

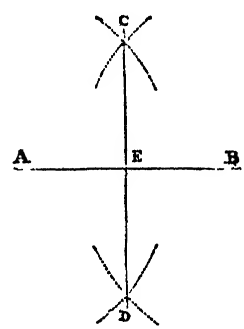
3. *From a point at the end of a given straight line to erect a perpendicular.*



Let AB be the given straight line, and A the point at the end. Take any point D and from D as centre, at the distance DA, describe the circle EAC, and join CD, and produce it to E. Join EA.

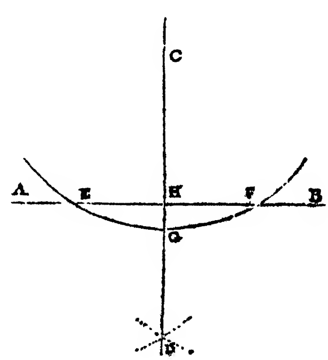
EA shall be the perpendicular required.

4. *To bisect a given straight line.*



Let AB be the given straight line; it is required to bisect it. Upon AB describe the equilateral triangles, ACB and ADB; join the vertices C and D by the line CD, intersecting AB at E. E shall be the point of bisection.

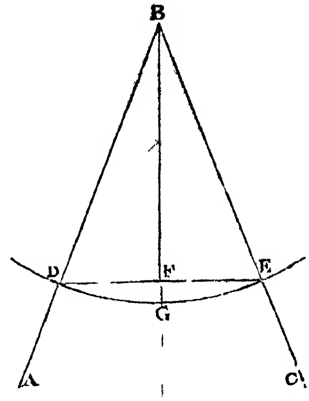
5. *To let fall a perpendicular upon a given straight line, from a given point above it.*



Let AB be the given straight line, and C the given point. From C take any distance CG, and describe the circle EGF, intersecting AB in E and F; bisect EF in H, and join CH.

CH is the perpendicular required.

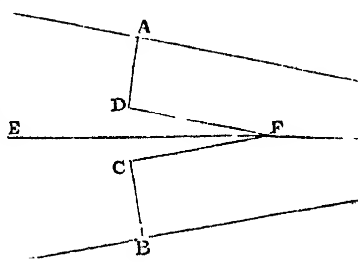
6. *To bisect a given angle.*



Let ABC be the given angle; take any point D in AB, and from B, as centre, at the distance BD, describe the circle DGE; join DE, and bisect it in F; join BF.

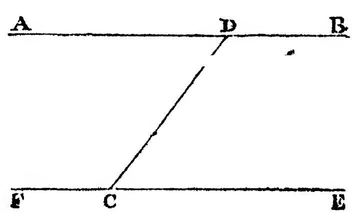
BF shall bisect the angle ABC.

7. *To bisect a given angle, when the inclination of the two sides can only be obtained, and not the vertex of the angle, included between them.*



Let A and B be the two sides, such, that they cannot be produced. Take any points, A and B, and draw the equal perpendiculars AD and BC; through D and C draw DF and CF parallel to A and B respectively; the angle DFC will be equal and similarly situated to the angle at the vertex, which will be bisected by the line EF *(produced)*, that bisects the angle DFC.

8. *To draw a line parallel to a given line.*

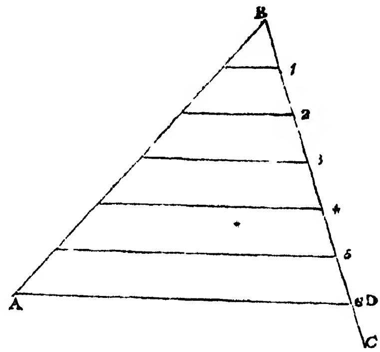


Through the point C to draw a line parallel to the given straight line AB.

Take any point D in AB, join CD, and at the point C make the angle DCE equal to angle ADC; produce EC to F.

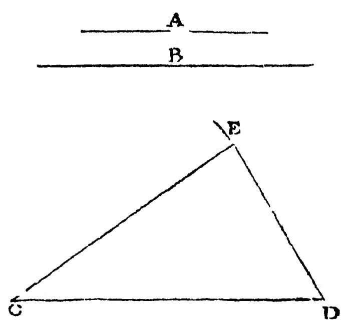
EF shall be parallel to AB.

9. *To divide a given line, AB, into any number of equal parts.*



At the point B, making any angle with AB, draw the unlimited straight line BC; take the required number of (six) equal measurements of any length from B towards C, ending at D; join DA, and through the several points on BD, draw lines, parallel to AD, to the line AB; these lines will intersect AB equally, and will be of the required number.

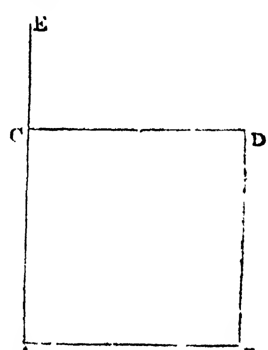
10. *Upon a given base, to describe a triangle, whose other two sides shall be respectively equal to two given lines, any two of which lines, however, must be greater than the third.*



Let A and B be the given lines, and CD be the given base; any two of them being greater than the third. It is required to describe upon CD a triangle, whose other two sides shall be equal to A and B.

At the centre C, with the distance CE, equal to B, describe a circle; and at the centre D, with the distance DE, equal to A, describe another circle, intersecting the first in E; join EC and ED. The triangle CED shall be the triangle required.

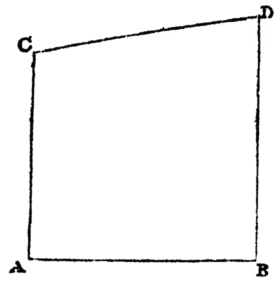
11. *To describe a Square on a given line AB.*



From the point A erect a perpendicular to AB; make AC equal to AB; and through the point C draw CD parallel to AB; make CD equal to AB, and join DB.

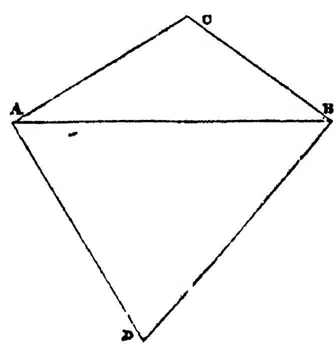
ACDB shall be the square required.

12. *To construct a trapezoid, having its two perpendiculars and its base given.*



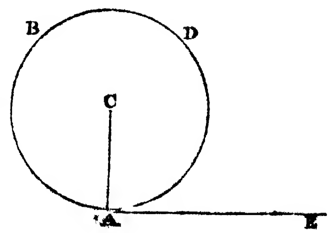
At the points A and B, of the base AB, erect two perpendiculars of unequal lengths of the length required, and join CD.
ACDB shall be the figure required.

13. *To construct a trapezium, ACBD, having the sides and the diagonal given.*



Conceive the trapezium divided into two triangles, ACB and ADB, having the common base AB, which is the given diagonal. Draw the base AB, and upon it, on their respective sides, construct the required triangles (prop. 10) ADB, ACB; having the sides AD, DB; AC, BC, of the required length.

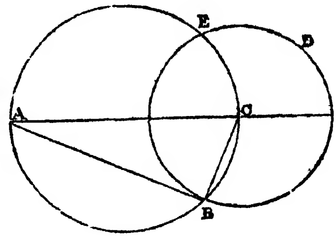
14. *From a given point A, in the circumference, to draw a tangent to the circle ABD, whose centre is C.*



Join CA, and make EA perpendicular to AC; EA shall be the tangent required.

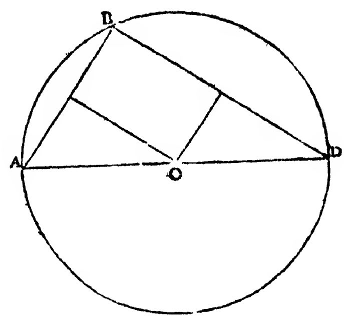
15. *From a given point A, without the circumference, to draw a tangent to the circle BED.*

Join AC, and upon AC describe the circle AECB. Join AB. AB shall be the tangent required; for ABC is a right angle, being in a semicircle, and therefore AB is at right angles to CB, which is the radius, and is therefore the tangent required.



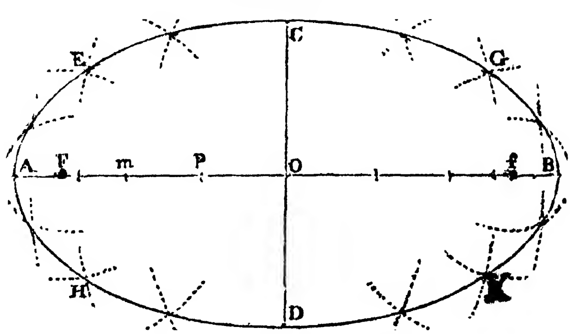
16. *Through the three given points A,B,D, not in a straight line, to describe a circle.*

Join AB and BD, and bisect them; erect perpendiculars till they meet in C; C will be the centre of the circle.



17. *To construct an ellipse, whose transverse and conjugate diameters are given.*

METHOD 1.



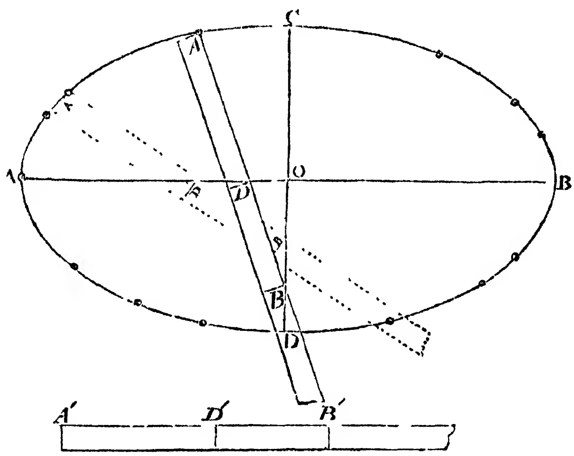
Let AB be the transverse diameter; bisect it in O; erect OC and OD perpendicular to AB, and equal each of them to the semi-conjugate; from C, as a centre, at the distance of the semi-transverse, describe the arcs intersecting AB in F and f. F, f are the foci. Then taking any point whatever m, and from F and f as centres, with distances Am, Bm, describe the arcs intersecting at E, G, H, K; thes

four points of intersection are points in the curve; again, by taking any other point p, and proceeding in the same way, four other points may be obtained.

By increasing the number of points taken upon the semi-transverse Ao, any number of points in the curve, that may be required, can be determined.

Through these points the circumference of the ellipse must be afterwards drawn.

METHOD 2.*

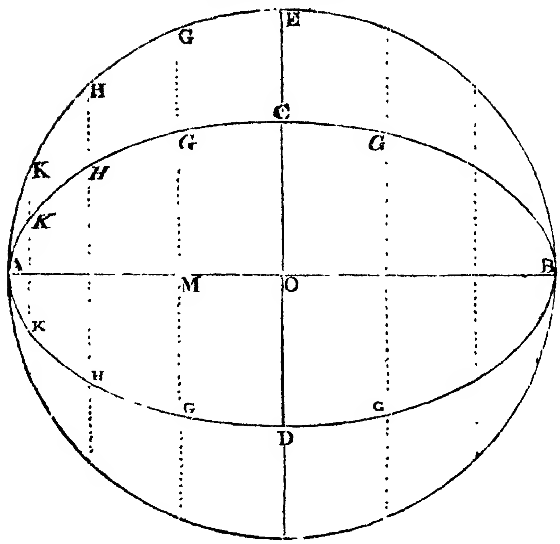


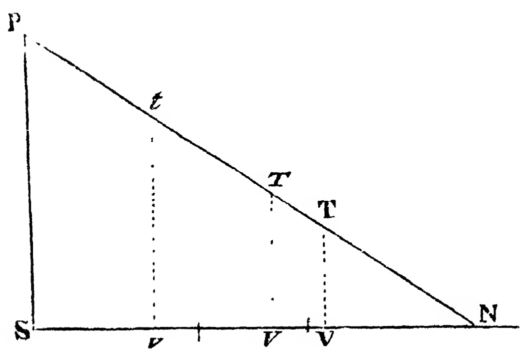
Having constructed the transverse and conjugate diameters, take a piece of card-board, with a fine clear edge, about the length of the transverse, and mark off upon it, from the same end, both the semi-transverse and semi-conjugate, so that AB=AO, and AD= CO; then, by keeping the semi-transverse mark B always in the semi-conjugate line CD, and the semi-conjugate mark D always, at the same time, in the semi-transverse line AB, and moving the point B upwards from D to C, and the point D sideways from O to A, the end of the card A will describe points of the curve between C and A.

By adopting the same method in the other quarters of the ellipse, the whole of the curve can be determined.

* This is a purely mechanical process, called striking the ellipse by the trammel. It is, however, a very correct and simple method, and one very much in use among Architects and Engineers.

METHOD 3.





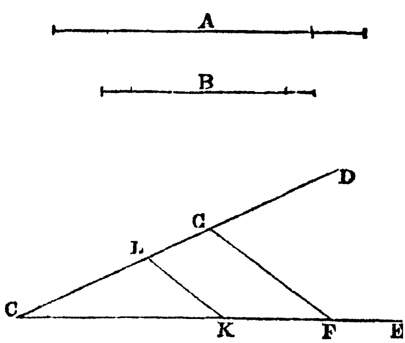
Construct the transverse and conjugate diameters as before, and upon the transverse, as diameter, describe a circle; then, because the ordinates of this circle are, to the corresponding ordinates in the ellipse, as the semi-transverse is to the semi-conjugate, we obtain a very simple method of determining the points in the curve.

Describe any right-angled triangle PNS, such that PN shall be to PS as the semi-transverse is to the semi-conjugate, then all lines TV, *TV*, *tv*, drawn parallel to PS, will have the same proportion to all lines TN, *T*N, as PS has to PN, or as the semi-conjugate has to the semi-transverse. Draw any ordinate to the circle, GM, and in the triangle PNS, cut off N*T* equal to GM; let fall the perpendicular *TV*, (practically, *place one leg of your compasses in T, and*

expand them till, describing an arc, the other leg touches the lines SN,) and measure off this distance (M*G*) on the line MG; M*G* is the corresponding ordinate of the ellipse, and *G* is a point in the curve. Produce GM to G; making MG equal to M*G*; this is also another point. By taking equal abscissa from O to B, two other points, *G* and G, on the other side of DC, may be obtained also.

The other points *H,K,L,* are determined in the same way, by measuring the ordinates of the circle on NP, and letting fall perpendiculars, for the corresponding ordinates of the ellipse.

18. *To find a third proportional to two given lines.*

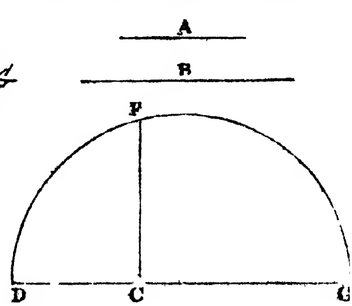


Let A,B be two given lines. Draw any two unlimited lines, CD and CE, making any angle between them. From CE cut off CF, equal to A; and from CD, CG, equal to B, join GF; again, take CK, equal to CG, and through K draw KL parallel to GF. CL is a third proportional to A and B, because, by similar triangles,
CF : CG : : CK or CG : CL.

19. *To find a fourth proportional to three lines.*

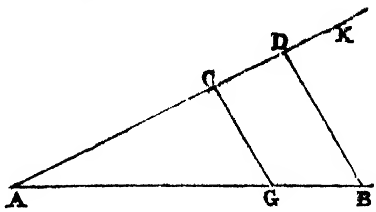
Proceed as in the above, but instead of taking CK, equal to CG or B, take CK equal to the third line; then CL in this case also becomes the fourth proportional.

20. *To find a mean proportional between two lines A and B.*



Draw any line DG; make DC equal A, and CG equal B; upon DG as diameter, describe a circle; erect the perpendicular CF, which is the mean proportional required. For DC. CG,=CF².
∴ DC : CF : : CF : CG.

21. *To divide a given line, AB, into proportional parts.*



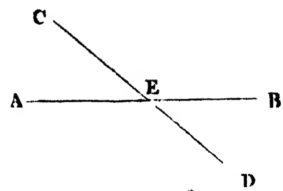
Through A draw any unlimited line AK, and take AC and CD of the required proportions; join DB, and through C draw CG parallel to DB. AG and GB, are the parts required.

For, by similar triangles, AC: AD :: AG: AB, *convertendo.* AC: (AD—AC) :: AG: (AB—AG);

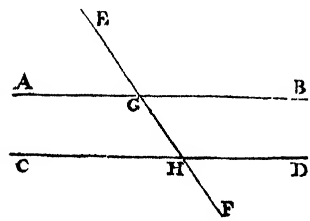
i.e. AC :: CD :: AG: GB.

USEFUL THEOREMS.

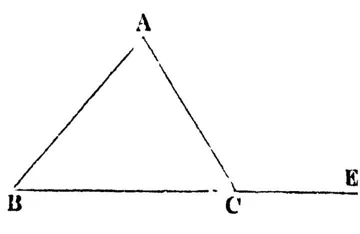
1. If two straight lines cut one another, the opposite angles are equal, and the four angles are together equal to four right angles; *i.e.*, the angle AEC is equal to the angle DEB, and the angle CEB to the angle AED. *(Euclid* I, 15.*)*



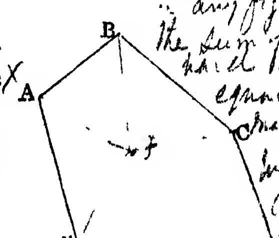
2. The alternate angles, made by a line cutting two parallel lines, are equal; and the exterior and interior angles are equal; and the two interior angles are, together, equal to two right angles; *i.e.*, the alternate angles AGH, GHD are equal; the exterior EGB equal to the interior GHD; and the two interior, BGH, GHD, equal to two right angles. *(Euclid* I, 29.*)*



3. If any side of a triangle be produced, the exterior angle is equal to the two interior opposite angles; and the three interior angles are equal to two right angles; *i.e.*, the angle ACE is equal to the two at A and B; and the angle ACB, together with the two at A and B, are equal to two right angles. *(Euclid* I, 32.*)*

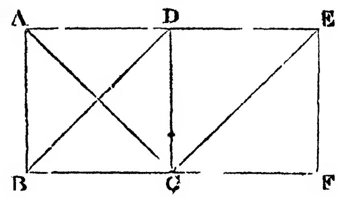


4. All the interior angles of any rectilineal figure are equal to four less than twice as many right angles as the figure has sides; *i.e.*, in the five-sided figure ABCDE, all the interior angles at A, B, C, D, and E, are equal to four less than twice five right angles; that is, are equal to six right angles or 540 degrees. *(Euclid* 1, 32, *Cor.)*



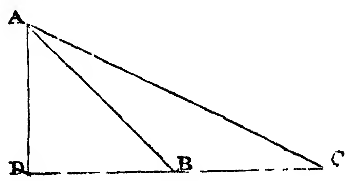
∴ any figure whole the sum of all the inside angles is equal to twice as many right L wanting (4 the ...

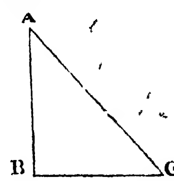
5. The greatest angle of every triangle is opposite the greatest side. (*Euclid* I, 18.)



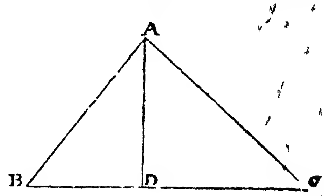
6. Parallelograms, upon equal bases and between the same parallels, that is, having the same perpendicular height, are equal to each other; *i.e.*, the parallelogram ABCD=the parallelogram DBCE.

Triangles, alike situated, are also equal; *i.e.*, the triangle ABC=the triangle CFE. (*Euclid* I, 35—38.)

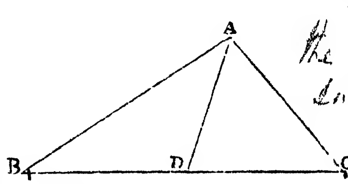




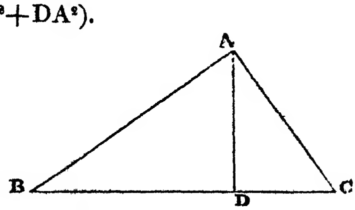
7. In every triangle, the square of the side, subtending any angle, is greater than, equal to, or less than, the square of the sides containing the angle, according as the angle is obtuse, right-angled, or acute; and the excess or deficiency is equal to twice the rectangle of one of the sides containing the angle, and that portion of it intercepted between the angle and a perpendicular, drawn to it from the opposite angle; *i.e.*,—



In the *obtuse*-angled triangle;
$AC^2 = AB^2 + BC^2 + 2CB \cdot BD$.
In the *right*-angled triangle;
$AC^2 = AB^2 + BC^2$.
In the *acute*-angled triangle;
$AC^2 = AB^2 + BC^2 - 2CB \cdot BD$.



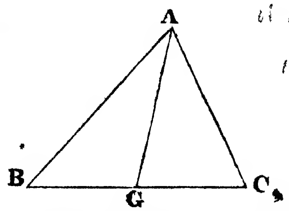
8. In any triangle, the sum of the squares of any two adjacent sides is equal to twice the square of half the other side, together with twice the square of the line drawn from the vertex to the point of bisection, *i.e.*, $AB^2 + AC^2 = 2\,(BD^2 + DA^2)$.



9. In any right-angled triangle, if a line be drawn at right angles to the hypothenuse, this line is a mean proportional between the segments of the hypothenuse; and the base and perpendicular are, respectively, mean

proportionals between the hypothenuse and the segment adjacent to them; *i.e.*, BD. DC=DA², CB.BD=BA², and BC. CD=CA². (*Euclid*—VI, 8).

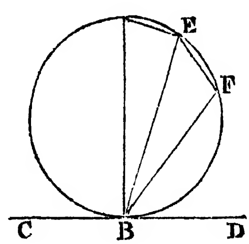
10. The rectangle of any two sides of a triangle is equal to the rectangle of the segments of the base, together with the square of the line, bisecting the included angle; *i.e.*, BA.AC=BG.GC+GA² (*Euclid* VI, *B*), and equal to the rectangle of a perpendicular drawn from this angle and the diameter of the circumscribing circle. (*Euclid* VI, C.)



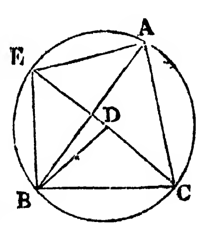
11. In every parallelogram, the squares of the two diagonals are together equal to the squares of all the sides.

12. Of any quadrilateral inscribed in a circle, the rectangle of the diagonals is equal to the two rectangles contained by its opposite sides. (*Euclid* VI, *D*.)

13. The angle, in a semicircle, is a right angle; in a segment, less than a semicircle, greater than a right angle; in a segment, greater, less; *i.e.*, AB being the diameter, the angle AEB is a right angle; the angle EFB, being in a segment less than a semicircle, is greater than a right angle; and the angle BAE is less. (*Euclid* III, 31.)



14. Angles, standing upon equal circumferences, are equal, whether they be in the centre or the circumference; *i.e.*, if the circumference BE be equal to the circumference BC, the angles EAB, BAC, in the circumference; and the angles EDB, BDC at the centre, are equal; and the angles EDB, BDC are respectively double the angles EAB, BAC. (*Euclid* III, 20 *and* 27.)



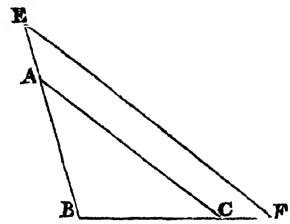
15. The two opposite angles of any quadrilateral figure, inscribed in a circle, are equal to two right angles; *i.e.*, the angles BEA and ACB, and the angles EBC, EAC, are equal to two right angles (*Vide* figure in Theorem 14). (*Euclid* III, 22.)

16. If a line touch a circle, any angle made between this line at the point of contact, and a line cutting the circle, is equal to the angle in the alternate segment; *i.e.* (See Fig 13), the angle CBE equals the

angle EFB, and the angle CBA equals the angle AEB. (*Euclid* III, 32.)

17. Parallelograms, and triangles of equal altitude, are as their bases; and of equal bases, are as their altitudes. (*Euclid* VI, 1.)

18. Equiangular triangles are similar, or have their homologous sides proportional. (*Euclid* VI, 4.) Let the triangles ABC, EBF, be equiangular; AB : BC :: EB : BF and BA : AC :: BE : EF and BC : CA :: BF : FE. (*Euclid* VI, 4)



19. Similar triangles have their areas, proportional to the square of their homologous sides. (*Euclid* VI, 19.)

20. Circumferences of circles are proportional to their radii. Areas of circles, to the squares of their radii. And, generally, all lineal measurements are proportional to their respective units of measurement.

21. All superficial areas are proportional to the compound ratio of the sides, that measure their areas; and, when these sides are similar, to the duplicate ratio of any one of them.

22. All solids also are proportional to the compound ratio of their measuring sides, and, when similar, to the triplicate ratio of any one of them.

but one integer, the index is 0: if they are all decimals, bring the first significant decimal figure into the fraction of an integer, and the power of 10, contained in the denominator, will in all cases be the index, with a negative sign—or, the index will be the number of figures, that this significant figure is removed from the unit place of the whole number, thus—

log.	5151	=	3·711892
log.	515·1	=	2·711892
log.	51·51	=	1·711892
log.	5·151	=	0·711892
log.	·5151	=	—1·711892
log.	·05151	=	—2·711892
log.	·005151	=	—3·711892

Examples.—What are the logarithms of the following numbers?

log.	3·1420	=	0·497206	+ then 0 in [illegible]
log.	41·36	=	1·616581	
log.	8·910	=	0·949878	
log.	647·1	=	2·810971	
log.	·000672	=	—4·827369	
log.	3·004	=	—0·477700	

If it should be found that the logarithms of the given number cannot be exactly found in the tables, the simple rule of proportion will obtain it.

Thus—As the difference between the next lower and the next higher number: the difference between the next lower and next higher logarithm :: so is the difference between the next lower and the given number : the difference between the next lower logarithm and the logarithm required.

For example—to find the logarithm of 235·756:—

The log. of the next lower number	2357	is	·372360
And of the next greater	2358	is	·372544
Their difference being			·000176

Then say, as 100 : : ·000176, : : 56 : ·000098 which, added to the log. of 235·7, which is 2·37236, is equal to 2·372458, the logarithm of the number required.

This method may be useful to a beginner, but to any one, who understands decimals, I should recommend the following in preference.

In taking the same example, observe that the logarithms of the next lower and next higher numbers have the common figures 372, and a difference of 176 only, between the next three figures.

Now, by the principles of decimals, each (decimal) figure is itself a decimal to the one immediately preceding; the figures ·56, therefore, in the given numbers, are decimals to the preceding figure 7; the measure, of the difference between 7 and 8, or of an unit of difference, being 176, the figures to be annexed to the next lower logarithm must be less than 176, and must depend upon the proportion, that the given decimal ·56 bears to an unit.

Multiply, therefore, this difference, or measure of the unit (176), by the given decimal ·56, and you obtain the proportion of the 176, to be added to the next lower logarithm.

Ex.	difference of logarithm	=	176	
	decimal of numbers	=	·56	
			1056	
			880	
The figures to be added to			98·56	
the next lower logarithm		=	2·372360	
The required logarithm		=	2·372458	= log. 235·756

What are the logarithms of the following numbers?

log.	3421·56·	=	3·534228
log.	2987·245	=	3·475271
log.	·0342172	=	2·534244

To find the number, answering to a given logarithm.

Seek in the tables for the given logarithm, opposite which, in the left hand column, is the number answering to the given logarithm.

What are the numbers corresponding to the following logarithms?

2·843855	=	log.	698·00
—3·530200	=	log.	·00339
1·301030	=	log.	20·000
4·600973	=	log.	39900
—2·714338	=	log.	·05180
5·804412	=	log.	637400
—1·937518	=	log.	0·86600

When the given logarithm cannot exactly be found in the tables, seek for the next lower logarithm; opposite which will be found the next lower number corresponding thereto; then say,—

As the difference between the next lower and the next higher logarithm, is to the difference between the next lower and the next higher number, taken as an unit, so is the difference between the next

lower logarithm and the given logarithm, to the difference between the next lower number and the number required, or to the decimal of that unit, to be annexed to the number of the next lower logarithm.

For example, to find the number answering to the logarithm 4·127860, the next lower logarithm is ·127753; the difference of this and the next greater is 323; and the number answering to the former is 1342.

Then, as ·323 : 1 : : ·107 : ·333, which, annexed to the first number, will be 1342333, and, pointing off according to the index, is 13423·33, the required number; or, briefly, the decimal to be annexed, is the decimal of a vulgar fraction, whose *denominator* is the difference between the next lower and next higher logarithms (323), or the logarithmic measure of an unit; and the *numerator* the decimal of that unit, or the difference between the given logarithm and the next lower (107); viz. $\frac{107}{323} = \cdot333$.

Examples.—What are the numbers corresponding to the following logarithms?

— 3·246418 = log. 0017636727
3·517694 = log. 3293·889
— 8·106457 = log. 0·00000001277783

Multiplication of numbers, by logarithms, is performed by finding the logarithms of the numbers; adding them, and finding the number answering to their sum. *Division*, by subtracting them.

Examples.—Divide 56·05 by ·075·

log. 56·05 = 1·748576
log. ·075 = —2 875061

log. 747·4 = 2·873515

Multiply 50 by 75 = 3750
Multiply 48 by 24 = 1152
Divide 4·167 by 38·56 = 0·108065
Divide 42·167 by ·061089 = 690·255217

Involution.

Find the logarithm of the number; multiply it by the index of power required, and find the number answering to the product.

Examples.—Required the cube of 5·68.

log. 5·68 = 0·754348
index = 3

log. 183·345 = 2·263044

What is the square of 4·678 = 21·883678
Required the cube of 321·5 = 33230963·375
Find the 4th power of 4·23 = 320·15587041

EVOLUTION.

Divide the log of the number by the index of the root, and find the number answering to the quotient.

EXAMPLES.—What is the cube root of 57·08 ?

log. 57·08 = 1·75684
div. by 3 = log. 385 = 0·58594

Find the square root of 5·008 = 2·237856
What is the 5th root of 67·40 = 2·3213
Required the cube root of 6 = 1·81712092

Note.—That, in Involution, if the index of the logarithm of the number be negative (the logarithms of decimals being always affirmative), it will be necessary to multiply them separately.

EXAMPLE.—To find the cube of ·05

$$
\begin{array}{rr}
\text{log. } \cdot05 = & -\ 2\cdot698970 \\
 & 3 \\
 & 2\cdot096910 \\
-2 \times 3 = & -\ 6 \\
\hline
\text{log. } \cdot000125 = & 4\cdot096910
\end{array}
$$

Also, in Evolution, where the index is negative, and is not some multiple of the index of the root, add to each, viz. (the index and the log.), some number that will make the index a multiple required, and divide the two separately, as in Involution. make it the divisor [illegible] any no. [illegible]

EXAMPLE.—Find the 6th root of ·05.

$$
\begin{array}{rr}
\text{log. } \cdot05 = & -\ 2\cdot698970 \\
= & -\ 2\cdot + 698970 \\
\text{Add} & -\ 4\cdot + 4 \\
\hline
 & -\ 6\cdot + 1098970
\end{array}
$$

and dividing by 6 = — 1·183161, the number answering to which is ·01525.

EXAMPLES TO THE TWO LAST CASES.

Required the cube of ·105 = 0·0011576245
What is the square of ·00534 = ·0000285156
Find the 4th power of ·00062 = ·000014776336
What is the 4th root of ·365 = 777272325
Required the cube root of ·0003214 = ·068498635
What is the 10th root of ·0016 = 0·525305609
limit

If the given number be a fraction, to find the logarithm.

Rule.—Subtract the logarithm of the denominator from that of the numerator, and the remainder is the logarithm of the fraction required.

EXAMPLES.—Required the logarithm of $\frac{7}{8}$.

$$\log.\ 7 = 0{\cdot}84510$$
$$\log.\ 8 = 0{\cdot}90309$$

$$\log.\ \tfrac{7}{8} = -1{\cdot}94201$$

Required the logarithms of the following fractions, $\frac{1}{3}$, $\frac{4}{7}$, $\frac{5}{16}$, $\frac{18}{42}$,

$$\log.\ \tfrac{1}{3} = -1{\cdot}5228787$$
$$\log.\ \tfrac{4}{7} = -1{\cdot}7569620$$
$$\log.\ \tfrac{5}{16} = -1{\cdot}4748500$$
$$\log.\ \tfrac{18}{42} = -1{\cdot}6320232$$
$$\log.\ \tfrac{1}{324} = -3{\cdot}4894550$$
$$\log.\ \tfrac{5}{7842} = -4{\cdot}8045432$$

MENSURATION OF PLANES.

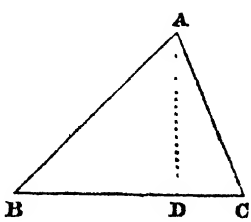
PROBLEM I.

To find the area of a triangle, when the base and perpendicular height are given.

Rule.—Multiply the base by $\frac{1}{2}$ the height.

EXAMPLE 1.—What is the area of a triangular field, whose base is 5 chains 50 links, and height 3 chains 20 links?

base = 5·50 chains
$\frac{1}{2}$ height = 1·60 chains
33000
550
10 | 8.8000 sq. chains
·88000 acres
4
roods 3·52
40
perches ~~14~~·080



A. R. P.
Answer 0. 3. ~~14~~

EXAMPLE 2.—Required the area of a triangle, whose base is 7 chains 25 links, and perpendicular height 90 links.

A. R. P.
Answer 0. 1. 12

EXAMPLE 3.—How many square yards are contained in a triangular plot of ground, whose base and height measure respectively, 8 chains 50 links, and 5 chains 50 links?

base = 8·5 chains
height = 5·5 chains
425
425
2 | 46·75
10 | 23·375 sq. chains
2·3375 acres
multiply by the sq. yards in 1 acre = 4840
area = 11313·5 sq. yards.

Note.—[The statute English measure of an acre is ten square chains, (each chain being 66 feet or twenty-two yards,) or 4840 square yards, or 43560 square feet. Areas are generally reckoned in acres, roods, and perches, square measures; 4 square roods being equal to an acre, and 40 square rods to a rood.]

PROBLEM II.

To find the area of a triangle, when the three sides are given.

Rule.—Take half the sum of the three sides, subtract each side severally from this quantity; then multiply this and the three remainders together, and take the square root for the area.

Example 1.—What is the area of a triangle, whose three sides are 30, 40, and 50 chains?

$$\frac{30 \times 40 \times 50}{2} = 60 = \text{the half sum.}$$

$$\begin{array}{ccc} 60 & 60 & 60 \\ 30 & 40 & 50 \end{array}$$

$$30 \times \underline{20 \times 10} \times 60 = 360000 \text{ sq. chains.}$$

$$\textit{Answer} = \sqrt{360000} = 600 \text{ sq. chains} = 60 \text{ acres.}$$

Example 2.—Required the area of a triangular field, whose three sides measure respectively 25 chains, 42 chains, and 56 chains?

A. R. P.
Answer ~~37 2 28·84~~ 49

Note.—Where the lengths of the sides are not full chains, as in the following example, the solution must be obtained by the general logarithmic expression, viz.:—

$$\log.\ area = \tfrac{1}{2}\cdot \log.\ s \times \tfrac{1}{2}\cdot \log.\ \overline{s-a} \times \tfrac{1}{2}\cdot \log.\ \overline{s-b} \times \tfrac{1}{2}\cdot \log.\ \overline{s-c}$$

where s = the semi-perimeter, and a, b, c = the sides.

Example 3.—The three sides of a triangular plot of ground are respectively 20 chains 40 links, 25 chains 20 links, and 30 chains 50 links. What is the area?

$$\begin{array}{r} 20{\cdot}40 \text{ chains} \\ 25{\cdot}20 \\ \underline{30{\cdot}50} \\ 2 \,|\, \underline{76\ 10} \\ 38{\cdot}05 = s. \end{array}$$

$$38{\cdot}05 - 20{\cdot}40 = 17{\cdot}65 = \overline{s-a}$$

$$38{\cdot}05 - 25{\cdot}20 = 12{\cdot}85 = \overline{s-b}$$

$$38{\cdot}05 - 30{\cdot}50 = 7{\cdot}55 = \overline{s-c}$$

log. s = 38·05 = 1·5803547
log. $\overline{s-a}$ = 17·65 = 1·2467447
log. $\overline{s-b}$ = 12·85 = 1·1089031
log. $\overline{s-c}$ = 7·55 = 8·8779470

2 | 4·8139495

2·4069748 = log. 255·255 sq. chains = 25·5255 acres

			25·5255
			4
			2·1020
			40
	A. R. P.		
Area =	25. 2. 4	=	4·0800

Example 4.—Given the sides of a triangle, 24 chains 72 links; 38 chains 75 links; and 44 chains 68 links; to ascertain the area in square yards.

Answer ~~1~~231700 sq. yards.

PROBLEM III.

To find the area of a triangle, having two of its sides, and the included angle given.

Rule.—Multiply half the product of the two given sides by the natural sine of the given angle; or, where it is desirable to use logarithms, apply the following expression, viz.:—

$$\text{Log. area} = \log.\tfrac{b}{2} + \log. c + \log. \text{sine } A - 10.$$

Example 1.—What is the area of a triangle, whose two sides are 40 and 50 chains, and their contained angle 37 degrees 24 minutes?

Natural sine A, 37° 24′ = ·60738

$$\frac{40 \times 50}{2} = 1000$$

10 | 607·38 sq. chains

60·738 acres
4
2·952
40
38·080

A. R. P.
Answer 60. 2. 38

Example 2.—What is the area of a triangle, whose sides are 42 chains, 56 links, and 11 chains 28 links, and the included angle 46 degrees 15 minutes?

$$\text{log. } \tfrac{AB}{2},\ 21{\cdot}28 \text{ chains} = 1.3279716$$
$$\text{log. AC, } 11{\cdot}28 \text{ chains} = 1{\cdot}0523091$$
$$\text{log. sin. A, } 46^\circ\ 15' = 9{\cdot}8587561$$
$$12.2390368$$
$$-\text{radius} = 10$$

acres

$$\text{log. } 17{\cdot}339 = 173{\cdot}39 \text{ chains} = 2{\cdot}2390368$$

4
1·356
40
14·240

	A.	R.	P.
Area =	17.	1.	14

Example 3.—Required the area of a triangular field, two of whose sides are 456 feet, and 327 feet, and the included angle 17 degrees 15 minutes?

A.	R.	P.
0.	2.	$1\frac{8}{9}$

What is the area of a triangle, whose sides are 11 yards and 24 yards, and the included angle 165 degrees 46 minutes?

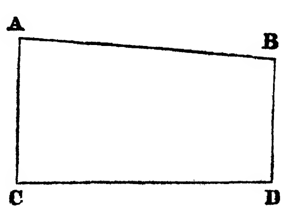
	A.	R.	P.
Answer	1.	0.	1

PROBLEM IV.

To find the area of a trapezoid.

Rule.—Multiply half the sum of the two parallel sides by the distance between them.

Example 1.—In a trapezoid, whose parallel sides are 7 chains 25 links and 8 chains 63 links, the distance being 11 chains 65 links, how many acres are there?



7·25
8·63
2 | 15·88

chains acres

$$7{\cdot}94 \times 11{\cdot}65 = 92{\cdot}50 = 9{\cdot}250$$

4
1·000

	A.	R.	P.
Area =	9.	1.	0

EXAMPLE 2.—What is the area of a trapezoid, the parallel sides of which are 14 chains 20 links, and 12 chains 35 links, and the distance between them 27 chains 25 links?

A. R. P.
Answer 36. 0. 28

EXAMPLE 3.—Required the area of a trapezoidal field, whose sides are 40 chains and 27 chains, and distance 15 chains and a half.

A. R. P.
Answer 51. 3. 28

PROBLEM V.

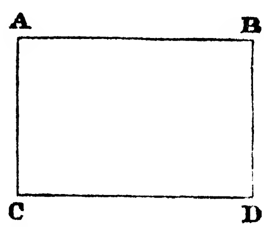
To find the area of a quadrilateral right-angled figure.

Rule. — Multiply the length by the breadth, the product will be the area.

EXAMPLE 1.—What is the area of a rectangular field, whose length is 35 chains 40 links, and breadth 24 chains 36 links?

chs.		chs.		sq. chs.		acres.
35·40	×	24·36	=	862·34	=	86·234
						4
						·936
						40
						37·440

A. R. P.
Answer 86. 0. 37



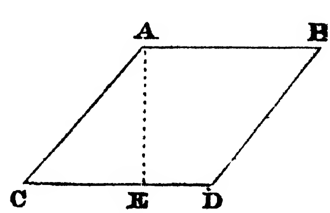
EXAMPLE 2.—Required the area of a right-angled parallelogram, whose length is 56 chains 24 links, and breadth 35 chains 42 links.

A. R. P.
Answer 99. 0. 32

PROBLEM VI.

To find the area of a parallelogram, whose angles are not right angles.

Let ABCD be the parallelogram, whose area is required; let fall the perpendicular AE. The measurement of the parallelogram is AE. CD, and AE is the sine of the angle at C, to radius CA; therefore the area = AC. CD. sine ∠ C.



Rule.—Multiply the product of the two adjacent sides by the sine of the angle between them.

Example 1.—Required the area of a four-sided regular field, whose sides are 20 chains 15 links, and 16 chains 89 links, and the included angle 30 degrees.

$$\sin \angle 30 = \frac{\text{rad}}{2} = \frac{1}{2} = \cdot 5$$

$$AC.\ CD = 20\cdot 15 \times 16\cdot 89 = \overset{\text{chains}}{340\cdot 3335} = \overset{\text{acres}}{34\cdot 03335}$$

Multiplying by sine $\angle 30 = \cdot 5$

	17·016675
	4
	·066700
	40
Answer 17. 0. 2 (A. R. P.)	2·668000

Example 2.—When the sides are 15 chains 25 links, and 21 chains 18 links, and the included angle 45 degrees, what is the area of the field?

A. R. P.
Answer 22. 3. 14

Example 3.—With the same sides, but with an angle of 105 degrees, what is the area?

A. R. P.
Answer 31. 0. 31

PROBLEM VII.

To find the area of a trapezium.

Rule.—Divide the trapezium into two triangles by the longest diagonal; take this as the common base of the two triangles, and multiply it by half the sum of the two perpendiculars, let fall upon it, from the opposite angles.

Example 1.—To find the area of a trapezium, whose diagonal is 20 chains, and the two perpendiculars 2 chains 50 links, and 3 chains 40 links?

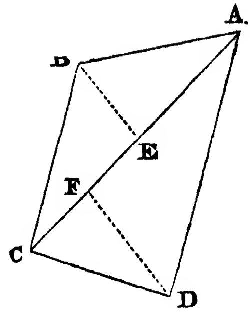
2·50
3·40
2 | 5·90
2·95 × 20 = 59 sq. chains = 5·9 acres.
4
3·6

A. R. P.
Answer 5. 3. 2

Example 2.—How many square feet of paving are there in a court yard, whose diagonal is 2 chains 64 links, and perpendiculars 95 links and 84 links? *Answer* 10292 square feet.

EXAMPLE 3.—In the field ABCD on account of obstructions, the following distances only could be taken, viz., A[illegible] 12 chains; BC, 8 chains; and the distances [illegible], 10 chains; and [illegible]C, 6 chains 50 links, (on the diagonal which was 15 chains,) required the area?

A. R. P.
Answer 8. 1. 33



EXAMPLE 4.—Given the distances AB, 6 chains; and AD, 8 chains. In consequence of obstructions, DB could not be measured; though the perpendicular points E, and F, upon it, were ascertained, and the lengths of the perpendiculars were found to be 3 chains and 4 chains. What is the area?

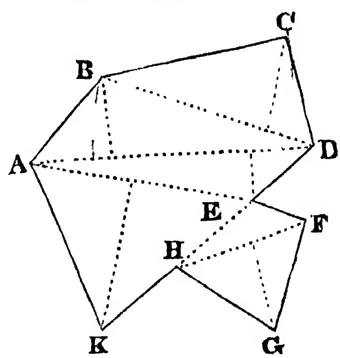
A. R. P.
Answer 3. 3. 38

PROBLEM VIII.

To find the area of an irregular polygon.

Rule 1.—Divide the polygon into trapeziums and triangles. Then find the sum of the areas of each, as in the following example:—

Let ACFK be an irregular polygon, whose diagonals BD, AD, AE, HF, and sides AB, BC, CD, &c., are given, viz.:—



BD = 7 chains

the diagonals { AD = 10·00, AE = 8·00, HF = 6·00

sides:

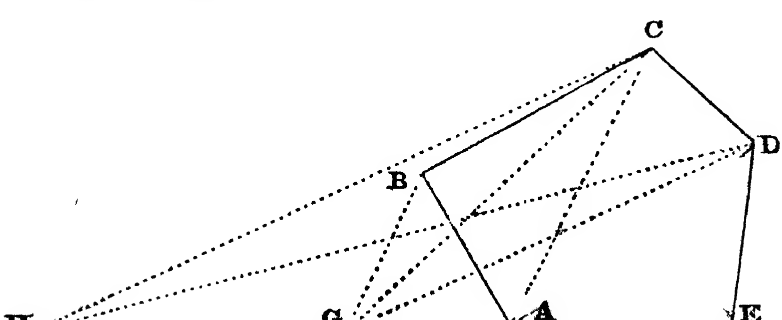
AB = 4·00
BC = 6·35
CD = 3·00
DE = 2·50
EF = 3·50
FG = 4·00
HG = 4·00
HK = 3·00
HE = 3·50
AK = 6·00

	chains.
Area of $\triangle$ ABD =	10·928
$\triangle$ BCD =	9·140
$\triangle$ ADE =	6·684
$\triangle$ EFH =	5·408
$\triangle$ FGH =	7·936
$\triangle$ AEK =	19·171
Total area of polygon =	59·267 square chains.
	5·267
	4
	·7068
	40
	28·2720

NOTE.—[The lengths of the sides of the figures have been preferred to those of the perpendiculars, because the lengths of the sides would have to be taken, in practice, for the sake of the offsets, and the sides, together with the diagonals, furnish sufficient data for the polygon; whereas the perpendiculars, though more easily measured, and more readily effective, in determining the areas, would be useless for ascertaining the irregularities of the hedges.]

Rule 2.—Construct carefully the given polygon, and reduce it geometrically to an equivalent triangle, as in the following example. Then measure the sides of this triangle, off the scale, by which the polygon was constructed, and find the area. This will be the area of the polygon.

Let ABCDE be the given polygon; it is required to reduce it to an equivalent triangle.



Join AC, through B; draw BG, parallel to AC; join GC; then the triangle GAC is equal to the triangle ABC, being on the same base

AC, and between the same parallels. And, therefore, GCDE, the four-sided figure, is equal to the original polygon. Again join GD; and through C, draw CH parallel to DG; and join HD. HDE is the triangle required.

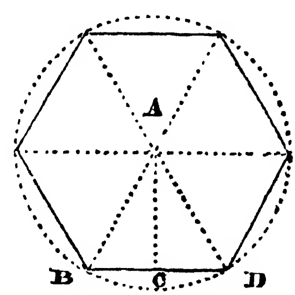
PROBLEM IX.

To find the area of any regular polygon.

Let the given polygon be divided into as many triangles as it has sides, by lines drawn from the centre of the circumscribing circle; then the *area of one of these triangles*, multiplied by the number of sides of the polygon, will give the area required.

To find this unit of area.

Let ABD be one of the triangles of a regular polygon, whose side BD = unity, then as BC × AC = the area of the triangle BAD, and BC $= \frac{BD}{2} = \frac{1}{2}$; and AC = cot· ∠ BAC to radius BC, which is equal to $\frac{1}{2}$ cot. $\frac{180^\circ}{n}$, (because BC $= \frac{1}{2}$ BD $= \frac{1}{2}$.) where n = number of sides of the polygon; therefore $\frac{1}{2} \times \frac{1}{2}$ cot $\frac{180^\circ}{n} = \frac{1}{4}$ cot $\frac{180^\circ}{n}$ = the required area of the triangle ABD.



Multiply this by n, the number of sides, and you obtain $\frac{n}{4}$ cot $\frac{180^\circ}{n}$ = the area of any sided polygon, whose side BD = unity. But areas of similar figures are to each other as the squares of their homologous sides. Therefore $L^2 \cdot \frac{n}{4}$ cot· $\frac{180^\circ}{n}$ = the area of any regular polygon, when L = the length of its side; hence is deduced the

1*st Rule.*—Square the side of the polygon, and multiply it by one fourth the number of sides, multiplied into the cot. of (180 degrees; divided by the number of sides).

EXAMPLE 1.—Let the side BD of the regular hexagon be equal to 20 chains; it is required to find the area.

$$L^2 . \frac{n}{4} \cot \frac{180^\circ}{n} = \text{area.}$$

$$\begin{array}{lrl}
\log. & 400 = & 2\cdot602060 \\
+ \log. & 6 = & 0\cdot778151 \\
+ \log. \cot. & 30^\circ = & \underline{10\cdot238561} \\
 & & 3\cdot618772 \\
- \log. & 4 = & \underline{0\cdot602060} \\
\log. 1039\cdot23 \text{ chs.} & = & 3\cdot016712 =
\end{array}$$

1039·23 chs. = 103·923 acres.

$$\begin{array}{r}
4 \\ \hline
3\cdot692 \\
40 \\ \hline
27\cdot680
\end{array}$$

A. R. P.
Answer 103. 3. 27

Rule 2.—Square the side of the polygon, as in the first rule, and multiply it by the tabular area, or multiplier placed, collaterally with it, in the following table.

EXAMPLE 1.—Taking the same data as in the preceding, making the side of the hexagon 20 chains, we have

No. of sides.	Names.	Areas, side = 1
3	triangle	0·4330127
4	square	1·0000000
5	pentagon	1·7264774
6	hexagon	2·5980762
7	heptagon	3·6339124
8	octagon	4·8284271
9	nonagon	6·1818242
10	decagon	7·6942088
11	undecagon	9·3656399
12	dodecagon	11·1961524

$20^2 = 400$

tabular area of the hexagon = 2·5980762

multiply by 400

sq. chs. = 1039·2304800

acres = 103·923

$$\begin{array}{r}
4 \\ \hline
3\cdot692 \\
40 \\ \hline
27\cdot680
\end{array}$$

A. R. P.
Ans. = 103. 3. 27 as before.

EXAMPLE 2.—Required by both rules, the area of the regular pentagon, whose side BD is 27 chains.

A. R. P.
Answer 125. 1. 27.

EXAMPLE 3.—What is the area of a regular dodecagon, whose side is 3 chains 49 links.

A. R. P.
Answer 136. 1. 19

PROBLEM X.

To find the circumference of a circle.

Rule.—Multiply the diameter by 3·1416.

Circumferences are to each other as their diameters, for $\pi = 3·1416 =$ *the semi-circumference, when radius is unity;* $=$ *circumference of the circle, when diameter is unity.*

Example 1.—What is the circumference of a circle, whose diameter is 30 chains?

Circumference $= 30\,\pi = 3·2426 \times 30 = 94·2480$ chains.

Example 2.—Required the circumference of a circle, whose diameter is 17 chains 40 links. *Answer* 54·66 chains.

Example 3.—What is the diameter of a circle, whose circumference measures 1,000 chains? *Answer* 318 chains 30 links.

PROBLEM XI.

To find the length of any arc.

As 360° : to the given degrees of the arc : : so is the whole circumference : to the length of the arc required.

Example 1.—What is the length of an arc of 20°, of a circle, whose circumference measures 850 chains?

As 360° : 20° : : 850 chains : x

$$x = \frac{850 \times 20}{360} = 47·22$$ *Ans.* 47·22 chs.

Example 2.—Required the length of the quadrant, the circumference measuring 300 chains. *Answer* 75 chains.

Example 3.—What is the circumference of a circle, when the arc of 30° measures 17 chains 20 links? *Answer* 206·40 chains.

PROBLEM XII.

To find the area of a circle.

Rule.— Multiply the square of the diameter by ·7854.

Areas of circles are as the squares of the diameters—

$$\therefore \text{ area of circle.} = \text{rad.}^2 \times \pi$$

$$= (2\text{ rad.})^2\frac{\pi}{4} = \text{diam.}^2\frac{\pi}{4}.$$

EXAMPLE 1.—What is the area of a circular field, whose diameter is 18 chains?

$$
\begin{array}{rr}
 & \cdot 7854 \\
18^2 = & 324 \\
\hline
 & 31416 \\
 & 15708 \\
 & 23562 \\
\hline
10 \,| & 254\cdot 4696 \\
\hline
 & 25\cdot 4469 \\
 & 4 \\
\hline
 & 1\cdot 7876 \\
 & 40 \\
\hline
 & 31\cdot 5040
\end{array}
$$

A. R. P.
Answer 25. 1. 31.

EXAMPLE 2.—The area of a circular plot of ground is required, whose diameter is 27 chains.

A. R. P.
Answer 2. 0. 19.

EXAMPLE 3.—What is the area of the circle, that can be described by a rope, measuring 5¼ chains, having one end fixed, as a centre?

A. R. P.
Answer 1. 2. 37.

PROBLEM XIII.

To find the area of an ellipse.

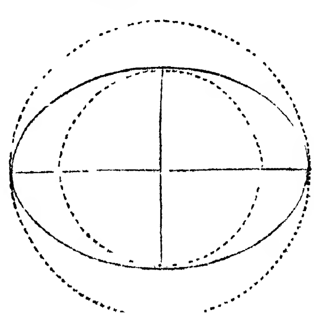
Rule.—Multiply the product of the major and minor axes by ·7854. An ellipse being a mean proportional between a circle described on its minor, and one on its major axis.

Let $b =$ *the diameter of the smaller circle ; and* a, *that of the greater ;*

$$\text{then} \begin{cases} \cdot 7854 \times a^2 = \text{area of greater circle,} \\ \cdot 7854 \times b^2 = \text{area of less circle,} \end{cases}$$

therefore $\sqrt{\cdot 7854^2 (a^2 b^2)} = \cdot 7854\, ab =$ *ellipse.*

EXAMPLE 1.—Required the area of an ellipse, whose transverse diameter is 12 chains, and conjugate, 8 chains 50 links.



sq. chains.
$\cdot 7854 \times 12 \times 8\cdot 50 = 80.\ 11.$

A. R. P.
Answer 8. 0. 1.

Example 2.—What is the area of the ellipse, whose major and minor axes are, respectively, 22 chains 40 links, and 15 chains 50 links?

A. R. P.
Answer 27. 1. 3.

Example 3.—Required the area of an ellipse, whose transverse diameter is 36 chains 20 links, and conjugate, 24 chains.

A. R. P.
Answer 68. 0. 37.

PROBLEM XIV.

To find the area of a parabola.

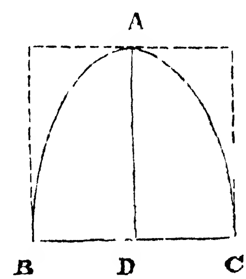
Rule.—Multiply the base by $\frac{2}{3}$ of the perpendicular height. The area of a parabola being equal to $\frac{2}{3}$ of its circumscribing rectangle.

Example 1.—Required the area of a parabola, whose base is 22 chains 50 links, and perpendicular height, 22 chains 50 links also.

$$22{\cdot}50 \times \tfrac{2}{3} = \tfrac{45}{3} = 15$$ sq. chains.

$$15 \times 22{\cdot}50 = 337{\cdot}50 = \text{area.}$$

A. R. P.
Answer 33. 3. 0.



Example 2.—What is the area of a parabola, whose base is 15 chains 25 links, and perpendicular height, 11 chains 30 links.

A. R. P.
Answer 11. 1. 38.

Example 3.—The base of a parabola is 12·50 chains, and its perpendicular height, is equal to its base. Required the area.

A. R. P.
Answer 13. 1. 26.

PLANE TRIGONOMETRY.

DEFINITIONS.

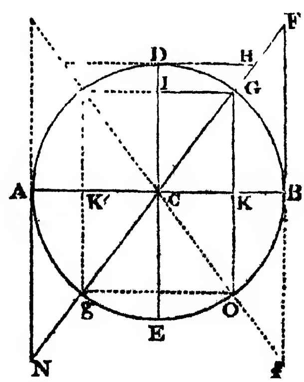
1. Angles are measured by the arcs of circles, cut off by the two sides, that compose the angles; *i.e.*, the angle GCB is measured by the arc GB.

2. The chord of an arc, is a straight line, joining the extremities of an arc, GO being the chord of the arc GBO.

3. The circumference of a circle, is divided by English Mathematicians into 360 equal parts or degrees; each degree, into 60 minutes; and each minute, into 60 seconds, which are thus marked: $4°, 3', 15''$; 4 *degrees*, 3 *minutes, and* 15 *seconds.* A quarter of a circle, or a quadrant, contains therefore 90 degrees.

4. The complement of an angle, is the difference between that angle and 90 degrees; therefore, *each of the acute angles of a right-angled triangle is the complement of the other.*

5. The supplement of an angle, is the difference between that angle and 180 degrees; or, measured on the circumference, the difference between the arc and the semi-circumference. And, as the sum of the three angles of any triangle is equal to two right angles, or 180 degrees, *each angle of any triangle has for its supplement, the sum of the other two.*



6. *The sine of an arc*, is a perpendicular, drawn *from* one extremity of the arc, upon the diameter of the circle, which passes through the other extremity, *therefore the sine of any two angles is equal to the sine of the third;* thus, in the annexed figure, GK, is the sine of the arc BG. Produce GK to O; GO is bisected; and the arc GB = the arc BO; therefore *every sine is equal to half the chord of twice the arc.*

7. The *Cosine* of an arc, is the sine of its complement, or, is that portion of the diameter, subtended between the sine and the centre, and measured *from* the centre *to* the sine. Thus, CK is the cosine of the angle GCB.

8. The *versed sine* of an arc; is that portion of the diameter, subtended between the sine, drawn from one extremity of the arc, and the other extremity, and is equal to (radius—cosine of arc,) being either BK or BK′.

9. The *tangent* of a circle, is a line drawn from any point of the circumference, at right angles to the diameter; the tangent of an arc, therefore, is a line drawn at right angles to the diameter *from one extremity*, and intersected by

10. The *secant*, which is a line drawn *from* the centre *through* the other extremity, or, in a *contrary* direction; thus BF is the tangent to the arc BG, and CF is the secant of the same arc.

11. The cotangents, cosecants, &c., are but the tangents and secants of the complements of the arc;

Thus DH is the cotangent, and CH the cosecant of the arc BG.

And in the same way GK,BF, and CF are respectively the cosine, cotangent, and cosecant of the *co*-arc DG; and BK, the co-versed sine of DG.

12. The supplemental arc ADG has the same sine GK, and equal tangents and secants; but the tangents and secants are of the opposite kind, being measured the other way; if the supplemental arc is greater than 90 degrees, they are negative, and those of the arc positive; if less than 90 degrees, the reverse.

CHAPTER I.

I. EQUIVALENTS.

The several triangles CKG, CBF, and CDH, are equilateral, and therefore similar; hence result, by proportion, the several values of the sines, cosines &c.

1. $GK : CG :: CD : CH$; or $GK = \frac{CG.CD}{CH}$

$$i.e.\ sine = \frac{\text{rad.}^2}{\text{cosec.}};\quad \text{or } cosec. = \frac{\text{radius}^2}{\text{sine}} = \frac{1}{\text{sine}},\ \text{where } R = 1.$$

2. $CK : CG :: CB : CF$; or $CK = \frac{CG.CB}{CF}$.

$$i.e.\ cosine = \frac{\text{rad.}^2}{\text{sec.}} = \frac{1}{\text{sec.}},\ \text{where } R = I:$$

$$\text{and } sec. = \frac{\text{rad.}^2}{\text{cosine}} = \frac{1}{\text{cosine}}.$$

3. CK : KG :: CB : BF ; or $CK = \frac{KG.CB}{BF}$;

i. e. $cosine = \frac{\text{sine rad.}}{\text{tan.}} = \frac{\text{sine}}{\text{tan.}}$. or $tan. = \frac{\text{sine}}{\text{cosine}}$.

4. KG : GC :: BF : FC : or $KG = \frac{GC.BF}{FC}$;

i. e. $sine = \frac{\text{rad. tan.}}{\text{sec.}} = \frac{\text{tan.}}{\text{sec.}}$.

5. GK : KC :: DC : DH ; or $GK = \frac{HC.DC}{DH}$

i. e. $sine = \frac{\text{cosine rad.}}{\text{cotan.}} = \frac{\text{cos.}}{\text{cotan.}}$; or $cot. = \frac{\text{cos.}}{\text{sine}}$.

6. BF : CB :: CD : DH ; or $BF = \frac{CB.CD}{DH}$

$tan. = \frac{\text{rad}^2}{\text{cot.}}$; $= \frac{1}{\text{cot.}}$; where $R = 1$:

Hence we have these general values.

1. $\text{sine} = \frac{\text{rad}^2.}{\text{cosec.}} = \frac{1}{\text{cosec.}} = \frac{\text{tan.}}{\text{sec.}} = \sqrt{\text{rad}^2.-\text{cosine}^2} = \sqrt{1-\text{cosine}^2}$.

2. $\text{cosine} = \frac{\text{rad.}^2}{\text{sec.}} = \frac{1}{\text{sec.}} = \frac{\text{cot.}}{\text{cosec.}} = \sqrt{\text{rad.}^2-\text{sine}^2} = \sqrt{1-\text{sine}^2}$.

3. $\text{tan.} = \frac{\text{rad.}^2}{\text{cot.}} = \frac{1}{\text{cot.}} = \frac{\text{sine}}{\text{cos.}} \times R = \frac{\text{sine}}{\text{cos.}}$.

4. $\text{cotan.} = \frac{\text{rad.}^2}{\text{tan.}} = \frac{1}{\text{tan.}} = \frac{\text{cosine}}{\text{sine}} \times R = \frac{\text{cosine}}{\text{sine}}$.

5. $\text{sec.} = \frac{\text{rad.}^2}{\text{cosine}} = \frac{1}{\text{cos.}} = \sqrt{\text{rad.}^2+\text{tan.}^2} = \sqrt{1+\text{tan.}^2}$

6. $\text{cosec.} = \frac{\text{rad.}^2}{\text{sine}} = \frac{1}{\text{sine}} = \sqrt{\text{rad.}^2+\text{cotan.}^2} = \sqrt{1+\text{cotan.}^2}$

7. and versed sine $= 1-\text{cos.}$:

8. The tan. of 45° = chord of 60° = sine 90° = radius :

And *generally*, if, in a right-angled triangle, the *base* be taken as the actual measurement of the radius, the hypothenuse and perpendicular become the actual lengths of the secant and tangent (to the same radius); and, if the *hypothenuse* be taken as radius, the perpendicular and base become the actual lengths of the sine and cosine, to the angle, at the point, considered as the centre of the circle.

II. VALUES OF THE SINES, COSINES, &c.

In the different quadrants of the circle.

Let A be any arc in the first quadrant, BG; then $\pi - A$ or BDG′, $\pi + A$, or BDAg; $2\pi - A$, or BDAgO, will be the arcs in the other quadrants.

In the first quadrant.		In the second quadrant.
See figure page 36.	GK = sin. A	$\sin.\overline{\pi-A} = GK = \quad + \sin. A$
	CK = cos. A	$\cos.\overline{\pi-A} = CK' = -CK = -\cos. A$*
	FB = tan. A	$\tan.\overline{\pi-A} = Bf = -BF = -\tan. A$†
	CF = sec. A	$\sec.\overline{\pi-A} = Cf = -CF = -\sec. A$

* CK′ being measured in an opposite direction to CK is *negative*.

† Bf, being measured opposite to BF, is also *negative*; and Cf being measured, not *through* the other extremity (see def. 10), but, away from it, is *negative* also.

The values of the tan. $\overline{\pi-A}$ and sec. $\overline{\pi-A}$ may be thus verified:—

$$\tan.\overline{\pi-A} = \frac{\sin.\overline{\pi-A}}{\cos.\overline{\pi-A}} = \frac{\sin. A}{-\cos. A} = -\tan. A.$$

$$\text{and sec. } \overline{\pi-A} = \frac{\text{rad.}^2}{\cos.\overline{\pi-A}} = \frac{1}{-\cos. A} = -\sec. A.$$

In the third quadrant.		In the fourth quadrant.	
$\sin.\overline{\pi+A} = K'g =$	$-\sin. A$	$\sin.\overline{2\pi-A} = KO = -KG =$	$-\sin. A$
$\cos.\overline{\pi+A} = CK' =$	$-\cos. A$	$\cos.\overline{2\pi-A} = CK =$	$+\cos. A$
$\tan.\overline{\pi+A} = BF =$	$+\tan. A$	$\tan.\overline{2\pi-A} = Bf = -BF =$	$-\tan. A$
$\sec.\overline{\pi+A} = -(CF) =$	$-\sec. A$	$\sec.\overline{2\pi-A} = Cf =$	$+\sec. A$

Sec. $\overline{\pi+A}$ = CF, but *negative*, for it is not drawn through *g*, the other extremity, but away from it through G.

Sec. $\overline{2\pi-A}$ is positive, being measured through the other extremity O.

These values also, may be verified by their equivalents :—

$$\tan.\ \overline{\pi+A} = \frac{\sin.\ \overline{\pi+A}}{\cos.\ \overline{\pi+A}} = \frac{-\sin.\ A}{-\cos.\ A} = \tan.\ A;$$

$$\sec.\ \overline{\pi+A} = \frac{1}{\cos.\ \overline{\pi+A}} = \frac{1}{\cos.\ A} = \sec.\ A.$$

$$\tan.\ \overline{2\pi-A} = \frac{\sin.\ \overline{2\pi-A}}{\cos.\ \overline{2\pi-A}} = \frac{-\sin.\ A}{\cos.\ A} = -\tan.\ A;$$

$$\sec.\ \overline{2\pi-A} = \frac{1}{\cos.\ (2\pi-A)} = \frac{1}{\cos.\ A} = \sec.\ A.$$

III. NUMERICAL VALUES,

Of the sines, cosines, &c., of 30°, 45°, *and* 60°.

Chord of 60° = radius = 1;

$$\sin.\ 30° = \frac{\text{chord } 60°}{2} = \frac{1}{2} = \cdot 5$$

$$\cos.\ 30° = \sqrt{1-\sin.^2 30°} = \sqrt{1-\tfrac{1}{4}} = \sqrt{\tfrac{3}{4}} = \tfrac{1}{2}\sqrt{3}$$

$$\tan.\ 30° = \frac{\sin.\ 30°}{\cos.\ 30°} = \frac{1}{2} \times \frac{2}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

$$\sec.\ 30° = \frac{1}{\cos.\ 30°} = 1 \times \frac{2}{\sqrt{3}} = \frac{2}{\sqrt{3}} = \frac{2}{3}\sqrt{3}$$

$$\sin.\ 45° = \cos.\ \overline{90°-45°} = \cos.\ 45°$$

$$\text{rad.} = 1 = \sqrt{\sin.^2 45 \times \cos.^2 45} = \sqrt{2\ \sin.^2 45} = 2\ \sin.^2 45$$

$$\therefore\ \sin.\ 45° = \sqrt{\tfrac{1}{2}} = \tfrac{1}{2}\sqrt{2}$$

$$\cos.\ 45° = \sin.\ 45° = \tfrac{1}{2}\sqrt{2}$$

$$\tan.\ 45° = \frac{\sin.\ 45°}{\cos.\ 45°} = \frac{1}{1} = 1.$$

$$\sec.\ 45° = \frac{1}{\cos.\ 45°} = 1 \times \frac{\sqrt{2}}{1} = \sqrt{2}$$

$$\sin.\ 60° = \cos.\ 30° = \tfrac{1}{2}\sqrt{3}$$

$$\cos.\ 60° = \sin.\ 30° = \tfrac{1}{2} = \cdot 5$$

$$\tan.\ 60° = \frac{\sin.\ 60°}{\cos\ 60°} = \tfrac{1}{2}\sqrt{3} \times \tfrac{2}{1} = \sqrt{3}$$

$$\sec.\ 60° = \frac{1}{\cos.\ 60°} = 1 \times \frac{2}{1} = 2$$

IV. RELATIVE VALUES

Of the sines, cosines, &c., to different radii.

As the actual lengths of the sines, cosines, &c., of different angles, and to different radii, vary in a ratio, compounded of the ratio of their subtending angles and their given radii, so, when the *angles* are constant, they vary as their radii; and when the *radii* are constant, as their angles.

Let sine a = sine of any angle to radius (unity); and sine A = sine of the same angle, to radius (R); then sine a : sine A : : 1 : R;

$$\therefore \frac{\text{sine } a}{1} = \frac{\text{sine A}}{\text{R}}.$$

whence, to transform formulas, calculated to radius unity—to others, to radius R; wherever sine a, tan. a, &c., occur, their equivalents, $\frac{\text{sine A}}{\text{R}}$, $\frac{\text{tan. A}}{\text{R}}$, &c., must be substituted in their stead.

EXAMPLE 1.—*To find the value of the tan. A (to radius R,) in terms of the sine and cosine.*

$$\text{tan. } a = \frac{\text{sin. } a}{\text{cos. } a}$$

$$\frac{\text{tan. A}}{\text{R}} = \frac{\text{sin. A}}{\text{R}} \cdot \frac{\text{R}}{\text{cos. A}} = \frac{\text{sin. A}}{\text{cos. A}}.$$

$\therefore \begin{matrix}\text{tan. A} \\ \text{to rad. R}\end{matrix} = \frac{\text{sin. A}}{\text{cos. A}} \times \text{R}$, which also verifies the following proposition, that

$$\text{as sin. } a = \frac{\text{sin. A}}{\text{R}}$$

$$\therefore \text{sin. } a \times \text{R} = \text{sin. A},$$

or, that the actual value of the sine, cosine, &c., of any angle A, whose radius is any length, R, is equal to that of the sine, cosine, &c., of the same angle, *calculated to unity*, multiplied by the given radius, R.

EXAMPLE 2.—What is the numerical value of the tan. 45° to radius 500?

$$\text{tan. } 45° \text{ to rad. } 1 = 1.$$

$$\begin{matrix}\text{tan. } 45° \\ \text{to rad. R}\end{matrix} = \text{tan. A} = \text{tan. a} \times \text{R} = 1 \times 500 = 500.$$

EXAMPLE 3.—What is the value of the sec. 30° to radius 300?

$$\text{sec. } a = \frac{1}{\text{cos. } a} = \frac{1}{\text{cos. } 30} = 1 \times 2\sqrt{\tfrac{1}{3}} = \tfrac{2}{3}\sqrt{3}$$

$$\begin{matrix}\text{sec. } 30° \\ \text{to rad. } 300\end{matrix} = \text{sec. A} = \text{sec. } a \times \text{R} = \tfrac{2}{3}\sqrt{3} \times 300 = 200\sqrt{3}$$

CHAP. II.

gnd. per.
and called

RIGHT-ANGLED TRIANGLES.

CASE. I.—*When one acute angle and one side are given.*

There will be no difficulty, if the following be borne in mind, viz.:—

1. That, if the *hypothenuse* be made *radius*, each leg will represent the sine of its opposite angle to that radius.

2. And if either the *base* or *perpendicular* be made *radius*, the hypothenuse and other side will represent the secant and tangent of the angle at the centre, and *that the sides are proportional to the links ~~they represent~~.*

Hence (as in the first case) as radius : to the hypothenuse : : sine of either angle : to its opposite side.

And (in the second case) 1st, as radius : is to the *perpendicular* : : so is tangent of the vertical angle : to the base ; and also : so is secant of the vertical angle : the hypothenuse.

And 2nd, as radius : is to the base : : so is tangent of the angle at base : to the perpendicular ; and : : so is secant of the same angle : to the hypothenuse.

EXAMPLE 1.—Let the hypothenuse of a right-angled triangle be 3 chains 50 links, and the angle at the base 25 degrees 15 minutes. Required the perpendicular and the base.

	As radius	$=10 \cdot 000000$
log. of hyp.	to log. hypothenuse $3 \cdot 50$	$= 0 \cdot 544068$ } added after
	so is log. sine $25^\circ\ 15'$	$= 9 \cdot 629989$
	to log. perpendicular $1 \cdot 49$	$= 0 \cdot 174057$
	and as radius	$=10 \cdot 000000$
	to log. hypothenuse $3 \cdot 50$	$= 0 \cdot 544068$
	so is log. cosine $25^\circ\ 15'$	$= 9 \cdot 956387$
	to log. base $3 \cdot 16\frac{1}{4}$	$= 0 \cdot 500455$

Ans. Perpendicular $= 1 \cdot 49$ chains
Base $= 3 \cdot 16\frac{1}{4}$ chains.

Ex. 2.—Given the vertical angle $35^\circ\ 54'$, and the perpendicular 12·20 chains. Required the base and hypothenuse.

Ans. Base $= 8 \cdot 83$ chs.; Hyp. $= 15 \cdot 06$ chs.

Ex. 3.—What are the hypothenuse and perpendicular of a right-angled triangle, whose base is 25 chains 80 links, and the vertical angle $89^\circ\ 27'$? *Ans.* Perp. $= 24$ links ; Hyp. $= 25 \cdot 80$ chs.

CASE II.—*When two sides are given.*

As in all right-angled triangles, the square of the hypothenuse is equal to the sum of the squares of the base and perpendicular, if any two sides are given, the third can be found by one of the following formulas:—

$$\text{Hypothenuse} = \sqrt{\text{perp.}^2 + \text{base}^2}$$

$$\text{Perpendicular} = \sqrt{\text{hyp.}^2 - \text{base}^2} = \sqrt{(\text{H} + \text{B}).(\text{H} - \text{B})}$$

$$= \frac{\log.\ \overline{\text{H} + \text{B}}}{2} + \frac{\log.\ \overline{\text{H} - \text{B}}}{2}$$

$$\text{Base} = \sqrt{\text{hyp.}^2 - \text{base}^2} = \sqrt{(\text{H} + \text{P}).(\text{H} - \text{P})}$$

$$= \frac{\log.\ \overline{\text{H} + \text{P}}}{2} + \frac{\log.\ \overline{\text{H} - \text{P}}}{2}$$

In the case, however, of the base and perpendicular being given, it is sometimes useful to obtain the hypothenuse *trigonometrically*, thus:

As perp. : rad. :: base : to tang. angle at vertex;
then, as sine angle at vertex : base : : radius : hypothenuse.

EXAMPLE 1.—What is the hypothenuse, when the perpendicular is 2·50 chains, and the base 3·90 chains?

First hyp. $= \sqrt{2{\cdot}50^2 + 3{\cdot}90^2} = \sqrt{21{\cdot}4600} = 4{\cdot}63$ chains.
secondly, by the above formula,

by saying,	as log. perpendicular 2·50	= 0·397940
	is to log. radius	= 10·000000
	so is log. base 3·90	= 0·591065
	to tang. angle at vertex 57·20	= 10·193125
and then,		
	as log. sine angle at vertex 57·20	= 9·925222
	log. base 3·90	= 0·591065
	so is radius	= 10·000000
	to log. hyp. 4·63	= 0·665843

we obtain the hypothenuse 4·63, the same length as in the preceding.

The angles can be obtained from the first method, by the following proportion, viz.:—

As log. hyp. 4·63 chains	= 0·665581
: log. rad.	= 10·000000
: : log. perpend. 2·50	= 0·397940
: log. sine angle at base 32° 41	= 9·732359

and : log. cos. angle at vertex 57° 19′;

EXAMPLE 2.—Let AC equal 45 chains, BC 36 chains, and the angle A, 51° 25′; required the base AB.

As log. BC, 36 chains	= 1·556303
to log. sin. ∠ A, 51° 25′	= 9·893041
so is log. AC, 45 chains	= 1·653213
	11·546254
	1·556303
to log. sin. ABC, 77° 43′	= 9·989951
or to its supplement, 102° 17′.	

The angle ABC has therefore two values, depending upon whether it is greater or less than a right angle; and also the angle ACB, and its opposite side AB, which may be either AB or AD.

EXAMPLE 3.—The two sides of a triangle are AC, 50 chains; CB, 25 chains; and the angle ACB, 16 degrees 45 minutes; what are the other sides and angles? *Answer* Base 27·07 chs. ∠ B = 147.° 49′ ∠ A = 15.26.

CASE II.—*When the two sides and included angle are given.*

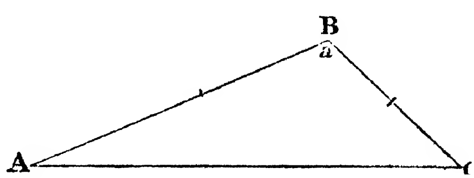
Rule.—Subtract the given angle from 180°, which will give the sum of the other two; then, x and y being any two angles,

$$x = \tfrac{1}{2}(x+y) + \tfrac{1}{2}(x-y),$$
$$\text{and } y = \tfrac{1}{2}(x+y) - \tfrac{1}{2}(x-y).$$

Then, to obtain $(x-y)$; say, as the sum of the two sides : is to their difference :: so is the tan. ½ sum of the two unknown angles, $(x+y)$: to the tan. ½ difference $= (x-y)$.

By substituting the values of $x+y$, and $x-y$, which are now both known, in the above formula, the values of x and y will be obtained.

In the triangle ABC, let the two sides, AB and BC (AB being greater than BC) be given, and the angle, at B $= a°$



then $180°-a°$ = sum of angles at A and C; and $(x-y)$ = difference.

$$\angle\ ACB = \frac{180-a}{2} + \frac{x-y}{2}$$

$$\text{and } \angle\ BAC = \frac{180-a}{2} - \frac{x-y}{2}$$

It is now reduced to the condition of Case 1st; proceed, therefore, according to the rules there given.

The following formula may sometimes be useful, in at once obtaining the base.

Find first the value of $(x-y)$ as above, then say—

As cos. $\frac{1}{2}(x-y)$: cos. $\frac{1}{2}(x+y)$: : AB+BC : AC

or,

As sine $\frac{1}{2}(x-y)$: sine $\frac{1}{2}(x+y)$: : AB−BC : AC

$$AC = \frac{\cos. \frac{1}{2}(x+y)}{\cos. \frac{1}{2}(x-y)} \cdot (AB+BC).$$

$$\text{or } AC = \frac{\sin. \frac{1}{2}(x+y)}{\sin. \frac{1}{2}(x-y)} (AB-BC)$$

that is,

as cos. $\frac{1}{2}$ (difference of angles),	or	as sine $\frac{1}{2}$ (difference of angles),
: cos. $\frac{1}{2}$ (sum),		: sine $\frac{1}{2}$ (their sum),
: : sum of the two sides,		: : difference of the two sides,
: to the base,		: to the base.

The advantage of using this formula is, that the logarithmic values of the *cosines* of $(x+y)$ and $(x-y)$ can be obtained at the same time as those of their tangents, which are wanted, in the first formula, to obtain the value of $x-y$; and the *separate* values of the angles are not required.

EXAMPLE 1.—Let the two sides AB, BC of the triangle ABC be 36·40 and 24·40 chs., and the included angle 16 degrees 26 minutes. Required the other side and the remaining angles.

$$\begin{array}{r} 180°\cdot 00' \\ 16\cdot 26 \\ \hline 2\,|\,163\cdot 34 \\ \hline 81\cdot 47 = \frac{x+y}{2} \end{array} \qquad \begin{array}{rl} 36\cdot 40 & \\ 24\cdot 40 & \\ \hline 60\cdot 80 & = AB + BC \\ 12\cdot 00 & = AB - BC \end{array}$$

$$\begin{array}{lrr} \text{As log. } AB + BC, & 60°80' = & 1\cdot 783904 \\ \text{is to log. } AB - BC, & 12\cdot 00 = & 1\cdot 079181 \\ \text{so is log. tan. } \frac{x+y}{2}, & 81°47' = & 10\cdot 840435 \\ \hline & & 11\cdot 919616 \\ \hline \text{to log. tan. } \frac{x-y}{2} & 53°49' = & 10\cdot 135712 \end{array}$$

$$\begin{array}{rl} 81\cdot 47 & \\ 53\cdot 49 & \\ \hline 135\cdot 36 & = \angle\ ACB \\ 27\cdot 58 & = \angle\ BAC \\ \text{Add } \angle\ ABC = 16\cdot 26 & \\ \hline 180\cdot 00 & \text{sum of the three angles.} \end{array}$$

The question being now reduced to the first case, the base may be obtained, as follows, by saying:—

As log. sine $\angle$ BAC, 27° 58′	=	9·671134
is to log. BC, 24·40	=	1·387390
so is log. sine $\angle$ ABC, 16° 26′	=	9·451632
		10·839022
		9·671134
to log. AC—14·72	=	1·167888

or, otherwise, by using the formula, above referred to, of the cosines of the sum and difference of the unknown angles, and saying:—

As log. cosine $\frac{1}{2}\,\overline{x-y}$, 53·49	=	9·771125
: to log. cosine $\frac{1}{2}\,\overline{x+y}$, 81·47	=	9·155083
:: is log. (AB + BC,) 60·80	=	1·783904
		10.938987
		9·771125
to log. AC, 14° 72′	=	1·167862

differing only in the two last places of decimals, which do not agree with the preceding, in consequence of the cos. $\frac{x-y}{2}$ (53°49,) not being the exact value of the logarithm, 10·135712; there is no difference, however, in their values, as to links, which is sufficiently near for practical purposes.

For finding the exact values of logarithmic sines, cosines, &c., and carrying the whole work out to seconds, see chapter on the "Theodolite and Trigonometrical Calculations."

Example 2.—Given the two sides 45 chains 44 links, and 40 chains 48 links, of any triangle ABC, and the included angle 117 degrees 50 minutes; what are the two other angles and the remaining side?

Ans. Base = 1~~58·91~~ chs. $\angle$ at B = ~~32.° 47.′ 30~~″. 33
73,49 $\angle$ at C = 29.° ~~22.′ 30~~″. 6

Example 3.—The included angle of a triangle is 45 degrees; the two sides are respectively 70 chains and 50 chains; what will be the length of the base? *Ans.* 49·50 chs.

Case III.—*When the three sides only are given.*

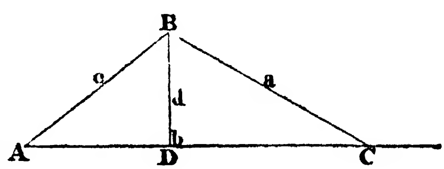
Let fall a perpendicular from the vertex upon the base, dividing the triangle into two right-angled triangles. We shall then have one side, and the right-angle known in each. And, by finding the lengths of the

segments of the base of the whole triangle, we obtain in each of the smaller right-angled triangles two given sides and an angle. Now, by saying,—

As the whole base, or sum of the segments
: is to the sum of the two other sides,
: : so is the difference of those two sides,
: to the difference of the segments of the base.

And, by adding half the sum to half the difference, we obtain the *greater* segment, which is always adjoining the *greater* side of the given triangle, and, by subtracting half the difference from half the sum, we obtain the *smaller* segment, adjoining the *smaller* side of the whole triangle.

The case having come under the form of two right-angled triangles, the required sides and angles must be worked out by the rules before given.



Let ABC be a triangle, whose three sides AB, BC, and AC, are given.

From B, let fall BD perpendicular to AC. In the two triangles ABD, CBD, AB and BC being known, and the angles at D right angles, we have to find first the two sides AD and DC; say,—

As AC : AB + BC : : BC—AB : x
when $x =$ AD$\sim$DC,
and because BC is greater than AB,
therefore DC is greater than AD;

then, by saying, (*see case of right-angled triangles*),—

as AB : rad. : : AD : cos. $\angle$ at A

we obtain the angle at A; and, by similar proportions, the remaining angles at C and B.—*The angles ABD, CBD, calculated from the smaller triangles*, should be together *equal to the supplement* of the sum of the angles at A and C.

EXAMPLE 1.—Let ABC be a triangle, having the sides AB, BC, and AC, respectively, equal to 25 chains, 36 chains, and 48 chains. It is required to find the angles.

Let fall the perpendicular BD, which will divide the given triangle into two smaller right-angled triangles.

Then, to find the difference of the segments of the base, say,—

$$\text{As AC} : \text{BC} + \text{AB} :: \text{BC} - \text{AB} : (\text{AD} \sim \text{DC})$$
$$\text{or, } 48 : 61 :: 11 : 13{\cdot}98\text{—}$$

and adding half the sum to half the difference, for the greater segment, and subtracting them, for the less, we have,—

$$\tfrac{1}{2} \text{ of } 48 = 24 \text{ ; and } \tfrac{1}{2} \text{ of } 13{\cdot}98 = 6{\cdot}19.$$
$$\text{where } 24 + 6{\cdot}99 = 30{\cdot}99 = \text{greater segment, DC,}$$
$$\text{and } 24 - 6{\cdot}99 = 17{\cdot}01 = \text{smaller segment, AD,}$$

It is now reduced to the case of right-angled triangles; next say,—

As log. AB, 25,	= 1·39794
is to log. rad.	= 10·00000
so is log. AD, 17·01,	= 1·23070
	11·23070
	1·39794
to log. cos. angle A, 47° 8′,	= 9·83276

And as log. BC, 36,	= 1·55630
to log. rad.	= 10·00000
is log. DC, 30·99,	= 1·49122
	11·49122
	1·55630
to log. cos. angle C, 30° 35′,	= 9·93492

Adding the angle at A and C, and deducting them from 180°, we obtain the angle at B.

$$47^\circ\ 8' + 30^\circ\ 35' = 77^\circ\ 43' \text{ ; and}$$
$$180^\circ - 77^\circ\ 43' = 102^\circ\ 17' = \text{angle B.}$$

EXAMPLE 2.—Given a triangle, whose sides are 25 chains 16 links, 24 chains 13 links, and 16 chains 17 links, respectively; required the angles. *Ans.* 74° 20′; 67° 25′; 38° 15′.

EXAMPLE 3.—A triangle, whose sides are respectively, 40, 50, and 60 chains, being given, it is required to find the angles.
Ans. 82° 49′; 41° 25′; 55° 46′.

EXAMPLE 4.—What are the angles of that triangle, whose sides are 498 chains 16 links, 464 chains, and 40 chains, respectively?
Ans. 125° 29′; 52° 11′; 2° 20′.

2ND METHOD.—When the nature of the question merely requires the determining of one of the angles, the following formula may be used, viz.:—

$\cos. \angle C = \frac{AC^2 + BC^2 - AB^2}{2\ AC\ CB}$ or, (vide fig. p. 49) making $AB = c$

$BC = a$, and $AC = b$, *corresponding to the angles to which they are opposite.* $\text{Cos. } C = \frac{a^2 + b^2 - c^2}{2\ ab}$

For $AB^2 = AC^2 + BC^2 \pm 2\ AC.\ CD$ (see Theor. 7, page 13.)

$$\therefore \pm CD = \frac{AC^2 + BC^2 - AB^2}{2\ AC}$$

now BC : CD :: rad. : cos. C

$\therefore \pm \frac{CD}{BC} = \pm \frac{\cos C}{R}$ and $\pm CD = BC \cos. C$ (when radius $= 1$)

$$\text{(by substituting) } BC \cos. C = \frac{AC^2 + BC^2 - AB^2}{2\ AC}$$

$$\therefore \cos. C = \frac{AC^2 + BC^2 - AB^2}{2\ AC.\ CB} = \frac{a^2 + b^2 - c^2}{2\, ab.};$$

Where cos. C is that of an angle, either greater or less than a right angle, whose value is expressed in terms of its sides; a and b being the sides that include it, and c the side opposite to it.

This formula, however, can only be used in its *present* form, when the sides are *small*, and when they can be calculated arithmetically, as in the case of the 1st Example, in the *first method*, the result of which we will now proceed to verify, by the present formula.

The given sides are 25, 36, and 48 chains.

Substituting these lengths in the above formula, we have,—

$$25^2 = 625\ ;\ 36^2 = 1296\ ;\ \text{and } 48^2 = 2304$$

$$\text{then } \cos. C = \frac{625 + 2304 - 1226}{2\ (1200)} = \frac{1633}{2400}$$

$$\frac{1633}{2400} = 0{\cdot}6804166 = \text{natural cos. } C$$

natural cos. $=$ cos. to rad. $(1) =$ cos. a.

log. cos. $=$ log. cosine to rad. $10^{10} =$ cos. A.

cos. A $=$ cos. $a \times$ R (see page 41)

$\therefore$ log. cos. A $=$ log. cos. a + log. 10^{10} (10)

By substituting natural cos. $a = 0{\cdot}6804166$

$$\text{log. cos. } a = -1{\cdot}832781$$
$$+\text{ log. R} = 10$$
$$47^\circ\ 8' = \text{log. cos. A} = 9{\cdot}832781, \text{ the same result as before.}$$

To find the angle C in the same triangle.

$$\text{cos. C} = \frac{AC^2 + BC^2 - AB^2}{2\ AC.\ BC}$$

$$\text{or cos. C} = \frac{2304 + 1296 - 625}{2\ (1728)} = \frac{2975}{3456} = {\cdot}85999$$

$$\text{log. } 85999 = -\ 1{\cdot}93492$$
$$+\text{ log. R} = 10$$
$$30^\circ\ 35' = \text{log. cos. C} = 9{\cdot}93492$$

To find the angle at B.

$$\text{cos. B} = \frac{AB^2 + BC^2 - AC^2}{2\ AB.\ BC}$$

$$\text{or cos. B} = \frac{625 + 1296 - 2304}{2\ (900)} = \frac{-383}{1800} = {\cdot}21278$$

$$\text{log. } {\cdot}21278 = -1.32793$$
$$+\text{log. rad.} = 10$$
$$\text{log.—cos. } 77^\circ\ 43' = 9{\cdot}32793$$

but this is the supplemental angle, being the value of —cos. B; therefore $180^\circ - 77^\circ\ 43' = 102^\circ\ 17' =$ cos. angle B.

It will be seen that these angles are the same as those which were obtained by the first method, where the same triangle was used.

WHEN THE SIDES, HOWEVER, ARE LARGE, it is necessary to convert this formula into a more convenient expression, for logarithmic calculation. viz.—

$$\text{Because, cos. C} = \frac{a^2 + b^2 - c^2}{2\,ab} \text{ (to rad. 1)}$$

$$\frac{\text{cos. C}}{\text{R}} = \frac{a^2 + b^2 - c^2}{2\,ab} \text{ to rad. R (see page 51)}$$

$$\therefore \text{ cos. C} = \text{R}\ \frac{(a^2 + b^2 - c^2)}{2\,ab}$$

and versed sine = rad.—cos.

$$\text{vers. sin. C} = \text{R} - \text{R}\ \frac{(a^2 + b^2 - c^2)}{2\,ab} = \text{R}\ (1 - \frac{a^2 + b^2 - c^2}{2\,ab});$$

$$= \text{R}\ \frac{(2\,ab - a^2 - b^2 + c^2)}{2\,ab}$$

$$\text{but sine}^2\ \tfrac{1}{2}\text{ C} = \tfrac{1}{2}\text{ rad (vers. sine C)}$$

$$\therefore \text{ sine}^2\ \tfrac{1}{2}\text{ C} = \frac{\text{rad}^2\ (2\,ab - a^2 - b^2 + c^2)}{4\,ab}$$

$$= \text{rad.}^2 \frac{(c^2 - (a - b)^2)}{4\,ab}$$

$$= \text{rad}^2 \frac{(c + a - b) \,.\, (c - a + b)}{4\,ab}$$

$$\text{sine } \tfrac{1}{2}\,C = \text{rad.} \sqrt{\frac{(s - a).\,(s - b);}{ab}}$$

where s = half the perimeter—

and for the *cos.* $\frac{1}{2}$ *C*, it may be proved in the same way, that

$$\text{cos. } \frac{C}{2} = \text{rad.} \sqrt{\frac{(s - c)\ s}{ab.}}$$

Assuming the same data as before, for the purpose of verifying the former results, and substituting their values, respectively, in the present formula *of the values of half the sines and cosines, we obtain,*—

$$\text{where } s = \tfrac{1}{2}\text{ perimeter} = \frac{25 + 36 + 48}{2} = 54{\cdot}50$$

$$\text{then sine } \tfrac{1}{2}\,A = \text{rad.} \sqrt{\frac{(54{\cdot}50 - 25)\cdot(54{\cdot}50 - 48)}{25 \times 48}}$$

log. 29·50		= 1·46982
log. 6·50		= 0·81291
—log. 25	= 1·39794	2·28273
—log. 48	= 1·68124	—3·07918
	2	—1·20355
		—1·60178
+ log. rad.		= 10·00000
log. sine $\frac{1}{2}$ A = 23° 34′		= 9·60178
2		

angle A = 47· 8, the same as before.

By the 2nd formula—that of cos. $\frac{1}{2}$ A = rad. $\sqrt{\frac{(s-a)\,s}{bc}}$, where A is the given angle required, and a is the side opposite to it.

log. 18·50		= 1·26717
log. 54·50		= 1·73640
		3·00357
—log. 25	= 1·39794	
—log. 48	= 1·68124	—3·07918
	2	—1·92439
		—1·96219
		10·00000
log. cos. $\frac{1}{2}$ A = 23° 34′		= 9·96219
2		

angle A = 47° 8′ as before.

A few Examples, without answers, are subjoined, which the student is required to calculate and verify, by the various methods above.

Example 1.—Given the three sides 40 chains, 90 chains and 60 chains, to find the opposite angles.

Example 2.—The three sides of a triangle are 24·50 chains, 31·60 chains, and 81·96 chains, what are the angles?

Example 3.—The sides of a triangular field are, respectively, 20 chains, 3·50 chains, and 45·20 chains. What are the opposite angles?

As a knowledge of some of the general relative values of sines, cosines, and tangents, in terms of each other, may be found exceedingly useful in practice to the Surveyor, some of the most useful ones are subjoined.

The most important formula, the basis, in fact, of the whole, is—

$$\text{sine } (a + b) = \text{sine } a. \cos. b + \cos. a \text{ sine } b$$
$$\text{sine } (a - b) = \text{sine } a. \cos. b - \cos. a \text{ sine } b$$
$$\cos. (a + b) = \cos. a. \cos. b - \text{sine } a \text{ sine } b$$
$$\cos. (a - b) = \cos. a. \cos. b + \text{sine } a \text{ sine } b$$

These are the values of the sines and cosines of the sum and difference of two arcs, in terms of the simple arcs.

By *adding* and *subtracting*, we obtain,—

$$\text{sine } (a + b) + \text{sine } (a - b) = 2 \text{ sine } a \cos. b$$
$$\text{sine } (a + b) - \text{sine } (a - b) = 2 \cos. a \text{ sine } b$$
$$\cos. (a + b) + \cos. (a - b) = 2 \cos. a \cos. b$$
$$\cos. (a - b) - \cos. (a + b) = 2 \text{ sine } a \text{ sine } b$$

which are the values of the sum and difference of the sines and cosines of the sum, and difference of the arcs.

Again, by making $a + b = m$

and $a - b = p$.

we have—

$$\text{sine } m + \text{sine } p = 2 \text{ sine } \tfrac{1}{2} (m + p) \cos. \tfrac{1}{2} (m - p)$$
$$\text{sine } m - \text{sine } p = 2 \cos. \tfrac{1}{2} (m + p) \text{ sine } \tfrac{1}{2} (m - p)$$
$$\cos. m + \cos. p = 2 \cos. \tfrac{1}{2} (m + p) \cos. \tfrac{1}{2} (m - p)$$
$$\cos. p - \cos. m = 2 \text{ sine } \tfrac{1}{2} (m + p) \text{ sine } \tfrac{1}{2} (m - p)$$

but m and p are *any angles whatever*, so that

$$\text{sine } a + \text{sine } b = 2 \text{ sine } \tfrac{1}{2} (a + b) \cos. \tfrac{1}{2} (a - b)$$
$$\text{sine } a - \text{sine } b = 2 \cos. \tfrac{1}{2} (a + b) \text{ sine } \tfrac{1}{2} (a - b)$$
$$\cos. a + \cos. b = 2 \cos. \tfrac{1}{2} (a + b) \cos. \tfrac{1}{2} (a - b)$$
$$\cos. b - \cos. a = 2 \text{ sine } \tfrac{1}{2} (a + b) \text{ sine } \tfrac{1}{2} (a - b)$$

This formula is useful in facilitating the transformation of the *product* of the sines of the sums and differences of angles into simple angles.

Now, by dividing the last class of formula, the one by the other, we have,—

$$\frac{\text{sine } a + \text{sine } b}{\text{sine } a - \text{sine } b} = \frac{\text{sine } \tfrac{1}{2}(a+b) \cos. \tfrac{1}{2}(a-b)}{\cos. \tfrac{1}{2}(a+b) \text{ sine } \tfrac{1}{2}(a-b)} = \frac{\tan. \tfrac{1}{2}(a+b)}{\tan. \tfrac{1}{2}(a-b)}$$

$$\frac{\text{sine } a + \text{sine } b}{\cos. a + \cos. b} = \tan. \tfrac{1}{2}(a+b)$$

$$\frac{\text{sine } a + \text{sine } b}{\cos. b - \cos. a} = \cot. \tfrac{1}{2}(a-b)$$

$$\frac{\text{sine } a - \text{sine } b}{\cos. a + \cos. b} = \tan. \tfrac{1}{2}(a-b)$$

$$\frac{\text{sine } a - \text{sine } b}{\cos. b - \cos. a} = \cot. \tfrac{1}{2}(a+b)$$

$$\frac{\cos. a + \cos. b}{\cos. b - \cos. a} = \frac{\cot. \tfrac{1}{2}(a-b)}{\tan. \tfrac{1}{2}(a-b)}$$

And by making $b = o$, we obtain the value of the tan. $\tfrac{1}{2}a$; cot. $\tfrac{1}{2}a$, tan.$^2 \tfrac{1}{2} a$, cot.$^2 \tfrac{1}{2} a$.

Again,—

$$\text{As tan. } \overline{a+b} = \frac{\text{sine } (a+b)}{\cos. (a+b)} = \frac{\text{sine } a. \cos. b + \cos. a. \text{ sine } b}{\cos. a. \cos. b - \text{sine } a. \text{ sine } b}$$

Dividing by cos. a. cos. b, we have,—

$$\frac{\dfrac{\text{sine } a}{\cos. b} + \dfrac{\text{sine } b}{\cos. b}}{1 - \dfrac{\text{sine } a. \cos. b}{\cos. a. \cos. b}} = \frac{\tan. a. + \tan. b}{1 - \tan. a. \tan. b}$$

$$\text{so also, tan. } (a-b) = \frac{\tan. a - \tan. b}{1 + \tan. a. \tan. b}$$

Make $b = a$ (in the first value)

$$\text{and tan. } 2a = \frac{2 \tan. a}{1 - \tan.^2 a} \text{ \&c., \&c.}$$

[NOTE.—For a fuller investigation of this subject, see Hall's Trigonometry, which I would strongly recommend to every practical man, who might have leisure to follow it up.]

LAND SURVEYING.

Part the First.

CHAP. I.

ON THE CHAIN.

As every measurement, whether of work, solidity, or superficial extent, must be measured by some unit, to which it can bear constant relation, it has been concluded in the case of land measurement, to make that unit an acre; the acre, or arpent, is the generally recognised unit of land measurement; it varies, however, considerably in different Counties.*

THE STATUTE ACRE in England consists of ten square chains, or one chain front by 10 chains deep, or of its equivalent rectangle ax, where a can be any number, and $x = \frac{10}{a}$ square chains; each chain containing 22 yards, or 4 poles, or 66 feet, or 100 links, and each link 7.92 inches.

The acre, therefore, is equal to 10 square chains

or $(4)^2$ poles $\times$ 10 $=$ 160 square poles.
or $(22)^2$ yds. $\times$ 10 $=$ 4840 square yards.
or $(66)^2$ feet $\times$ 10 $=$ 43560 square feet.
or $(100)^2$ links $\times$ 10 $=$ 100,000 square links.

---

* For the different values of the acre in different Counties of England, the subject, at the end of the book.

The chain, in common use, is called Gunter's chain, from its inventor, and is divided into ten equal parts, distinguished by a piece of brass, with notches; the brass at the first division, from either end, having one notch; at the second division, two notches; at the third, three; at the fourth, four; and at 50 links, or the middle of the chain, there is a round piece of brass.

The object of marking these divisions from either end of the chain, is to enable the surveyor to measure *either way* from each end; but it is productive sometimes of serious mistakes, as in consequence of 60 links being marked the same as 40, when the eye is not accustomed to distinguish the distance between them, the one is frequently mistaken for the other; and, in measuring with the chain, no mistake is so common as this to a young beginner; and I would particularly charge him to be careful, that in taking off a distance of, what he may suppose to be, 2 chains 45 links, he is not mistaking for it 2 chains 65 links; there is less likelihood of mistaking 30 links for 70, though a stupid inattentive assistant will sometimes even make that mistake.

Each of the above brass divisions of the chain is again sub-divided into other ten parts, or links. If therefore a chain = unity;

$$\text{each brass division} = \frac{1}{10}, \text{ or } 0{\cdot}1 \text{ chains;}$$

$$\text{and each link} = \frac{1}{100}, \text{ or } 0{\cdot}01 \text{ chains.}$$

The advantages of this arrangement is, that, in measuring a line, it matters not whether the distance be termed 7·32 chs. (*7 chains* 32 *links,*) or 732 links; and, as 10 square chains make one acre, or 10 times 100 links squared, it is only necessary to multiply together the length and breadth, given in links, of a piece of ground, whose area is required,

and set off four decimal places, when the integers will be square chains; or set off five decimal places, and the integers will be the required acres.

NOTE.—[In doing the chain up, after the day's work, begin always at the middle, folding two links up at once. When done up in this way, by taking hold of the two handles, and throwing the other part from you, you are safe from any entanglement of the chain, when you want to use it.]

To measure a straight line with a chain.

In measuring with the chain, it is requisite to have ten small arrows, or pins of strong iron wire, about a foot and a half long, and pointed at one end, to stick into the ground.

A piece of red cloth should also be tied to the ring of each arrow, so that they may be more easily perceived in the midst of grass or underwood.

Having determined upon the starting point and the direction of the line, it is always requisite to have the line carefully ranged, and generally one or two poles or flags set between the two end flags; these flags should consist of a straight pole about an inch or $\frac{3}{4}$ inch diameter, pointed and shoed at the bottom, and having a white and red flag, about nine inches square at the top, they are generally about six feet high, though in the taking of long base lines, they are occasionally made of from 10 to 15 feet.

On a large survey, a Surveyor should be at least furnished with three of the latter and half a dozen of the former.

There is a great deal of mechanical nicety in setting these poles, and perserving a correct line throughout. To obtain this,—Set up a flag at your starting point, and one at the termination of your intended line, and, having selected the distances between, where you are desirous of setting up flags, direct your assistants to hold the flags carefully upright and move as you direct them, one way or the other.

It is frequently the practice for persons, who are directing the ranges, to stand immediately behind the starting flag, and, by first looking on one side and then on the other, to range the intervening flags with the distant one: this is by no means a correct method; it is, in all cases, whether of directing the fixing of a flag by another between two fixed points, or the continuing of those points onwards oneself, an act of necessity, to stand some 10 or 20 paces off from the nearest flag, and, by contracting its visual breadth, to secure the correct position of the flag that is to be put down, by at once covering the three.

Having thus obtained a practical method of measuring along a base line, and not deviating at the fixing of every chain's length to the right or the left, we have but to secure a correct and equal extension of the chain at every length, to obtain the true length of the line required. Let one assistant take the lead, who is called the *leader*, having the ten pins and the chain handle in his left hand, and proceed towards the mark at the end of the line; while the other assistant, called the *driver*, holding the other end of the chain also in his left hand, keeps it close against the mark or starting point; when the leader has come to the end of the chain, let him turn round with his face towards the driver, and, taking one of the arrows in his right hand, let him pull the chain tightly, keeping it on the ground, and fix the arrow he held in his right hand in the ground.

As the chain is seldom straight at first, when the leader faces the driver, the driver must always keep *his* end to the ground, close to the mark; but the leader, previously to finally setting the arrow, should raise the chain with both hands, shake it to the very end, and before he puts his arrow down, see that it is perfectly straight.

The leader must also observe, as he is putting down the chain, to look towards the driver, to see if he is himself in the right direction, or whether the driver is noting him to the right or to the left; when he has succeeded in hitting the right direction, (which he knows by the driver calling out "*down*,") and, obtaining the proper measurement of the chain, by carefully complying with the directions above, and has put his first arrow down, he must *return* the cry of "*down*" to the driver, and, taking the chain up in his right hand, must proceed onwards until the driver, coming to the first arrow, cries out "*stop*," when the same process is repeated. The leader having a second time returned the cry of "*down*," the driver picks up the arrow, carrying it in his left hand, taking care, as long as he has two or more flags to cover before him, to direct the leader; when (the leader), however, has past all

but the last flag, the proper position of the arrows depends upon himself, which he must carefully do, by covering the backward flags.

When the leader has put down the ten pins, he calls out "*tally*," and the driver, dropping his end of the chain, comes up to the place, picks up the last arrow put down, counts over the whole of the arrows, and, putting his foot to the mark, returns them again to the leader, who finding they are the right number (10), proceeds with the chaining as before; the second "*tally*" is then called; then the third; each cry, to prevent mistakes, being repeated by the driver. I would here observe, that many errors have arisen from the omission of these apparently trifling superfluities, and much valuable time lost. It is this superfluity of care which distinguishes the man of practice from the novice—the latter *thinks* it so simple that he cannot make a mistake; the former *knows* that, from that very simplicity, error is more likely to ensue.

In this manner they continue, till the whole line is measured. Should it not be a complete chain to the end, it is usual for the leader to put his end of the chain down to the mark, and for the driver to read off the distance; and this is certainly the correct method, except there are offsets to be taken between the last full chain and the end of the line, when the driver had better put his end down, and the leader pull up, as the position of the offsets is measured from the driver's end.

It is advisable, at the end of each tally, to leave a mark in the ground, and note it in the field book, especially when the line is long, as the latter prevents mistakes, and the former makes the correction of it more easy.

It need only be added that the leader having put down his arrow, should, on proceeding, walk so as to keep the flags, that are before him, always covered, and, on turning round, should so arrange himself as to cover the starting point (as near as he can) with the last arrow put down; this will be saving the driver some considerable trouble in keeping him in the proper direction, and will, at the same time, give the leader himself a facility in keeping the course, when a hill, or other local obstruction, may hide the forward flags from the driver's view.

For practice—I would recommend some line to be selected of about half a mile, which should be carefully measured two or three times, until not more than an error of a link should be detected, in two or three consecutive trials.

NOTE.—[It is requisite, to ensure accuracy, that the chain should be frequently measured. If found incorrect, take care that the error be divided among the whole, so that *each* 10 links be of equal length; in each 10 links, it is better to cut off a little from several rings, than take a whole ring away.]

Professional Surveyors measure their chains regularly every morning, when they are on service.

CHAP. II.

THE OFFSET STAFF,

(For the Chain.)

Is a narrow slip of wood about 1½ inch wide by 1 inch thick, and generally 10 links long, divided into links; it is made of deal or some light wood, and should be furnished at one end with a small notch or hook, to put the chain through the hedges, and be numbered on both sides, from different ends. It is used for the purpose of measuring short distances, called *offsets*,* from the line to the hedges, &c.

As these offsets must be measured at right-angles to the chain, the Surveyor should stand on the opposite side of the chain to the hedge, or object to be measured to, and walking along the chain, looking at either end, mark where a perpendicular from the given object would fall upon the chain. These are but approximations, but practice will soon make them as practically accurate as is necessary.

* These offsets should never exceed one chain.

CHAP. III.

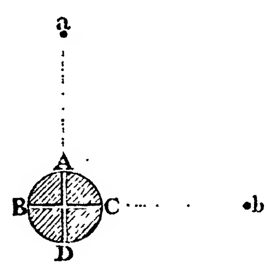
ON THE CROSS STAFF.

The cross staff is about 5 feet 6 inches high, pointed, and strongly shoed, to drive into the ground, and having at the top a circular piece of brass, or hard wood, so fixed as to be taken on and off.

The piece of brass is furnished with four brass pins, set at right-angles to each, and the wood head is divided into four divisions, by grooves, also at right angles, cut about half an inch down into the wood, which is about 2 or 2½ inches thick. This head is sometimes made with a spring, to move up and down the pole of the staff, and to remain firm at any height.

It is always requisite to test the accuracy of these grooves, which can be done in the following way:—

Let ABCD be a cross staff; and AD, BC, its sights or grooves; place a flag, *a*, in the direction of the grove AD, and another at *b*, in that of BC; half reverse the cross, so that DA point to *b*, and CB to *a*. Then, if the flags are still in the direction of the grooves, the instrument is correct.



The principal use of the cross staff is in the measurement, by perpendiculars, of straight-sided fields; and in the setting off of short perpendiculars from a base line; in the laying out of streets and building lots: and generally, in determining, by means of a perpendicular therefrom, the position of a house, mile-post, pond, or any conspicuous object, whose approximate situation alone is required, either to fill up or elucidate the plan, and whose distance from the measured line might exceed the limit of an offset.

It is especially useful where no great accuracy is required, when, in the computation of the areas, it is not requisite to take into account the irregularities

of the hedges, as, *in surveying a field*, it saves the necessity of measuring the whole way round, and makes a diagonal, with two perpendiculars from each of the opposite corners, all the admeasurements required.

In the measurement of straight-sided fields, when the perpendiculars are taken from the diagonal to the opposite corners, considerable trouble is sometimes experienced in determining the position of the perpendiculars on the diagonal; and two or three trials are often required, when accuracy is wished for. To remedy this, a little portable instrument was invented by a friend, which can be carried in the hand, and which at once determines the required position of the perpendicular. It is a small oblong piece of wood, of about two inches thick, with two pins placed vertically, having a small mirror, at an angle of 45°, which being held in the line of the diagonal, and turned towards the flag, which is set in the corner of the field, gives you, when your hand is at the required position, the reflection of the flag in the same line with the pins.

By holding this in your hand, keeping the pins in the direction of the diagonal, and walking along it till you perceive the reflected flag immediately in the line of the pins, the true position of the perpendicular can be correctly determined.

It is a simple but very useful instrument.

CHAP IV.

THE FIELD BOOK.

The field book should be of a convenient size for the pocket, having the left page ruled with a central column, and the right page left blank for remarks. The central column should be headed "*Chains*," on either side "*Offsets*," and the right page "*Remarks*."

The central column is intended for all actual lines measured, and, by commencing *from the bottom of the page*, the page becomes a smaller representation of the reality with the line measured from you, and the

offsets, at their respective distances on that line, taken at so many links to the right or to the left, as are actually on the ground. In keeping the field book, it *first* should ever be remembered, that the central column is virtually but one line, representing the chain, the space *within* the column being merely required for the several distances on the chain, whence the offsets are taken; and, *secondly*, that all offsets, read either way, outward *from* the central column, in the same way as they are measured outwards *from* the chain.

To preserve uniformity, as it is more natural to measure from left to right, the place measured *from*, is put on the left of the central column, at the bottom of the line, and the station measured *to*, is put at the top, to the *right;* the points of commencement and termination of the line can thus be immediately seen.

The book should be interleaved with blotting paper, and the entries, if possible, should be made in ink. The pages should also be numbered, before beginning for facility of reference.

If the direction of the line is determined by an angle taken by the theodolite, or the bearing of the line be given by the circumferentor, the angle of the former or the bearing of the latter is placed in the central column, immediately above the starting point, according to the examples below:—

When the line crosses a road, or hedge, &c., make corresponding lines in the field book, as, in the first example of Theodolite Surveying, at the several distances on the line: of 2 links; 1 chain 10 links; 1 chain 26 links; 11·20 chains; 11·40 chains; 13·21 chains; always considering the column as but a line.

In taking "offsets" to corners of fences, houses, &c., mark the relative position of the corner, as to the chain line (see distance 6·79 on line 6·90; and 0·10 on line 7·31, in the example of "CHAIN

SURVEYING") and generally be careful to make the field book, as much as possible, a *fac simile* of the ground itself, with each post, hedge, house, &c., placed on the book, as to the central column, considered always as a line, in the same position as they stand to the chain on the ground. And think not that any time is gained to the Surveyor by hurrying over the notes in the field. *A little care in the field will save much trouble in the office.*

Stations are generally expressed in the field book by the following character △, which, in the plan, is represented by a circle in pencil, drawn round the station point, which should be always that of a needle.

I would never recommend the use of letters for stations in Chain Surveying. In the first place, they are soon exhausted ; in the second, they in no way assist the memory. The BASE LINE, perhaps, had better, when referred to, be termed the *base line* AB, in contradistinction to the secondary lines, which are required in surveys of some extent, and are virtually base lines to their own portions of survey, and may be lettered CB, EF, &c., &c.

In all other cases, distinguish the lines by their lengths, and the points upon them, by the distance of those points from the zero end of the lines ;—thus, in the *first* example of the method of keeping the field book for chain surveying, "*from* 609 *on* 609, *to* 0 *on* 609," the line begins at 609 on 609, and runs to 0 on 690—that is, the measured line is a line, connecting the end of the line 609 with the commencement of the line 690.

And again, in the *second* example, "*from* 685 *on* 731, *to* 574 *on* 635," the point started from, is that point upon the line 731, which is 6 chains 85 links from the zero end of the line, and its termination, a point 574 on another line 635.

In *theodolite* surveying, it is better perhaps to use letters (see example), as the stations are but few, and mostly come within the exception above referred to, letters, in this case, as being usually applied to trigonometrical stations, may be, therefore, more characteristic and distinctive.

Surveyors sometimes take the bearing of the base line at the commencement of a survey, and enter it at once in the field book ; this enables them to plan the estate in reference to the meridian line. It must be remembered, that this is but the magnetic bearing, and must be entered as such.

The correction for the variation* must be subsequently determined, and the true bearing of the line inserted in the book.

EXAMPLES OF FIELD NOTES.

(*To be read upwards.*)

Chain Surveying.

	6·90	to 574 on 635
—D—	6·79	— ×
10 + 1	700	
10 + 5	500	
10 + 4	400	
10 + 3	300	
10 + 3	200	
10 + 2	100	
D 10 + 0	0·50	
D 7	0·00	D 3
from 685 on 731		

EXAMPLE II.

Maiden { —	7·31	— × } Lane
—	6·95	— ×
top of bank —	6·85	— × △ to 0 on 690
10 + 20	6·70	
10 + 30	6·00	
10 + 36	5·00	
10 + 70	1·00	
90	0·10	
from 609 on 690		

EXAMPLE. I

Theodolite Surveying.

	20·47	△
path —	13·21	— ×
hedge —	11·40	— ×
D —	11·20	— ×
	11·11	△
	2·00	D 41 + 12
H —	1·26	— ×
D. of Rd. —	1·10	— ×
to G.P. 33	0·80	
Road —	0·02	— ×
between 0 on 1077	123°23′	and 20·47 on 20·47
at 1077 on 1077		

EXAMPLE II.

	82° 55′	and △ A
	72° 24′	and △ E
between △ C at △ B	68° 28′	and △ D

	100°20′	and △ B
between △ D at △ C	14° 53′	and △ A

	180° 0	and △ C
	168° 49′	and △ B
between △ E at △ D	89° 42′	and △ A

from △ A	9·13 chs.	to △ B

	151°59′	and △ E
	86° 27′	and △ D
between △ B at △ A on base line AB	11° 32′	and △ C

EXAMPLE I.

of this subject, see "chapter on the

Surveying by the Circumferentor.

	17·84	to 0 on 755
	6·00	25 to fence
	250	30 to fence
	S.63·45W.	
	15·41	0
pond { 60+10	12·00	20 to fence
pond { 50+12	10·00	15 to fence
	2·00	10 to fence
	N.6·15E.	
	8·82	4
	1·00	·28
	S.75E.	
	7·55	6 to fence
	3·50	16 to fence
from corner	0·00	3
of field near	S.49°30′E.	
road		

As it is indispensable, for the proper keeping of the field book, that the common general and local divisions of property should be understood, I have subjoined a few remarks on that subject.

THE BOUNDARIES OF PROPERTY.

The usual boundary of the field, where the ditch is between it and the hedge, is the brow of the ditch, or that edge of the ditch which is furthest from the hedge. This, however, is not always the case, as sometimes it is the stem of the quickset, or the roots of the hedge; depending upon local custom.

The common allowance from the quick root, for the brow of the ditch, varies, in different places; being 4½ links, 6, 8, and sometimes as many as 9 links.

When between fields, it is ordinarily 5 links; when the ditch separates two contiguous properties, 6; and adjoining waste lands, moors, commons, roads, &c., generally 7 links. A wall is generally the division line between the properties, on which side soever the ditch may be placed. When there is a boarded fence between two fields, the fence belongs to the occupant of that side where it is clap-boarded, as the nails are considered to be driven home.

The *centre* of a stream running between two properties, is usually the boundary line.

In most parishes in England, a parish ditch is the boundary; the course of this ditch having been altered by time and circumstances, will account for the apparent freaks of the division of properties, which is so striking in the map; running across fields, dividing a pond, and putting one end of a street in one parish, and the other end in another.

It is particularly important in taking the notes of any survey, whenever any of the measured lines cross, or come near any of the division lines of the adjoining property, that these division lines should be noted in the field book. It at once localises the estate surveyed, and is oftimes of great use to the Surveyor afterwards.

NOTE.—[It were needless to observe, that, whether the contents of the ditch are, or are not, to be taken into account, depends upon the object of the measurements. No one would think of *including* them, in the measurement of a crop of wheat or barley, or of *omitting* them, when the area of each field is required, for the determining of the area of the whole.]

CHAP. V.

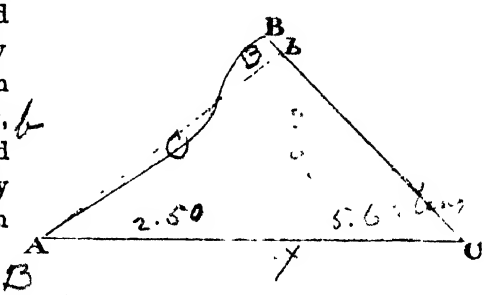
SURVEY OF A SINGLE FIELD.

Triangular.

HAVING given a description of the chain, offset-staff, &c., together with the method of ranging correctly, and the best plan of keeping the field book, we will now proceed to the several plans adopted in actual survey, according to the circumstances.

Let ABC be a triangular field to be surveyed.—The first thing to be ascertained, is, whether a plan is required, or whether the area alone is sufficient.

If only the area, observe whether the sides AB and BC are sufficiently straight and regular to warrant any point *b* to be taken, from which straight lines, drawn to A and C, would approximate sufficiently to the area of the given triangle ABC.

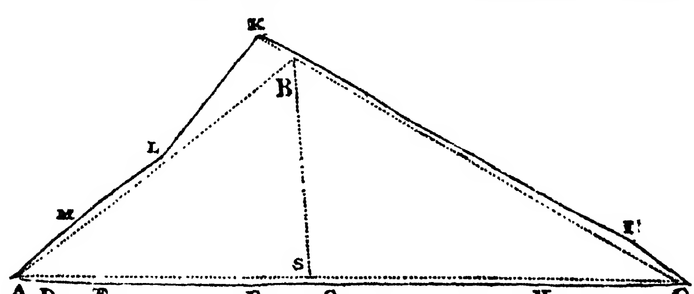


FIELD BOOK.

OFFSET.	CHAIN.	OFFSET.
perp. from 260 on 5·60	3·15	to B.
	5·60	to C.
△	2·50	
from △ A		

Having carefully selected the point *b*, it will only be requisite, in measuring along AC, to observe where a right angle from (b') would fall upon AC, note in the field book 2·50; leave a mark in the ground, and measure to C $=$ 5·60. Draw a line in your book, and measure from 2·50 to $b =$ 3·15; then $\frac{1}{2}$ (3·15)·(5·60) $=$ 8·82 square chains $=$ area of triangle ABC.

If a plan be required, or the hedges AB and BC be so irregular as to require offsets being taken, it will be requisite, for the sake of the *offsets alone*, to measure round the whole field along the three sides. Let the field now assume the form in the accompanying diagram:—



select any points A, B, C, commanding the longest and nearest lines to the several hedges of the field, and place flags at those points A, B, and C.

ΔS FIELD NOTES. ΔB

From 5.70 on 12·73.	4.25	to 8·50 on 9·27
×	6·80	0 + D (to 0 on 12·73)
	5·26	8 being Δ A
Δ	3·20	4
from 8·50 on 9·27.		
ditch—	9·27	— crosses 4 + D
Δ	8·50	3
	3·20	3
	1·20	14
	0.00	0 + D
from 12·73 on 12·73.		
D—	12·73	— × 0 + D to Δ C
	10·18	4 + D
	6·15	6
S Δ	5·70	4
	4·82	15
	1·60	5
	·56	4 + D
from Δ A	·00	0 + D

First observe, whether the point A be exactly at the ditch; if it is, put in the centre line 0, and in the right hand offset line 0; proceed till you come opposite 56 links, the first corner or bend D; take the offset (4) to D; this line, connected with A, gives the actual position of the ditch. Had there been no remark at 0, on the line AC, there would have been nothing to show where the ditch went to from the point D; proceed along the line AC, marking down the several lengths, as marked above, with their respective offsets, and selecting some point S, 5·70, to be noted as a station, whence to measure afterwards, a check or tye-line to B; note it Δ (a station) in the left hand column, as the tye-line will be to the left. Having completed the measurement of the line, put *where to* in the right offset column at the top, and draw *one* line over the whole; all *loose* lines, that is, lines not connecting certain points, have *one* line only drawn over them; *fixed* lines have *two*. Measure next from C to B, write from 12·73 on 12·73 on the left offset column in your field book, and observe, as before, what the position of the ditch is; it will, of course, be 0, as before, as the station was at the end of the previous line, and the line was measured up to the edge of the ditch, which is denoted by the ditch crossing, thus; ditch—(12·73)— ×

A B

C

and mark BA—the direction you are going in—whether to the right or the left, CB being the direction you have just come: BA the way you turn (to the left); proceed, therefore, measuring along CB, and marking the offsets, till you come to B, which enter in your field book 8·50 △, proceed with the measurement to the hedge, making 9·27; put down in your centre column 9·27, offset 2 to right, and draw a line, thus:—

hedge——(9·27)——× *i.e. hedge crosses.*

From B measure to A, and as B is 8·50, write in your book, in the left hand column, from "△ 8·50 *on line* 9·27," and proceed as before to A, which is a point 0, or zero, on line 12·73, mark this on the right hand column; and, as this completes the triangle, draw *two* lines above it, instead of one.

Then, from point 5·70 on line 12·73, measure carefully the tye-line to △ B, being 8·50 on 9·27, as the whole check of the accuracy of the measurements of the two sides AB, BC depends upon the correctness of SB; and draw *two* lines over it in the field book, writing *check* line across it.

Note.—[The reader's attention had here better be directed to the object, had in view, in the measurement of this line BS; it is not for the sake of determining the correctness of the point S, but that of the lines AB and CB. If either of those lines be made longer than it really is, the point B, on the line BS, correctly measured, would be found in the plotting to fall within the triangle; if shorter, it would fall without; but the position of the point S might be materially altered, without affecting the correctness of the length of BS. Yet the reason of the inutility of the line BS, as a check upon S, becomes the cause of its efficiency, in verifying AB and BC, as, whether S should be 5·90 or 5·50, when it is called 5·70, is not of much moment, as the *length* of BS would scarcely be affected by it.]

It is sometimes necessary and always desirable, that the field notes should be sufficiently distinct, as to show at once, when referred to, on which side of the ditch of a field the hedge is. The black line on the plan is the *ditch* line—the hedge line is seldom noted; to obtain the ditch line, when the hedge is between the measured or chain line, and the brow of the ditch, the offsets would have to be measured through the hedge to the brow of the ditch. To do this at every offset, would be too troublesome; the plan, therefore, generally adopted, is, at the first offset, to measure up to the roots of the hedge (say 12 links), and then from there, to the brow of the ditch ($+\overset{D}{8}$), and assume this ($+\overset{D}{8}$), as the average distance, to be added to the hedge offset, throughout the line.

This plan furnishes us at once with a simple method of determining the position of the hedge, thus :—the offset (12 + D) shows, that the hedge is *beyond* the field; 12 being *up to* the brow of the ditch, denoted by (+ D)—the distance from there to the roots of the hedge not being wanted, as not belonging to the field under measurement. And the offset $(12 \overset{D}{+} 8)$ shows the hedge to be within the field, as the offset, or distance of the chain from the boundary or brow of the ditch, is not 12 links, but 12 + 8 or 20; the distances being taken separately for the reasons above.

By adopting this plan, the Surveyor can at once define the position of the hedge. When several fields are measured, and the lines cross from one field to another, it is useful to mark, both where the hedges and the ditches cross.

To find the areas.

In finding the *area* of the field, there are two or three practical methods indifferently adopted.

THE FIRST METHOD.

That of dividing the field into a triangle (whose sides are the measured lines,) and into trapeziums, (whose heights are the measured offsets,) as being the most correct, though certainly the most tedious and troublesome, shall be the first described.

NOTE.—[The example is worked out fully for the use of the student, who may choose out of the three methods for himself.]

EXAMPLE 1.—To find the area of the previous figure from the field notes.

1. To find the area of the triangle ABC.

$$
\begin{array}{rl}
AB = & 6{\cdot}80 \\
BC = & 8{\cdot}50 \\
AC = & 12{\cdot}73 \\
2\,| & \underline{28\ 03} \\
& 14{\cdot}01 = S = \text{semi-perimeter.}
\end{array}
\qquad
\begin{array}{rrr}
\multicolumn{3}{l}{\text{Area} = \sqrt{S\,(S-a)\,(S-b)\,(S-c)}} \\
14{\cdot}01 & 14{\cdot}01 & 14{\cdot}01 \\
\underline{6{\cdot}80} & \underline{8{\cdot}50} & \underline{12{\cdot}73} \\
7{\cdot}21 & 5{\cdot}51 & 1{\cdot}28
\end{array}
$$

log. 14·01 = 1·146438
log. 7·21 = 0·857935
log. 6·51 = 0·741152
log. 1·28 = 0·107210
2 | 2·832735
log. 26·69 chs. = 1·426368

not field

10 | 26·69
A. 2·669
4
2·676
40
27·040

ABC

Area of triangle = 2. 2. 27. (A. R. P.)

TO FIND THE AREAS OF THE ~~TRAPEZIUMS~~ of Tra. half

On line AC.

Rule.—Multiply the lengths, by half the sum of the heights, for each area.

Lengths	Perpend. Heights.
56	0 / 4
104	4 / 5
322	5 / 15
88	15 / 6
45	6 / 6
403	6 / 4
255	4 / 0

·56 × ·02 = ·0112

1·040 × ·045: 5200, 4160 = ·046800

3·22 × ·10 = ·3220

·880 × ·105: 4400, 8800 = ·092400

·45 × ·06 = ·0270

·403 × ·05 = ·2015

2·55 × ·02 = ·0510

·0112
·3220
·0468
·0270
·0924
·2015
·0510
·7519 chains.

10 | ·7519
·07519
4
·30076
40
12·03040

Areas = 0. 0. 12. (A. R. P.)

On line BC.

Lengths.	Perpend. Heights.
1·20	0 / 14
200	14 / 3
530	3 / 3
·77	4 / 3

1·20 × ·07 = ·0840

5·30 × ·03 = ·1590

2·000 × ·085: 10000, 16000 = ·170000

7·70 × ·035: 3850, 2310 = ·026950

·0840
·1590
·1700
·0269
4·399 sq. chs.

10 | 4·399
·04399
4
·17596
40
7·03840

Areas = 0. 0. 7. (A. R. P.)

On line BD.

Lengths.	Perpend. Heights.
3·20	17 / 4
2·06	4 / 8
1·54	8 / 0

8·200 × ·405: 16000, 128000 = 1·296000

2·06 × ·06 = ·1236

1·54 × ·04 = ·0816

10 | 1·4812
·14812
4
59248
40
23·69920

·1236		
·0816		
1·2960		A. R. P.
1·4812 sq. chs.	Areas =	0. 0. 23.

	A.	R.	P.
Area of triangle =	2.	2.	27
Trapeziums on AC =	0.	0.	12·03
on BC =	0.	0.	7·03
on BD =	0.	0.	23·69
Area of field =	2.	3.	29·75 acres.

EXAMPLE 2.—Take, for practice, the field notes of the Fields Nos. 1 and 2, Plan No. 1, at pages 78 and 80, and calculate the areas, checking them by the following method.

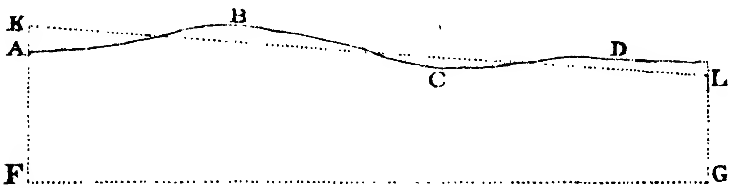
NOTE.—[Though this method is seldom requisite in small surveys, yet it is important that the reader should be acquainted with the best means of ensuring accuracy in cases of importance.]

THE SECOND METHOD.

A common method in practice, in computing the areas of irregular-sided fields, is to have a piece of transparent horn—and by *giving* and *taking*, as it is termed—to draw a straight-sided polygon, equal to the given irregular one, and, by means of the compasses and scales, to measure the lengths of the new lines, and from these lengths to calculate the area. This is a sufficiently close approximation in skilful hands; though I would counsel young beginners not to attempt it at first, until, by computing the area, by means of triangles and offsets, on several occasions, they have had opportunities of testing their own correctness.

EXAMPLE.—Let ABCDE be the irregular outline of a hedge or ditch, by placing along it a straight-sided piece of transparent horn; the position of the line KL can be determined, such that the area KLGF shall be

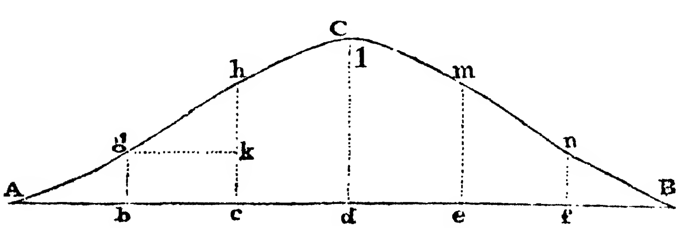
equal to the area of ABCDEFG; so that the pieces, taken in at A and C, shall be equivalent to the pieces given up at B and D.



NOTE.—[I would recommend the beginner to find by this method the areas of the preceding figure, and of the fields in the several figures in the work, comparing them with the calculated areas.]

As it is sometimes desirable to calculate in the field, without plotting, the area of an irregular figure, which has been surveyed by the circumferentor; and, as the space included within the lines of survey can always be calculated separately, without reference to the areas between these subsidiary lines and the natural boundaries, I subjoin an useful practical method of determining this area, when the offsets are supposed to be taken at regular distances.

To approximate to the areas of offsets, let AB be a straight line, near the curvilinear hedge ACB, and let it be required to determine the area, included between the line AB and the hedge.



In measuring from A to B, take at every chain's length, *Ab*, *bc*, *cd*, &c., the offsets *bg*, *ch*, *dl*, *em*, &c.; then the whole area ABC, shall

be equal to the sum of $gb + ch + dl + em + fn$, multiplied into their common distance of one chain, or 66 feet.

For the area of $Abg = \frac{1}{2} Ab.\, bg = \frac{d}{2}(bg)$, where $d =$ the distance.

$$gbch = bc\left(\frac{bg + ch}{2}\right) = d\left(\frac{bg + ch}{2}\right) = \frac{d}{2}(bg) + \frac{d}{2}\, ch$$

$$hcdl = cd\left(\frac{ch + dl}{2}\right) = d\left(\frac{ch + dl}{2}\right) = \frac{d}{2}(ch) + \frac{d}{2} dl$$

$$ldem = de\left(\frac{dl + em}{2}\right) = d\left(\frac{dl + em}{2}\right) = \frac{d}{2}(dl) + \frac{d}{2}(em)$$

$$mefn = ef\left(\frac{em + fn}{2}\right) = d\left(\frac{em + fn}{2}\right) = \frac{d}{2}(em) + \frac{d}{2}(fn)$$

$$nfB = \frac{fB}{2}(fn) = d\left(\frac{fn}{2}\right) = \frac{d}{2}(fn).$$

$$\therefore \text{Area} = d\,(bg + ch + dl + em + fn).$$

EXAMPLE 1.—Let $d =$ one chain, and bg, ch, &c., respectively 10, 15, 17, 14, 9 links, what will be the area of the figure?

```
  10                 ·65 sq. chs. = ·065 acres.
  15                                  4
  17                               ·260
  14                                 40
   9                      Poles  10·400
 ·65
   1
 ·65 square chains.
```

EXAMPLE 2.—Given the several offsets 15, 25, 40, 10, 60, 30, 25, 8, 18, 9, 8, 4, 6, 0, taken at one chain's distance. Required the area.

	A.	R.	P.
Ans.	0.	1.	0.

TO MEASURE A FOUR-SIDED FIELD.

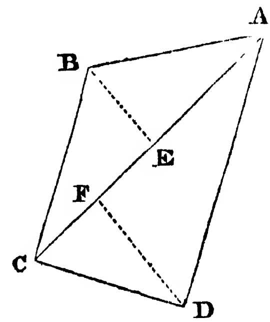
This must, in all cases, be divided into two triangles. If *the boundaries are irregular*, each hedge must be taken by means of a subsidiary line and offsets, and a diagonal line, drawn from the two opposite corners, that are *most remote* from each other.

If the boundaries are *regular*, the diagonal line alone need be measured, together with the lengths of the perpendiculars to each of the other corners.

To compute the Area.

Find the area of each of the triangles that compose the trapezium, separately, adopting one or the other of the different methods of computation before enumerated, and their sum is the area required.

Let ABCD be the given trapezium, and first let it be straight-sided; measure AC and BE, and FD then the area will be equal to $\frac{BE+FD}{2} \times AC$; for it is equal to $AC.\frac{BE}{2} + AC.\frac{FD}{2} = AC.\left(\frac{BE}{2}+\frac{FD}{2}\right) = AC.\left(\frac{BE+FD}{2}\right)$



Now let the sides be *irregular.—It will be necessary to measure all round for the sake of the offsets;* one other measurement ties the whole in, and you have two triangles, ACB and BDC, upon a common base, CB. To prevent errors to young beginners, I would recommend them, in all cases, for some time, taking the tye-line from the base to the vertical angles, till they get into practice; thus, in the last case, in measuring from C to A, when nearly opposite (as well as the eye can tell) to D, take any station F; and, when opposite to B, any station E; after completing the diagonal CA, measure BE, FD; this will at once prove the accuracy of the measurements in the field of the several lines AB, BC, CD, AD, and will, at the same time, guard against any error in the plotting at home.

The following field notes, accompanied with a plan (being Field No. 1, in Plan No. 1,) of a four-sided field, near Maiden Lane, surveyed in the above manner, are given as an example of the method of keeping the notes in the field, and of plotting the work at home.

It were advisable that the student should plot every one of the examples of field notes *himself*, for practice, and compare them with the plans given.

FIELD No. 1, PLATE I.

From 230 on 873 check line	3·67	to 480 on 485
From 490 on 873 check line	4·71	to 574 on 635
	8·73	to 619 on 624
	4·90	△
△ from 34 on 485	2·30	
	7·16	to 34 on 485
10+24	7·00	
10+21	600	
10+19	500	
to gate post 22	392	
10+11	300	
10+ 7	200	
10+ 7	1·00	
to top of bank 10+8 from 574 on 635	0·00	
Maiden { D—	635	— × } Lane
D—	597	— ×
top of bank—	587	— ×
4	500	
4	400	
2	300	
D+2	200	
D+5	100	
to 2d gate post 12	0·08	
from 719 on 724		

70 40 D—	7·24	— ×
8	7·19	
7	600	
5	500	
6	400	
6	300	
to gate post 10	2·79	
D+4	2·00	
D+5	1·00	
from 480 on 485		
D —	485	— ×
+ 13	480	△
+ 28	400	
+ 32	300	
D 10 + 27	200	
D 10 + 28	1·00	
D 10 + 10	0·34	△
10 + 15	0·00	
D — Maiden		— × —Lane.

Commencing on top of bank by the road side, at the south-east corner of the field.

THE PLOTTING OF THE ABOVE NOTES.

First lay off the measured lines, independently of the offsets.

Draw any line AB, indefinitely in pencil, as your guiding line, so placed, as to throw the plan into a favourable position on the paper. Now, for the length of this line, by looking at the field notes you

will find that the whole line is 4·85; but there is a △ at 24, and another at 480; 480—24, or 4·56, is the length of this subsidiary line, forming one of the lines of triangulation; write in pencil 456 against this line.

The next line runs to the right, from 480 on 485, (which 480 from 0 is equivalent to 456 from 24,) and is 724 long; but the station is at 7·19; draw any line, making the supposed angle with AB, and mark 7·19 against it.

The third line also turns to the right, and though continued to 635, across Maiden Lane, has its station point at 5·74; mark this line also, in its supposed position.

The fourth line does not say to the right or to the left, because it runs from a station 574 on 635, the end of the last line, to a previous station, 24 on 485, which was the starting point; this line is 716 long; mark this distance against it.

The next measured line is from 24 on 485, or the starting point, and runs to another previous station, 619 on 624; its total length is 873, and there are two stations upon it, 230 and 490; draw this line in its proper position, and mark off the stations, 230 and 490.

The next two lines are check-lines, the one (471) from 490 to 574 on 635; the other 3·67 from 230, to 480 on 485; mark these also.

Having placed roughly these several lines, with their given lengths, in their supposed position, proceed to plot them off correctly by triangles, marking in every case, on the plan, the direction the line was measured in on the ground, (see plan). AB, of course, is the base of the whole. Upon AB lay off a triangle, whose other two sides are 7·19 and 8·73, laying off upon 8·73 the stations 2·30 and 4·900. Then, to verify the correctness of the work thus far, measure the distance of the station 2·30 from the point B; this, if the work be correct, should be 3·67, the length of the dark line.

Next, upon the line 873, lay off another triangle, whose sides are 574 and 7·16; the distance of the previous station 490, from the vertex of this triangle, should, if the work be right, be found 4·71. The whole field is now plotted.

METHOD OF SURVEYING AND PLOTTING TWO FIELDS TOGETHER.

The field in the previous example having been already surveyed, the adjoining field was added to it,

which is to be plotted from the accompanying field notes.

FIELD No. 2, PLATE II.

from 500 on 868 check line	5·35	to 685 on 731
	8·68	to 609 on 609
from 574 on 635	500	Δ
	690	to 574 on 635
├ D —	6·79	— ×
10 + 1	600	
10 + 5	500	
10 + 4	400	
10 + 3	300	
10 + 2	100	
to top of bank 10	0·50	0 to D
to top of bank 7	0·00	3 to D in field
from 685 on 731		

Maiden { D —	7·31	— ×	} Lane.
D —	6·95	— ×	
top of bank —	6·85	— ×	Δ
10 + 20	6·70		
10 + 36	500		
D 10 + 70	100		
..... 90	0·10		
from 609 on 609			
D 15	609	Δ	
6 + 4	5·82		
D + 4	500		
D + 10	400		
D + 16	300		
D + 22	200		
D + 27	100		
to gate post 40	0·50		
35 + 21	·32		
pond			
pond + 20	·12		
from 719 on 724		produced.	

☞ Having the previous notes of Field No. 1, the following notes were taken for the survey of the adjoining Field No. 2.

Produce the line 7·19 to 609 further, which is a station—then, turning to the right, agreeably to the field book, mark off the distance 685, which is the station point in the next line 731; from this point, 685, the distance to a known corner in the first field, (being 574 on 635), is 690.

Now, because 574 on 635 is a known fixed point, and the line 7·19 of the last survey, is a fixed line, its production is also fixed, and the end 609 is a fixed point; the line joining 5·74 on 635, and this point is, therefore, a fixed line without being measured; upon this base, therefore, describe a triangle, whose other two sides are 685 and 690, and their intersection is also a fixed point. By measuring the diagonal line 869, its distance, determined in position by joining two fixed point, is checked by its measured distance.

Again, by taking the distance 500, upon this diagonal, and measuring the check-line 535, this line, measured from a point in a fixed line, to the intersection of two other lines, becomes a check upon the vertex of the triangle.

Having found the plotted check-lines agree with the measured distances, draw the subsidiary lines carefully in ***red ink***, and proceed to lay off the offsets.

TO LAY OFF THE OFFSETS.

Take the first Field.

The offsets, on the line AB, are all to the left, mark off, therefore, the several offsets in their proper places, observing, in the case of the second offset, that $\overset{D}{10}+10$ is ten links up to the hedge, which, with the ditch, is ten links wide. This determines the position of the hedge to be *within* the field; without the $\overset{D}{10}$, or, had it been (D+10), the hedge would be *beyond* the boundary of the field. At 4·85, the cross ditch of the field intersects the line.

In the next line the offsets are still to the left, and are marked (D+5), showing that, in this case, the hedge is without the field. At 7·19, there is an offset of 8, to where the edge of the side ditch intersects the side of a pond; at 7·24 the cross ditch of the field intersects the line as before. The width of the pond 40 links, and its length 70 links, are marked.

In the next line, the offsets are still to the left, and the (D+5) shows, that the hedge, in this case, is beyond the field. As this line, if produced, will cross Maiden Lane, it is produced across for the purpose of determining its width and position. In every case, in addition to the mere measurement of the field, it is advisable to annex such collateral localities as roads, turnpikes, ponds, &c., as may determine the relative position of the field. At 0·08 there is an offset of 8 links (on the left) to the second gate post. In the following line 716, where the offsets are "still to the left" at 392, there is an offset of 22 to "gate post" only. Observe—that it is usual to take to the *first* gate post, and allowing the average width of gate to be 15 or 16 links, to determine the position of the gate by the direction of the line. It is difficult to fix the position without some fixed rule, such as the above, of always taking the *first* point of the gate, as you come to it, on the line, whence the offset is taken.

The first offset on the line 716 is (10+8), always on the left. This 8 is up to the bank, bounding the lane, and 10 is the distance to its top. This 10 is repeated throughout, the average width having been taken. The other three lines have no offset to them.

OFFSETS OF SECOND FIELD.

The first line is 609, and the first offset is at ·12 (still on the left), being (pond+20), or, 20 up to the edge of the pond+20 across; the second offset at ·32, is (35+21), that is, 21 up to the pond +35 across the pond, which, by the accompanying diagram, ends there; the distance ·32 being taken, in order to show that there was the end of the pond.

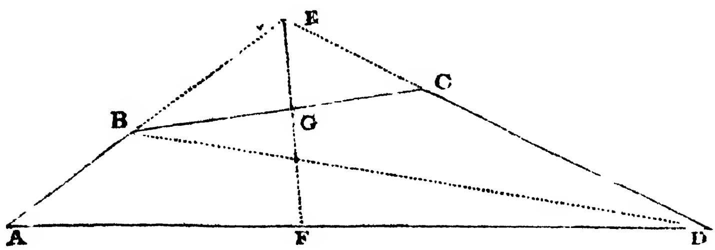
The next offset is at 50, that is, 40 to first gate post; at 1·00, the offset is (D+27), that is, 27 to the brow of the ditch, which being beyond the boundary of the field, the width of that and the hedge is not required; the other offsets are regular to the distance 5·82, which has an offset of $\overset{\text{D}}{(6+4)}$, that is, 4 up to the hedge, and 6 through the hedge, which is now within the field, to the brow of the ditch; the previous offset was (D+4), that is 4 to the brow of the ditch (the hedge being then beyond the field), the ditch therefore *changes* at this point, denoted in the diagram by . There is here also a cross hedge, on the left, which the diagram shows too.

In the next line, at ·10, there is an offset, taken to the corner of the field, ·90 links; and the relative position of the sides, to the offset line, is expressed by the diagram in the notes, which should ever be, as much as possible, a ground plan of the locality.

In line 690, the first offset taken is at the station point or 0·00, 7 to the left, and 3 to the right: showing, that this point is between the top of the bank by the road, and the *field* ditch at the bottom; at 50 links, the line touches the ditch, having 0 offset to it.

There are no offsets to the two following lines.

There is a form, however, of a four-sided field, which it would be dangerous to measure in the manner before referred to.



Let ABCD be the field. To measure BD as a diagonal, and to let fall perpendiculars thereon (supposing the field to be *straight sided*), would be troublesome, and might be productive of error; and, to divide the field into two triangles, with BD for their base, subtended, in the one case, by so obtuse an angle as C, and, on the other, by so acute an angle as A, would be equally hazardous. One good tye line, from C to BD, might be taken; but a tye line from A, except upon the production of DB, which might be impracticable, would be useless.

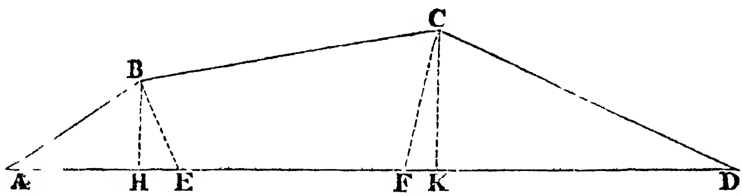
It would be better, according to the circumstances of the locality,—

1st.—Either to produce DC and AB to their intersection at E; and, taking all the requisite measurements, for the computation of the triangle AEC, take, at the same time, those for the area of BEC, and substract the smaller from the greater, for the required aréa.

Example—Let AD be the base measured, and take as a station, any point F, opposite to E; from A measure along AB, towards E; at the distance AB, in the field book, mark "*fence of field* crosses," (and also "out of field") and continue the measurement to E. From

E measure towards D, marking (out of field) till you come to C, where, at the distance EC, say fence of field crosses, and proceed to F. From the station at F, towards E, measure to CG, (where fence crosses,) and continue to E; measure CB.

2nd.—If the hedges be so high or thick, that you cannot range beyond—or, if there be any other local objection, I would prefer the following method:—



In measuring from A to D, take a point E, about the same distance from A as B is, and mark it a △; and at F, take any other point, making FD about equal to CD, marking it in the field book as △, and proceed to D; measure the other three sides, which must be done, for the sake of the offsets. Then, from E measure to B; and, from F to C, the points B and C become the vertices of two triangles, ABE, and FCD, whose bases, AE and FD, being given, as well as the lengths of their respective sides, the vertices B and C are given, and therefore the line BC, between them, without being measured, is given also; by measuring the actual distance between B and C, BC becomes a check-line. Other check-lines BH, and CK, to the several triangles upon the base AD, can be taken or not, as the importance of the survey may or may not render it necessary.

THE LINEAR MEASUREMENT OF AN ANGLE BY THE CHAIN.

It must have been observed by the student, that this method of chain surveying is but a practical application of Euclid's problem, (see Problem x, page 5,) "for constructing an angle, equal to a given angle,"

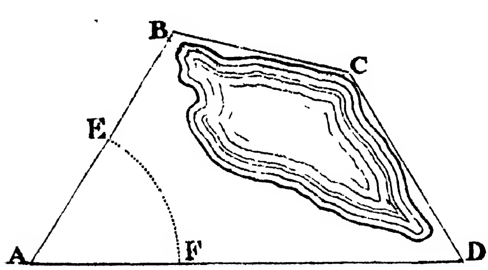
viz., by completing the triangle, and making a second triangle equal to the first.

In *chain surveying*, an angle is not measured by the number of degrees and minutes it contains, but by the length of the base that connects its sides, either completing the field (if it be of a triangular shape), or, as a diagonal, forming the base of another triangle, or of a new system of triangles.

The length and position of this connecting side, or linear measurement of the angle, should not be taken indifferently, but should be so selected, as to connect sides, as nearly equal as possible to each other, and to the line that connects them, forming an equilateral triangle, where possible, or an isosceles.

Again, when practicable, the largest triangle should be taken, though it is not actually necessary, as is shown in the following example. Field

Let ABCD, be the given ~~angle~~.



It is not actually necessary, to make AB, AD, the two sides of a triangle, whose base, BD, shall determine the measure of the angle at A. Any points on AB, and AD, may be taken at pleasure, as E, and F. The distance EF, will determine the angle at A, and the position of the sides AE, AF. These lines are but parts of the straight lines AB, AD; and, therefore, AB, AD, are determined; and the points B, and D, upon them.

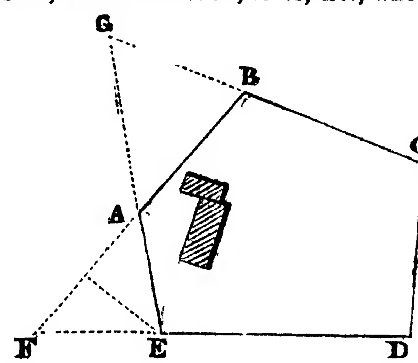
There might be many local obstructions to measuring from B to D, and AD might be disproportionately long to AB. By making AF

as nearly equal as possible to AE, you obtain a far more favourable measure of the angle at A than BD might be.

THE MEASUREMENT OF AN ANGLE, EXTERNALLY, BY MEANS OF THE OPPOSITE, OR SUPPLEMENTAL, ANGLE.

It sometimes happens, that an angle cannot be measured internally, from local obstructions, such as trees or buildings intervening between the sides that include it. In this case the following plan should be adopted.

Let ABCDE be any enclosure, such as a wood, river, &c., whose angles cannot be measured internally; or, a farmstead, which, from the position of the buildings, must be measured from without.



Assume the side AE, as the base line, and, for the sake of the offsets, measure all round the enclosure, taking the lengths of the sides, ED, DC, CB, BA. The nature of the obstructions prevents the measuring of the side AD, in the triangle ADE, which would give the angle AED; and, also, the length of BE, in the triangle ABE, to obtain the angle BAE. Neither of these angles, therefore, can be obtained in the usual way.

Though the angle BAE, however, cannot be *directly* determined, there are three different angles, from which it can be deduced, viz.:— its own opposite angle, GAF, or either of the supplemental angles, FAE, BAG. Some one or other of these may be free from local obstructions.

First, let the supplemental angle, GAB, be free; produce the range EA to G, where CB, also produced, intersects it; measure AG, AB, and GB; AGB is a triangle upon the base AG; AE is determined in position, and, therefore, its production AG, is so also; and GB, and AB, being measured, B becomes determined; and the line GB and its *Production* BC, and therefore C; but E is determined; an imaginary

line, EC, connecting them, becomes so also. This being the base of the triangle CDE, whose two sides, CD and DE, are measured, D becomes determined, and therefore the whole figure is determined, by measuring the supplemental angle GAB, making, in all cases, its sides, BG and AG, the *production* of the sides, CB and EA respectively.

For the purpose of *obtaining the offsets*, it would be necessary to measure every side; there are, therefore, but two, not subsidiary, viz., the supplementary sides, AG, BG, which have to be measured, *purely as such.*

The same plan can be adopted, with any number of angles, modified according to circumstances, and having always regard to the using of as few supplementary lines as a due attention to accuracy will permit.

CHAP. VI.

SURVEY OF FIELDS OF MORE THAN FOUR SIDES.

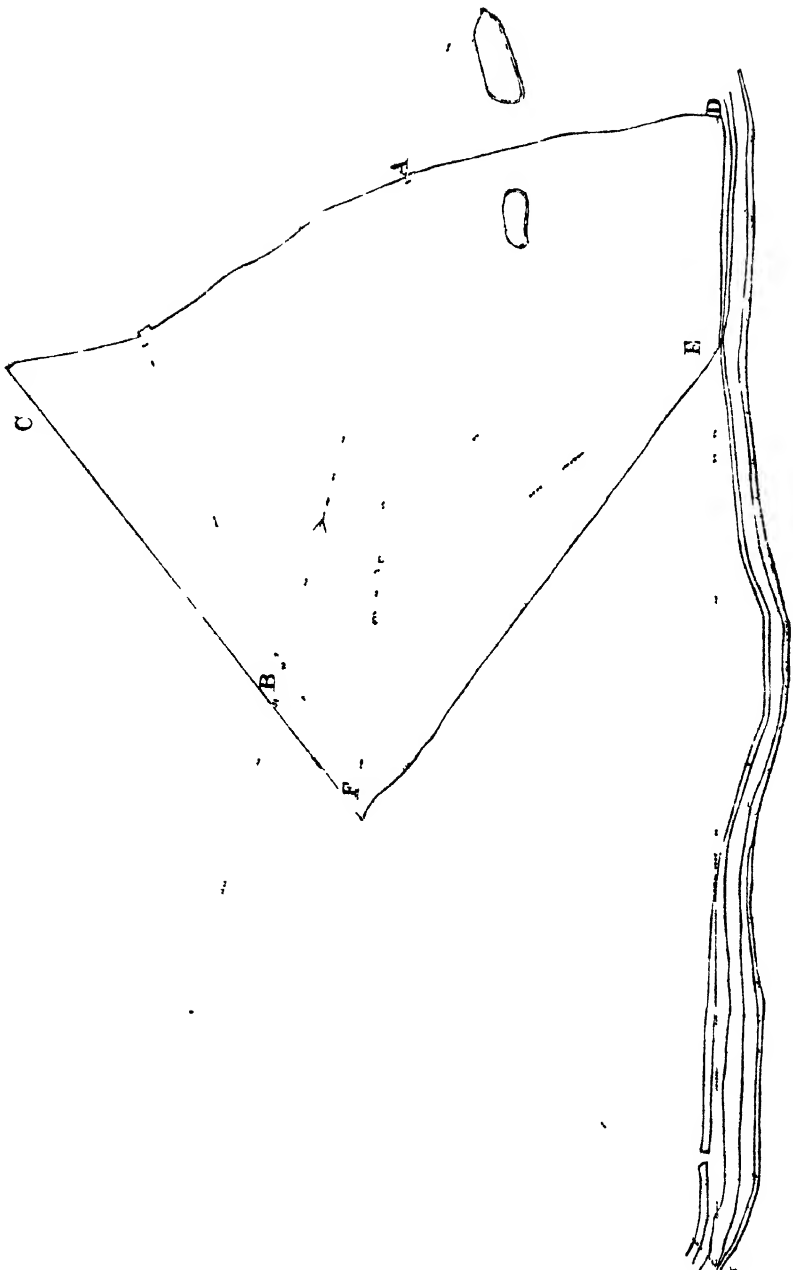
Many-sided fields are of so many various forms, that it is impossible to give any specific rule for all.

The chief point in this, as in most cases, to be secured, is, that as the *whole circumference* of the field *must* be measured for the *sake of the offsets*, a selection should be made of the fewest and the most favourable supplementary or diagonal lines, with the best points for check-lines, to the corners.

The following field notes, of a five-sided field near Highgate, are given as an example, which the student is recommended to plot for himself, and compare with the accompanying plan.

From 608 on 608	6·80	to 11·97 on 11·97
	6·08	to 2766 on 3400
pond 70 + 76	4·55	
70 + 75	425	
	400	22
	3·00	30
	2·00	20
	1·00	21+D
from 456 on 462		
D—	4·62	—×2+$\overset{H}{8}$+R
△	4·56	
	2·00	10
	1·90	12
	0·26	11 to G.P.
	0·10	10 to G.P.
	0·00	10+$\overset{H}{8}$+R
from 1197 on 11·97		
△	11·97	10 >
	11·50	18
	7·00	18
	6·00	15
	500	15
	300	18
	2·00	20
	1·00	20+B
	0·48	12 to G.P.
	0·32	12 to G.P.
	0·00	12+D
from 10·10 on 10·32		
D—	10·32	—×o+D
△	10·10	
	8·50	
	7·46	to16,63 on B.L.
path+40	3·00	0
	2·00	0
	0·00	2+D
from 8·67 on 8·67		

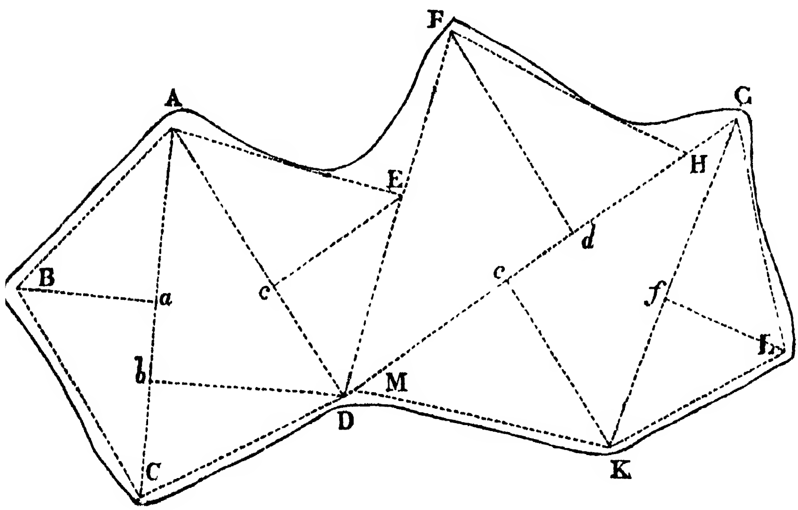
△	8·67	110 } gateway
	8·43	106 } gateway
	7·00	40
	6·08	24 to stile
	6·00	24 to stile
	5·00	17
	4·80	20
	4·00	24
	3·00	25
	2·00	30
	1·00	22
	0·00	10+D
from 27·66 on 34·00		
to dwarf pollard blazed } 2	34·00	△
path—	33·60	—×
	31·40	pond 40+10
	31·00	pond 40+60
	30·00	pond 100+60
D—	27·80	—×
	27·66	△
	20·00	△
path—	17·90	—×
D—	16·63	—× △
BASE-		LINE.


A
B
C
D
E
F
G

I have subjoined, for the practice of the student, a few examples of the best methods that might, under different circumstances, be advantageously adopted.

Example 1.

In the survey of many-sided fields, it must be remembered, that the whole field can always be divided into as many triangles (less two) as the figure has sides. Thus in the annexed diagram,



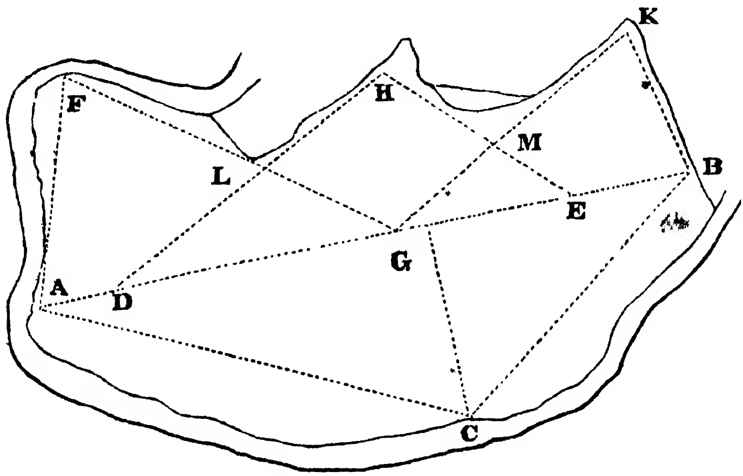
the whole figure ABCKG, by drawing the diagonals, AC, AD, FD, DG, and GK, is divided into triangles.

I would not, in most cases, recommend this method, for it is violating one of the first principles of surveying, viz., that of working from *whole* to *part*.

Instead of confining the whole field within the limits and error of one triangle, this method not only depends upon a *number* of triangles and the errors of each, but the error of any one of the triangles is not confined to itself, but is carried through the whole. Thus the triangle GKL, (being correct in itself,) depends upon the correctness of the triangle GMK; the two triangles, GKL and GMK, upon the triangle DFH; DFH upon AED; the triangle AED upon ADC; and ADC, and all the preceding, upon the correctness of the one triangle ABC.

Example 2.

In the survey of the Field, in the accompanying diagram, a different arrangement has been adopted.



One base line, AB, has been taken through the centre of the field, and the various triangles have been based upon it, determining the several points required: each point in this arrangement depending solely upon the accuracy of the admeasurement of the triangle to which it belongs.

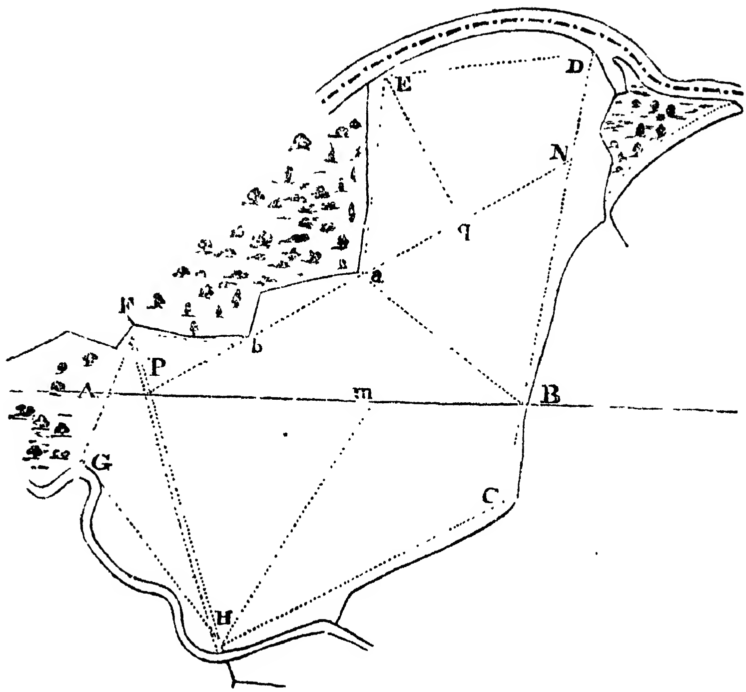
For example, take the point C, by the side of the river—this point is the vertex of the triangle ACB, whose base is the base line AB, and whose accuracy is secured by the check-line GC. The points F, H, K, also, are the vertices of the triangles, AFG, DHE, GKB, whose bases are respectively AG, DE, GB, portions severally of the base line AB. The distances DL, ME, not being required for the determining the vertex H, become check-lines upon all three.

The base line AB, and the check lines GC, LG, ME, are the only supplementary lines required.

Example 3.

The next example is that of a very irregular field (selected for practice sake from the centre of a survey),

through which the base line (AB) runs in a certain direction, which direction is required to be known.



Through B draw any *loose* line CBND, so placed as to command the offsets along the whole of that side of the field, selecting upon it such a station N, as to be in the same straight line with P *b a*, produced. The distance of the line PN fixes CD, and PN itself; the other side of the triangle is also fixed, and the point *a* is a fixed point, and therefore the line *a*B. By *measuring a*B, the correctness of the whole triangle is determined.

Again, *a* being a fixed point, as well as D, an imaginary line, joining them, is also fixed, which is the base of the triangle E*a*D; by measuring the sides *a*E, ED, the position of E is determined; the line E*q* is a check-line in this case; thus far E*q*, *a*B, and *a*N are the only supplementary lines required.

As *b* and A are fixed points, measure *b*F and AF, and you obtain the point F; the only supplementary line required being P*b*.

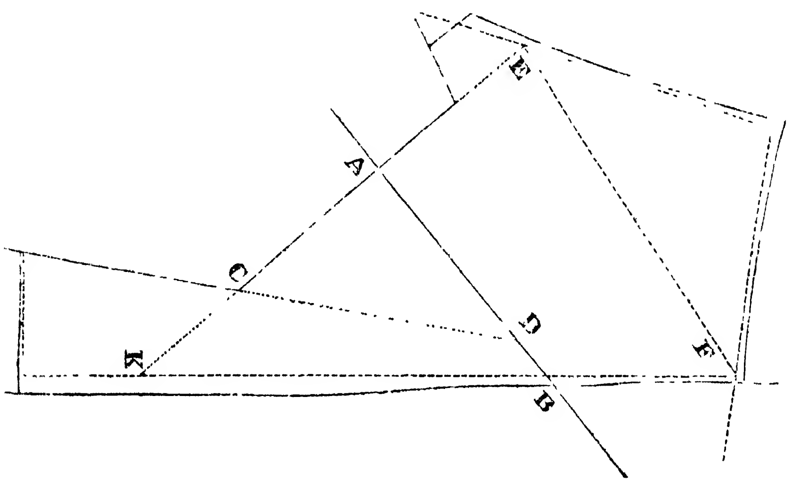
By producing FA to G, G becomes a fixed point; but C is a fixed point; the imaginary line of connection, CG, is also fixed; and the

measurements GH, HC, determine the position of H. All these lines are required for the offsets; by measuring, along the footpath, the line HP, you have a check upon the whole of this latter part of the work.

All the supplementary lines, required for the survey of the whole field, are P*b*, *a*N, E*q*, *a*B, and PH.

Example 4.

This example is that of a field under similar circumstances, of a "base line running through it," but the general position of the field is different, and a different arrangement required.



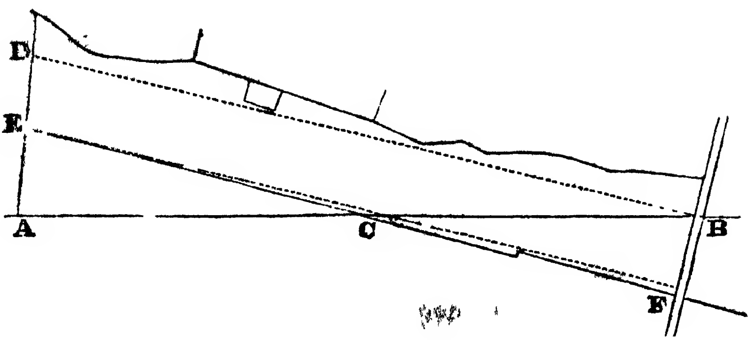
The same principle is still kept in view, of making the sides of the triangles serve the purpose of taking the offsets.

In this case there is but one supplementary line CD, and two check-lines, BF, and CK, or lines of verification to the whole.

Example 5.

This example is that of a long narrow slip, crossed at one corner by the base line.

The best method of surveying it, if circumstances will allow, is given in the accompanying diagram.



By measuring CF and FB, on the base CB, CF is determined as to its relative position to CB, for the side BF measures the angle BCF. Produce FC to E; the point E is determined, and the supplementary line EB. By measuring ED and BD, the other two sides, they both become fixed lines also.

As the accuracy of the whole, however, depends upon the correctness of the measurement of the angle BCF, it would be advisable to produce DE to A, and measure AC, and then obtain the measures of the opposite angle ACE, as a check upon the other.

CHAP. VII.

TO SURVEY SEVERAL FIELDS TOGETHER.

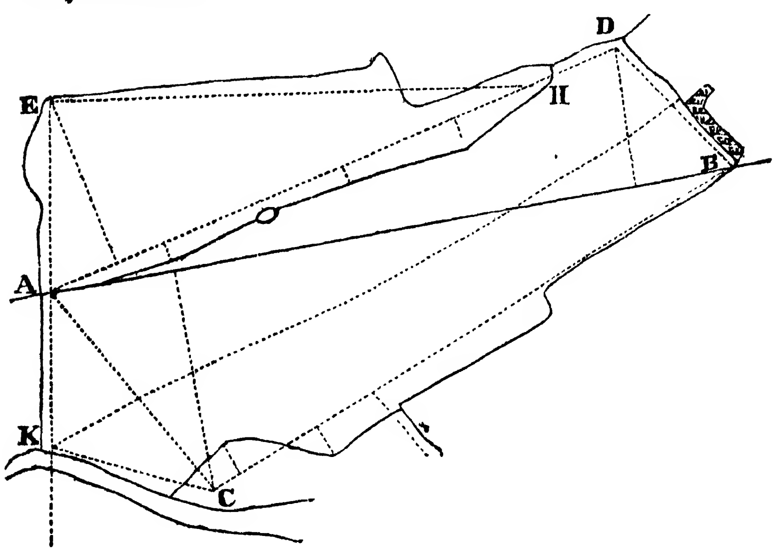
Walk over the ground first, and carefully examine into the relative position of the fields, and the most eligible points, for fixing the main and subordinate triangles, having reference always to making the lines of the triangles *subsidiary* to the measurement of the hedges; at the same time, taking care, that these triangles are nearly equiangular, right angled, or isoceles; and that they are bounded by fixed points, such as posts, stiles, houses, corners of fences, &c. There should be as few as possible supplementary lines: lines

that are merely used in determining the triangles, and do not also serve for taking the offsets.

I subjoin a few examples of this kind, with field notes (of the two last), plans, and descriptions of the mode adopted in making the survey, and the reasons for selecting and arranging the several lines.

EXAMPLE 1.

In this case, there are two contiguous fields irregularly bounded.



Let the base AB (which may be taken arbitrarily) pass in a diagonal direction through the corners of the larger field.

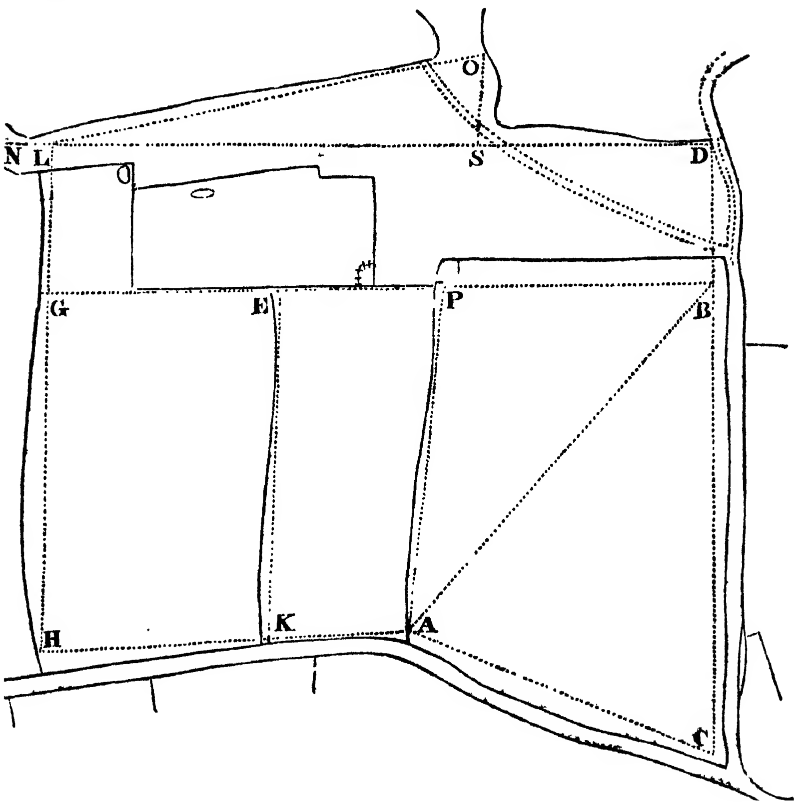
By basing upon AB, the triangles ACB, ADB, you obtain the position of the three hedges of the larger field, one of which is common to the adjoining field.

By basing upon AH, the triangle AEH, the whole of the second field is measured, and the side EA is a fixed line; its production to K is also fixed; and, as C is fixed, being the vertex of the triangle ACB, the line KC, without measurement, is determined; when measured, *which it must be for the sake of the offsets to the river*, its *measurement* is a base of verification to the whole.

Check-lines had better be taken in each of the triangles, for the sake of correctness. These, from the absence of offsets, are soon measured, and add very little to the trouble or time of the survey.

Example 2.

This is a case of four or five adjoining fields. Take a base line AB; upon AB, describe the triangle ACB; produce CB to D.



Upon the other side of AB, describe the triangle APB; produce BP to G.

The imaginary line AG, being a fixed line, describe upon it the triangle AHG; produce HG to L; join DL, and produce it to N; and upon LS, a portion of the line DN, describe the triangle LOS. By taking proper offsets along the several lines, all the fields will be surveyed.

The methods above given, the student must bear in mind, have been supposed *free* from those *local obstructions* which will often materially change the arrangement. He must keep in view the difference he will find in laying down a method of survey, *when all the plan is before him, and he has a bird's eye view of the whole, which is supposed to be perfectly flat and free from every obstruction*; and, when, ignorant of the locality, unable to comprehend its content at a glance, he has also to contend with the local difficulties of hills, vallies, woods, rivers, houses, &c.; in fact, of all that constitutes, in all cases, the difference between theory and practice.

NOTE.—[The various methods of overcoming these difficulties will be found fully explained in Part the Second.]

CHAP. VIII.

SURVEY OF A PORTION OF A PARISH.

EXAMPLE.—This example is that of a larger portion of country, comprising some 80 or 100 fields, 6 or 8 roads, a village, river, canal, &c., and a line of rail road running through a corner of it, towards the village. It is an actual survey of a portion of the county of Derby, in the parish of Willington.

A base line AB has been selected, contiguous to an intended line of railway, and running through the whole of the survey, intersecting the road HC, at C; the road GK at D; and the road MO at E, which distances are carefully marked in the field book. Where the base line crosses the hedges also, at the most favourable place for running cross lines along the hedges, stakes must be put in, and the points carefully noted, taking offsets *en route* to any hedges or corners of hedges, fences, or other objects, that may be within an offset distance.

The method actually adopted would be, in order to avoid any needlessly going over the same ground twice, to commence at A, measure AN, NVH, then HC, CV, and the fields within the block VCH, and NC.

From H, the next line measured would be HG, observing carefully where the best stations could be taken for the cross-hedges, on the same side of them, as the stations were selected in the base line.

Then measure GD and HD, observing, in measuring HD, to have the range of the line carefully defined, where the several hedges cross: so as accurately to define the several points in the line CD, where the cross hedge lines, from AB to HG, intersect. By this plan, all these cross-lines are check-lines.

Produce GD, to K, in the same straight line; measure KC, taking notice as before, where the cross hedges come, and on their proper side, and *complete* the block HGKC.

Then measure CL and LK.

Now return to C, and produce HC to F, where it intersects the base line; marking the several points P, Y, and X, upon it, and the several cross hedges.

Then from D produce HD to M; and from M, measure a line in range with EX, which produce to O; join OB; then complete the block GXMD, and the triangular piece KDM.

There now remains but the part adjoining the village.

From P, measure PRS, and join ST; then produce VR to *b*, and join *ba*; the lines *ba*, *a*T, TS, SR, R*b*, will tie the whole of the houses in.

This must always be the plan adopted in the survey of a village, or farmhouse, or homestead; to confine

all the areas within one triangle, whose three sides should severally pass through the principal points of the place.

Having given the method, adnpted in practice, for saving time in the survey of the plan, we will proceed to explain the nature and use of the several main lines.

The line ME, of the triangle DME, is the measure of the angle MDE; but CH, in the triangle CHD, is the measure of the opposite and equal angle CDH. Therefore the measured and the determined distance, agreeing or disagreeing, of either the side CH or ME, is a proof of the correctness or incorrectness of the angle at the vertex D.

Produce ME, the *fixed* line, to O; any points upon this production are also *fixed*. The point X, which is in a range with the road HP, is fixed; and H, being a fixed point, the lengths of HX is determined.

Its measurement becomes a line of verification to the opposite angle HMX, or (MDE, being supposed correct,) of the supplemental angle DEM: which is the angle that this new line MD makes with the base line. *The line OB, if the nature of the ground will permit its being measured, measures the opposite angle BED, and is another check upon its correctness.*

Again, MX is the measure of the angle MHX; and GD is also the measure of the same angle.

The actual distance of GD, compared with its computed or determined distance, is a check upon the correctness of the length of MX.

Having determined the correctness of these triangles, there can be no error of any moment in the *filling in*.

In fact, all the lines used for the measurement of the offsets to the cross hedges, are only so many additional check-lines to the triangles, or measures of the angles at their vertices.

EK being *determined* by the previous measurements, its measured distance is a check upon the angle CDK, and, therefore, upon the direction of the line KC, relative to the base line AB.

The correctness of the triangle, CLK, is secured by the common check-line to its vertex Lc.

The triangle AHC, having in AC a portion of the base line, depends upon the correctness of the measured distances AN and NC.

A being thus a fixed point, as well as H, measure the line NH, and as H has been previously assumed correct, HN is a measure of

the angle NCH, which is the supplemental angle to the two known angles HCD, ACN; the length VC is a check upon the distances CN and CH.

Now returning to the other parts of the survey, the line HZ, produced to the base line at F, is an additional verification *of the whole of the triangulation.*

To ensure a correct survey of the village, observe that the line MO*a* passes close to one side of it.

From P, drawing PRS through R, and joining ST, you have *known* lines *close to the village*, on *another* side; producing YR to a point *b*, such that a line *ba* shall pass close to the third side of the village, you surround the whole with a fixed triangle. All errors must be confined *within* this limit; and all lines, for the measurement of the streets or lanes, carried through to either of the sides of this triangle, are, as in the case of the cross hedge-lines, in the first part of the survey, virtually but so many corroborative checks of its accuracy.

CHAP. IX.

SURVEY OF FIELDS NEAR MAIDEN LANE.

Plate No. 3 is a plan of the point of junction of Maiden Lane, with the Junction Holloway Road, and of six or seven fields in one of the corners.

The extent of the survey was settled in the first instance, and the whole of the ground carefully examined, and the arrangement of the lines, as much as circumstances might afterwards allow, predetermined.

The plan does not take in the whole of the field notes, as the roads surveyed extended much further than the page would admit of being introduced; the notes are, however, retained, as the student will not, of course, be under similar restrictions.

FIELD NOTES OF SURVEY.

	6·34	to 16·61 on 1728
	6·33	15+p+21+15
	6·17	9+p+6+15
		D
	5·95	6+p+9+16
		D
	5·70	2+p+24+15
		D
	5·44	30+15
p+p—	5·38	—+
p+5	4·90	28 to end of fence
	4·51	12+waterfall
p+10	4·38	8+fence+D
	4·23	6+20
D—	4·22	—+ +D
D		D
13	4·16	20
D—	4·10	—+
D—	400	—+ D
D		
3	3·90	
p+11(D—	3·70	—+
	326	20+15
p+5	297	20+20
	2·32	20+15
path+10	200	30+15
	1·44	25+15
	1·23	33+20
		D
	0·94	28+15 to paling
path+9	0·70	
from 544		
on 609		

	3·54	to 583 on 609
	327	3
	300	4
	2·62	10 to paling
	2·50	10+ fence +8
	2·39	58+26 to G.P.
	209	88 to G.P.
	205	60
	1·93	23+18
	1·68	4+p+22+fence+(
path—	1·65	—+ +path
	1·00	9+10+3 to paling
from 860		
on 1067		

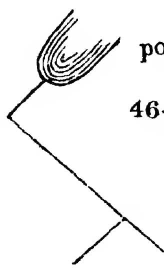
paling—	6·09	—×13—
D—	5·95	×
	5·90	2+18
	5·83	△
	5·80	1+D
△	6·44	
path—	5·39	—×
	5·00	2+D
	4·48	to 487 on 10·67
	4·47	0+D
pond—	0·61	—×
13	0·60	33
pond—	0·31	—×
pond		
12+4	0.30	
from 578		
on 578		

	579	to off. 23 at 8·20 on 19·55 (page 107)
4	570	
8	500	
+2	1·55	
D+6	0·00	
from 0·13		
on 734		

	7·34	to off. 21 at 483 on 968
	700	2+D
	4·52	8 to G. P.
	4·36	10 to G. P.
	1·56	17 to 1031 on 1728
△	0·13	
	0·12	18+D
from 578		
on 578		

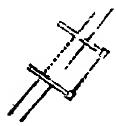
		to off. 98 at 908 on 1728 (page 105)
	5·78	
	5·75	29+
	5·60	26+pond
	5·43	32+pond (35)
	5·25	40 to G. P.
	300	18
	2·50	14
	2·10	10
	0·72	4
	0·55	0
	0·25	6
	0·13	12
from 616		
on 1306		

	13·06	to offset 14 at 147
10+30	13·02	on 1955
D		
10+76	6·83	
	6·16	△
D		
10+73	6·02	4+D
D—	6·02	—×
	0·08	8+D
from 1063 on 1067		
pond		
46+p+0		
	10·67	
	10·63	△
15	10·50	
stile—	10·46	—×0 to post
8	10·00	6
2	9·36	
P		P
3	9·00	8
path+path—	8·74	—×
to paling 10+6	8·61	
△	8·60	
D—	4·88	—×
	487	△
from 1443 on 1728		
	9·68	to offset 26 at 16 ch.
D+7+44+0	9·35	on 19·55
haystack+2	905	
46	900	
30	800	
7	600	
to fence 21	4·83	
D+12	479	
16	400	
18	300	
5	200	
0	1·46	
D+3	1·00	
from 17·28 on 17·28		
page 105		



		to offset 27 at 710 * on 12·30 (p. 104)
	5·56	18+13 *
	541	32
	400	26
	300	22
	2·70	8+oval
	2·40	
	1·00	13+hedge
from 765 on 11·60 page 104		
△	4·06	to 100 on 1160
	4·05	11+p+15
	3·72	27
	3·26	7+
	2·95	3+p+26
	1·18	24
path—	0·93	—×
from 574 on 577		
	↑	
fence—	5·77	—×
△	5·74	
	5·67	3+16 (D) ↑ on fence
p—	5·58	—×22+16 (D)
	500	2+18
(30) D—	4·50	4+16
30 (D) D—	412	10+12—×
	400	12+14
	300	9+16
	256	30+15
	200	41+14
	1·00	50+12
	0·95	342+0
	0·80	21+18
p —	·57	—×
	0·30	11+10 (P) +8+12 (D)
D—	0·29	—×

		stile D
	0·20	16+12+13+15
from 765 on 1160		



	11·60	to 1728 on 1728
16+3	11·56	14+p
10+6	11·00	17+p
12+7	10·50	15+p
D		
12+10	10·00	8+2 path
D		
12+13+p—	9·53	—×
10+11+2+7	900	
12+23+p+26	8·00	
10+23+p+26	7·90	
D		
to post 20+18+p	7·70	
+25	765	△
D—	737	—×
	720	27+D
37	600	
+path+35	400	
+path+21	200	
△	1·00	
to post stile 16	0·76	
stile—	0·69	—×
16+6	0·65	
R		
to G. P. 21+11	0·36	
to G. P. 41	0·17	
from 1230 on 12·30		

lamp	△	post
	12·30	
76	12·24	
	11·76	30+22
	11·70	4 to pump
	10·92	12+20
	10·50	16+13
	9·32	20+9
H+9+69	7·77	
	7·21	25+9
	7·10	27

Shrubbery

		R
	5·00	5+30+H
		R
4+15+10+34	3·00	10+28+H
R		R H
11+10+25	1·00	17+20+10
46	0·58	
11+14	0·51	
34+12	0·14	
from 1026 on 1026 along High Road		to Kentish Town
to 2 G. P. 0	10·26	
*to turnpike G. P. 7	10·09	*offset 50 at 19·15 on 19·15
to fence 15	9·82	
to fence 10	9·78	
hedge—	9.73	—×
	9·64	18 to G. P.
	9·58	15 | 18
\ fence—	9·49	—×
	9·46	2
	9·34	20
from 1728 on 1728		
	△	0
	17·28	0 to 2 brick pier
	17·14	11 to pier of small bridge
path—	17·00	—×
	16·93	8 to further post
	16·86	0+stile
to tree 0	10·32	
D—	10·31	—×
	9·08	90 to G. P.
to tree 0	8·39	
D—	8·35	—×
from offsets 14 at 147 on 1955		
	7·53	to 2166 on 2624
	300	28 to 2624
from 300 on 23·46		

Holloway & Highgate Road ×

	23·46	0 to sign post
	23·32	5 to G.P.
	22·00	5 to garden gate
R		
	21 00	10 to G.P.
10+44+3	20·00	2+8+2
Rd. —	19·00	—×
	17·90	19+14+28
	17·64	19+15+28
	17·44	20 to paling
—15+40	16·66	
		P
	16·37	7+5+8+2
		R P
	14·60	12+6—8+4 to G.P.
to G.P. 20+30	13·95	
24+25	12·00	
	10·00	27+4+8+12
		CHURCH LANE
	9·50	26+4+10
25+15	8·92	
	5·34	40+4+12 to G. P.
15+4	4·42	40+4+11 to G. P.
Road—	3·70	—×
	3·00	△
		R P
	1·00	7+44+4+15
from 2624 on 2624		

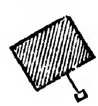
NOTE.—[The two following lines not in the engraving.]

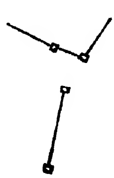
	12·58	to 159 on 159
	12·00	20
△	10.16	40
	6·50	76+D
	2·00	23+57
fence—	·70	—+
to fence 4+hedge—	0·90	—× 90
from 2166 on 2624		
	△	
	26·24	R
15+0	25·55	40+5+13
R—	25·54	—×
R		R
20+16	24·00	20+5+14
R		R
18+35	22·52	3+6+15
Road—	22·35	—×
to G. P. 20+36	22·34	[from gate.
△	21·66	0 to 9th lamp-post

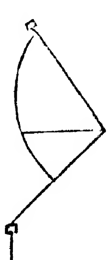
	21·46	14 to on fence
4+20+36+6	21·27	
to G. P. 20+40+5	20·62	
	20·00	8+5
	19·09	16 to G.P.
20+35+4	18·93	2+13 to G. P.
R		
15+30+10	18·00	2+11
Road—	16·20	—×
12+30	1500	3+12+5
14+20	13·00	13+4+13
14+10	10·00	24+5+13
10+9	9·00	28+4+12
12+7	8·00	30+6+13
R		R
20+6	7·00	30+6+13
16+11	6·00	26+5+12
R		R P
15+15	500	20+5+10
R		R P
17+20	400	19+4+10
R		R P
D+10+26	3·00	10+10+10+D
Road—	2·32	—×
R		P
D+13+30+3	2·00	10+13+D
R		
D+28+30+10	1·57	
path—	1·56	—×
gate fence—	·82	—× 12 to stile
from 19·55 on 19·55		Along the Junction Holloway Road.

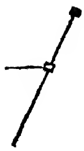
from 217 on 19·55	3·73	to 638 on 663

△	19·55	0
	19·26	25
	19·15	50
post by hedge 47	19·07	
	19·00	40+20
	18·72	37
	18·52	18









to G. P. 30	18·42	
to post 15+20	17·84	10+10 to stone
17+15	16·00	26 to G. P.
	15·86	15+10 to G. P.
11+15	15·47	△ 24 to fence
7+13	14·00	20+6
	12·24	20+8 to G. P.
	12·08	20+6 to G. P.
	8·20	10+13 to ↑ on fence
3+28	600	
3+25	500	10+3
20	3·00	10+3
	2·17	△
2+25	2·00	10+2
R		R
D+3+27	1·70	10+2+D
	1·47	14 to on fence
	1·13	22 to fence
D+4+18	1·00	16+6+D (20)
25	0·00	
from 159 on 159		

from 159 on 159	2·14	to 5·74 on 6·63
		R
D+0	1·59	△30+5
D		
6+12	1·53	
to G.P. 10	1·31	
to G.P. 12	1·15	R
5+8	1·00	23+5
from 638 on 663		

check-line. from 116 on 330	0·32	to 518 on 544

	3·30	to 215 on 663
from 397 on 544	1·16	△

hedge —	6·63	— ×
R		
3+28	639	0+7
△ road—	6·38	— ×
R		R
16	600	10+6
△	574	
5	500	26+8
D+0	460	26+8+D

3+8	4·00	24+10
4+10	300	20+8
4+16	2·60	15+10
△	2·15	
R		R
D+10+20	2·00	8+7+D
road—	1·00	—×
R		
G+33+0+3	0·55	6+D
from 518 on 544		

hedge of road	5·44	—×
	5·34	7 to G. P.
△	5·18	15 to G. P.
R		
5+30+6	5·00	20+D
R		
D+8+30	4·61	
road	4·60	—×
3+18	4·00	12+30
△	3·97	
3+16	3·90	
8+15	3·40	20+30
	3·00	15+25
to G.P. 17+18	2·96	
to G.P. 20+20	2·80	
30+20	2·00	15+20
	1·01	△
20+13	1·00	18+10
R		R
D+25+10	·800	20+15
		R
D+10+0	·00	33+20+D
from 23·41 on 2341		being old sign post. in previous survey.

NOTE.—[The commencement of this line will not, from want of room, be found inserted in the plan.]

For the assistance of the student, we will briefly go over the different selections of the lines, and the arrangement of the notes, making such special comments upon any local obstructions that may have occurred, as may enable the student to meet the same under similar circumstances.

The first line commences at a point 23·41, or 23·41, which is the end of an old line; this point is a sign post, selected as of certain reference and known position. This position is defined at starting, being at 0·00, close by the road side—that is, having the whole of the road (33) on the right side, and 0 (of the road) on the left; the 10 links on the left, and the 20 links on the right, are the width of the grass, common in country lanes, between the road and the hedge.

The station, 101, is a point required for the purpose of connecting this present new line with the line above referred to (23·41). This is, of course, not wanted for our present purpose. At 280, 20+20 to G. P. (or gate-post), at 29 18+17 to G. P. Reference is seldom made to a gateway *twice*. It is usual to mark the position of the *first* post of the gate you come to.

The station 397 is wanted for the purpose of measuring the bend in the road, the remaining distance being taken as one of the sides of the triangle. The distance 215, on the next line, being taken as the second side, and the base of the triangle being the distance 3·30, which will be found immediately after the next line, 663, viz., from 397 on 544 (330) to 215 on 663. There is a *check*-line to this, for the sake of accuracy, from 116 or 330 (·32) to 518 on 544. This gives the direction (relatively) of the line 6·63. At the end of this, the lane again bends to the left, for a distance of 1·59 chains. The position of this line is also fixed by the measured base of 2·14 chains; from the end of 159, to a point 571, on the former line 663. The road now turns quickly to the right, and runs straight for nearly a quarter of a mile, to the turnpike (19 chains 55 links). This sharp turn in the road is measured by the subsequent base line of 3 chains 73 links, from 217 on the new long line, to the station 638 on the second line 6·63.

It is upon this long line, 19 chains 55 links, that all the triangulation is based, and the relative positions of the several roads in the plan determined. The small triangles, used for the windings of the

lane, which came first in the field notes, should be plotted last in the plan. The surveying of a winding road, by means of the chain, is very inaccurate, and should, where possible, be avoided. It, however, seldom happens, that when extreme accuracy in the delineation of a road is required, that this mode of measurement becomes compulsory. One average line, through the whole of its windings, can mostly be taken, when desirable; and when an approximation, as to position (which is frequently the case), is all that is wanted, the present method is sufficiently correct.

In the taking of the offsets upon this base line 19·55, great care is requisite in selecting the most suitable. It often happens, in practice, that stations for the side triangles cannot be taken at the base line itself, but an offset points near to it, such as a post in a fence, a gate post, the corners of a fence, or any other fixed point that can at any time be referred to. For instance, the first station here selected is that of a broad arrow on the fence (offset 14 at 147), for the purpose of measuring the cross hedge.

The next is that at an offset of 23 at 8·20 chains. The third station is that of the offset 24, at 15 chains 47 links.

At 16·00, then, is the common offset of 26 to gate post; at 17·84, 35 on left to a post at the corner of the field; at 1842, 30 to first left hand gate post; at 18·52, 18 links to a post in the corner of the field to the right; at 1872, 37, on the right, to where a small garden enclosure commences; at 19·00, 40 + 20, *i.e.*, 40 up to the enclosure + 20 to the corner; at 19·07 to stile post on left, by side of hedge; at 19·15, 50 to first right hand turnpike gate post; at 19·26, 25 links to second gate post on right; at 19 55, to the further corner of turnpike.

From this corner you turn to the left, on the high junction Hol- loway Road, for a distance of 26 chains 24 links.

The *direction* of this line is obtained by measuring the line 12 chains 58 links, from the station 21·66, on the line 2624, to the end of the line 159.

The number of the offsets on this line will depend altogether upon the degree of nicety required. At 1·57 the offsets on the left hand are $10 + \overset{\text{R}}{30} + 21$ and D, that is 10 up to the road, which is 30 links wide, beyond which there is a grass plot of 28, up to the ditch. At 2·00, the line is on the right 10 links from the path, and on the left 3 links from the road. At 16·20 the road ×, that is, the line

crosses the road, the position of the road in the previous or subsequent offset point will define the side of the road; for instance, at the previous offset, 15 chains, the line is only 3 links from the road; at the subsequent offset there is a space of 10 links between the line and the road on the left. The side of the road, therefore, crossed at 16·20, must be the right side of the road, and you cross out of it. The station selected at 21·66 is a lamp post, being the ninth from the gate. At 22·38, the line crosses the road again, and, as at the next distance you have road on each side of the line, it now crosses *into* the road. At 25·54, the line crosses a side of the road again, and, as in the next station 25·55, the whole of the road is on the right, there being no portion of it (o) on the left, the line crosses the right side, *out* of the road. At 26·24 the line ends. The road, now taking a new direction, another line of 23 chains 46 links is taken, making an angle with the preceding, which angle is measured by the line 7·53 chains, drawn from 309 on 23·46 to 21·61, on 56·24.

There are no particular observations needed on this line; at 9 chains 50 links there is a lane, called Church Lane, to the right; the position of its corners, and their distances from the line, are carefully defined. At 16·37, and at 16·66, on either side, the hedge ceases, and the wood fence begins. Between 17·64 and 17·90 there is a house on the right, 62 links off the line; at 23·46 the station is a sign-post, at the corner, where the Junction Road comes out into the high Holloway Road.

This completes the Holloway side of Maiden Lane. We now return to the Kentish Town side, to the survey of the fields.

From offset 14, at 147 on 1955, turning to the right, a line is measured to the second brick pier of a culvert, being a distance of 17·28; on this there is an offset to a post in the corner of the hedge of the first field, viz., at 9 09, 90 to G.P.

From 17·28 another line is again measured, of 10 chains 26 links, to second gate post of turnpike, which is a known point of offset upon the line 19·55.

This completes the first triangle, which passes obliquely through three or four fields

Again, from this point 10·26, turning to the right, along the same Junction Holloway Road, but in a *contrary* direction, measure 12 chains 30 links, to a lamp post.

From the end of this line measure 11·60 to 17·28 on 17·28, to the second pier of culvert, above referred to. In this line we meet

with a path, running through the field, the position of which it is necessary to define.

Instead of saying, at 200 distances, 20 links to path, &c., I would recommend a plan (which I have adopted in this instance) of 21+path,
D
or, as at 7·70, of 25+p+18+20: the intersection of p, shows the position of it, without interfering with the system and simplicity of the arrangement.

Upon this line 11·60, there is a third triangle based, for the purpose of obtaining the position of the boundaries of the field notes; the sides of this triangle are respectively 5 chains 74 links, and 4 chains 6 links.

On this line, 577, the first offsets taken are at 30 links, being 16 links after the stile, which is 12 links wide, and 13 links from the ditch, which is 15 links across.

At 4·12, ditch crosses—and, from being on the right side of the line, runs 30 links to the left, forming, as the following offset at 4·50 shows, a small pond, whose length on the line is 31 links, and width 30 links to the left. The offsets, now, go on regularly to the end.

In the next line, 406, which brings us back again, close upon the high road, at the distance of 1·18, there is an offset of 24 to the paling; at 2·95 (3+p+20), that is, 3 to the path, which is 20 links off from the corner of a small enclosure; at 3·26, the offset of 7 links is taken to the widest part of the oval; at 3·72, you come to the end of it, which is 27 links off from the line; the position of the main fence is fixed by the two offsets, 20 and 27, and the beginning and end of the enclosure by their distances on the line; the size of the oval is known by the distance 7. At 4·05 there is an offset of 11 to the path+15 to the corner of a garden, on the other side of the field.

The offsets are now carried up to where the triangulation was left off. Let us now, therefore, proceed with the triangulation.

The next line we come to is the line 556, which, turning to the right, runs from a known point, 765 on 1160, alongside of a thick quickset ornamental hedge, to another known point, viz., an offset 27 at 710 on 12·30; this line, therefore, acts as *a check-line*. The first offset, at 100, shows the position of the hedge; the second distance, 246, the commencement of an oval enclosure; the next, 270, shows that, at its widest part, it touches the line; at 3·00 the oval ends, and is 22 links off the line.

At 541, another half-oval begins, which, at its widest part, at the end of the line, is 18 links off, its conjugate axis being 13 links; this point, which is 31 links from the present line, is the offset point 27, on the old line 12·30. This line completes the Field No. 6.

The next line runs from the brick pier at the corner of No. 5, along the boundary hedge, between Field No. 5 and Fields Nos. 1 and 4. It connects the point 17·28 at the pier, with another known point, at the other end of the field near Maiden Lane, being offset 26 at 16 chains, on the line 19·55. This line finishes Field No. 5.

The following line, 10·46, is a loose line, being one of the two sides of another triangle, based upon a previous line 17·28, and selected for the purpose of surveying fields Nos. 2, 3, and 4. It runs to a stile post at the corner of field No. 3.

The other side of the triangle is the line 13·06, which extends from the same stile post to a known point, viz., the offset 14 at 147, on the line 19·55, being the corner, on Maiden Lane, of field No. 2.

On the line 10·67, there are two stations taken, 487 and 8·60; the object of them will be explained presently. The offsets on this line are as usual, except at the end, where they differ somewhat; for example, at 10·46, the line touches the stile post on the line, denoted by (0+post), and stile crosses at 10·50—there is an offset at 15 links, to the angle of the adjoining property. At 10·07 there is an offset to another angle in the fence.

On the line 13·06, the chief thing to be observed, is, that at 6·02 the line crosses the ditch between fields Nos. 2 and 3, and that the corner of field No. 2 is (10+73) links, to the left of the line, where it crosses, while the corner of field No. 3, is 4 links to the right. The offsets of $73 \overset{D}{+} 10$, $67 \overset{D}{+} 10$, denote, that on this side of the field the hedge is *within* the boundary of the field, as was explained above. There is but one $\triangle$ on this line, viz., 6·16.

The next line measured, 578, is from this $\triangle$, 6·16, to a previous known point, viz., offset 90 at 908 on 1728. This line is a check upon the line 10·67, and at the same time it gives the position of the boundary between fields Nos. 3 and 2.

From 578, along the ditch, between 4 and 1, the line 734 is measured to a known offset, viz., 21 at 483 on 968. This line is a check upon the measured lengths of fields Nos. 1 and 2, as to their

sides adjoining Maiden Lane. At 156 the line is 17 links distant, from a previous point 1031, on a fixed line 1728. There are, therefore, *two lines* drawn half way across to the central, to show that it is not a *loose* line, and the *half way* denoting that the line, in its fixed direction, is produced further.

Next, from a point on the last line (0·13), a line is measured along the boundary, between 1 and 2, to a known offset on Maiden Lane, being offset 23 at 8·20 on 19·55.

Then, from the end of 578, a line is taken between fields 3 and 4 to a previously known point (487 on 1067) and produced to 609 (denoted by the two lines half way across).

Upon this line there is a △ (544), selected for the purpose of obtaining the winding of the old flat ditch, that runs at the bottom of the field No. 4, and another △ at 5·83, for the corresponding boundary of No. 3.

The next line, 354, is a line connecting a point, 860 on 1067, with this last △ 5·83, having offsets to the fence on the right.

And the last line is a line measured from the first of the △s on the line 609, (1544), to a known point on the line 1728.

The offsets, on this and the preceding line, are many and somewhat complicated, but certainly intelligible (with a little trouble) to the student, who has made himself master of the preceding explanations.

The Areas of the Fields in the Survey, Plate III, *are respectively as follows :—*

	A.	R.	P.
No. 1 contains	4.	3.	0.
2 . .	4.	2.	25.
3 . .	3.	0.	32.
4 . .	3.	1.	28.
5 . .	5.	3.	0.
6 . .	2.	3.	8.
7 . .	11.	2.	28.
Total area	35.	2.	1.

CHAP. X.

REDUCTION OF CUSTOMARY TO STATUTE MEASURE,

and vice versa.

THE statute length of the perch is 16 and a half feet, but it is different in various counties of England.

In Devonshire and Somersetshire, the customary perch, that is, the local measure of the perch, is less—being but 15 feet.

In Cornwall, it is more, 18 feet; while in Lancashire, it increases to 21; and in Staffordshire and Cheshire it is as much as 24 feet.

This is a *lineal* difference. There is, also, in some counties of England, a *superficial* difference in the measure of an acre; an acre, in Wiltshire, containing only 120 square statute perches, instead of 160.

The Wiltshire customary acre is, therefore, one quarter less than the statute acre, and the rood one quarter less than the statute rood.

As property is frequently bought and sold by the customary measure of the county wherein it lies, the Surveyor is often called upon to reduce it from one to the other.

DIFFERENT VALUES OF THE ACRE.

The number of (statute) square yards in an acre will, of course, vary with the length of the customary perch of the county.—(An acre consisting of ten square chains or of 160 square perches.)

In the statute acre, a square perch is 272·25 square feet, and the acre, therefore, is equal to

$$\begin{aligned} 272{\cdot}25 \times 160 &= 43560 \text{ square feet,} \\ &= 4880 \text{ square yards.} \end{aligned}$$

In the acre of Devonshire or Somersetshire, as the square perch contains 15×15 square feet, or 225 square feet,

$$\begin{aligned} \text{the number of sq. feet} = 225 \times 160 &= 36000 \\ \text{and of yards} &= 4000 \end{aligned}$$

In Cornwall, where the perch is 18 feet,

$$\begin{aligned} 18 \times 18 = 324 \times 160 \text{ feet} &= 51840 \text{ sq. feet,} \\ \text{or } & 5760 \text{ sq. yards.} \end{aligned}$$

The Lancashire perch is 21 feet long; the square perch, therefore, must contain $21 \times 21 = 441$ square feet, which will make the acre to contain 70,560 square feet, or 7840 square yards.

The customary acre in Cheshire and Staffordshire is the largest of the whole, each perch being 24 feet; the acre will consist of $24 \times 24 \times 160$ square feet, which is equal to 92160 square feet, or 10240 square yards; while the Wiltshire acre consists only of $\frac{3}{4}$ the statute acre, or 3630 square yards.

By dividing the square feet, in a customary perch, by that in the statute perch, you will obtain the measure, the customary is of the statute acre, taken as an unit; and, by multiplying statute square links by this number, you can at once bring the statute acre into the corresponding customary form:—

$\frac{225}{272 \cdot 25} = \cdot 826447$; the measure of the Devonshire acre in terms of the statute acre.

$\frac{324}{272 \cdot 25} = 1 \cdot 19$, that of Cornwall; $\frac{441}{272 \cdot 25} = 1 \cdot 62$, of Lancashire.*

$\frac{576}{272 \cdot 25} = 2 \cdot 1157$, that of Cheshire.

$\frac{120}{160} = \frac{3}{4} = 0 \cdot 75$, that of the Wiltshire acre.

DIFFERENT LENGTHS OF THE CHAIN.

Because the area of any acre $A = 10x^2$, when x equals a chain's length in feet; therefore $x = \sqrt{\frac{A}{10}}$ in sq. feet, where A is the area in sq. feet of the given acre, whether statute or customary:—

The Devonshire acre contains 36000 square feet;

$$\text{the chain} = x = \sqrt{\frac{36000}{10}} = \sqrt{3600} = 60 \text{ feet.}$$

The Statute acre contains 43560 sq. feet.

$$x = \sqrt{4356} = 66 \text{ feet.}$$

The Cornwall acre contains 51840 sq. feet.

$$x = \sqrt{5184} = 72 \text{ feet.}$$

The Lancashire acre contains 70560 sq. feet.

$$x = \sqrt{7056} = 84 \text{ feet.}$$

The Cheshire and Staffordshire acre $=$ 92160 sq. feet.

$$x = \sqrt{9216} = 96 \text{ feet.}$$

* This is the same as the Irish perch.

As the chain is divided into 100 links, whatever may be the length of the link, and, as in every case, an acre equals 10 square chains, the area of an estate in any county can be *at once* found in the customary measure of that county, by using the proper length of chain, without first calculating it, by statute measure, and then having the trouble to bring it back again to customary.

If a chain be divided into 100 links, or a in feet $=$ 100 links; then each link $= \frac{a}{100}$ feet $=$ the decimal of the number of feet.

		Feet.		Inches.	
The statute link, therefore,	=	·66	or	7·29	in length.
The Devonshire link	=	·60	or	7·20.	
The Cornwall link	=	·72	or	8·64.	
The Lancashire link	=	·84	or	10·08.	
The Staffordshire link	=	·96	or	11·52.	

To reduce Statute Measure to Customary, or one Customary to another.

Rule 1.—Bring the acres, roods, &c., in every case, to square perches; multiply these by the number of square feet in the given perch to bring them into square feet (a foot being the common unit of measurement of both statute and customary measure), and divide by the number of square feet in the required perch; this will bring it into perches; raise these perches to roods and acres and the result is the area in acres, roods, and perches, of the customary measure required.

Or *Rule* 2.—Multiply the given area in acres by the measure, the statute perch is of the customary perch, and the product will be in customary acres, which reduce to their proper value.

Or *Rule* 3.—Bring the given area into acres and decimals, and divide by the measure the customary is of the statute acre (considered as an unit.)

EXAMPLE 1. *(by the 1st rule.)*—Reduce 25 acres, 2 roods, 26 perches, statute measure, to the customary measure (Derbyshire) of 15 feet to a perch.

$$\begin{array}{rrr}
\text{A.} & \text{R.} & \text{P.} \\
25. & 2. & 24. \\
4 & & \\
\hline
102 & & \\
40 & & \\
\hline
4096 & \multicolumn{2}{l}{\text{square perches.}}
\end{array}$$

The square feet in a statute perch $= 272{\cdot}25 \times 4096 = 1115136$ square feet.

$$15 \times 15 = \frac{1115166}{225} = 4956 \text{ customary perches.}$$

$$\begin{array}{r|l} 40 & 4956 \\ \hline 4 & 123\text{—}36 \\ \hline & 30.\ 3.\ 36. \end{array}$$

30. 3. 36. Derbyshire measure.

Example 2. *(by the 2nd rule.)*—How many Cornwall customary acres are there in 45 acres, 2 roods, and 27 perches, statute measure?

$\frac{272{\cdot}25}{324} = {\cdot}84 =$ the measure, the statute perch is of the Cornwall perch, considered as an unit.

$$\begin{array}{r|l} 40 & 27{\cdot}00 \\ \hline 4 & 2{\cdot}675 \\ \hline \end{array}$$

$$45{\cdot}66875 \times {\cdot}84 = 38{\cdot}36175$$

$$\begin{array}{r} 38{\cdot}36175 \\ 4 \\ \hline 1{\cdot}44700 \\ 40 \\ \hline 17{\cdot}88000 \end{array}$$

A. R. P.
Ans. 38. 1. 17.

Example 3. *(by the 3d rule.)*—How many acres, in the county of Stafford, are required to make a farm of 100 statute acres?

The Staffordshire measure of an acre is 2·1157.

$$\frac{100{\cdot}000 \text{ acres}}{2{\cdot}1157} = 47{\cdot}426 \text{ Staffordshire acres.}$$

$$\begin{array}{r} 4 \\ \hline 1{\cdot}704 \\ 40 \\ \hline 28{\cdot}160 \end{array}$$

A. R. P.
Ans. 47. 1. 28.

To bring Customary into Statute Measure.

Reverse each of the preceding rules.

Example 1.—How many statute acres are there in 28 acres, 3 roods, and 15 perches, of Devonshire measure?

$$\begin{array}{r|l} 4 & 15{\cdot}000 \\ \hline 4 & 3{\cdot}375 \\ \hline \end{array}$$

28·84375 Devonshire acres.

The measure of the Devonshire acre $= {\cdot}826447 \times 28{\cdot}84375 = 23{\cdot}8378$

$$\begin{array}{r} 4 \\ \hline 3{\cdot}3512 \\ 40 \\ \hline 14{\cdot}0480 \end{array}$$

A. R. P.
Ans. 23. 3. 14.

EXAMPLE 2.—A farm in Cornwall consists of 100 acres, of local measurement, how many statute acres are there?

100 acres.
Cornwall acre measure = 1·19

Ans. 119 acres of statute measure.

EXAMPLE 3.—In 30 acres, 3 roods, 36 perches, Derbyshire measure, how many statute acres?

A. R. P.
Ans. 25. 2. 16.

EXAMPLE 4.—A gentleman, wishing to purchase a farm in Lancashire, which is 486 acres and 2 roods, of the statute measure, is desirous of knowing how many acres of customary measure there are in it? *Ans.* 300 acres.

EXAMPLE 5.—In Cheshire, there is a farm of 240 acres 2 roods, of statute measure; a recent purchaser wishes, by the purchase of a portion of the adjoining land, to increase his farm to 350 customary acres: what will it cost him to do so, at the rate of £40 per statute acre? *Ans.* £20,000.

EXAMPLE 6.—A nobleman wishing to farm 400 acres of customary measure, in the county of Wiltshire, is desirous of knowing what it will cost him, at the rate of £30 per statute acre?

Ans. £9,000.

MISCELLANEOUS EXAMPLES.

How many acres of Lancashire measure are there in 250 statute acres?

A. R. P.
Ans. 154. 1. 11.

How many statute acres will make up a farm of 300. 2. 30. acres, of Wiltshire customary measure?

A. R. P.
Ans. 225. 2. 2.

The base of a triangular field measures 6 chains 25 links, of a Cheshire chain, and the perpendicular, 4·84. How many statute acres are there in it?

A. R. P.
Ans. 3. 0. 32.

A rectangular field measures, by a Lancashire chain, 12 chains 45 links, and its perpendicular breadth, 8·20 links. How many acres would a farm of the same size contain in the county of Devonshire?

Scotch Measure.

The acre in Scotland consists, as in England, of 10 square chains, (each chain divided into 100 links,) and is reckoned in acres, roods, and *falls*, which are equivalent to the English perches; 40 falls making one rood, and 4 roods one acre. The Scotch chain, however, is 8 feet longer than the English, being 74 feet instead of 66.

The acre being 10 sq. chains $= 10 \times 74^2 = 54760$ sq. feet.

The chain or 100 links $=$ 74 feet, therefore 1 link equals ·74 feet $=$ 8·88 inches.

As 10 sq. chains $=$ 160 square perches

$$\frac{54760 \text{ feet}}{160} = \text{one square perch}$$

therefore one square perch or fall $=$ 342·25 sq. feet.

As the Scotch fall $=$ 342·25 sq. feet,
and the English perch $=$ 272·25 sq. feet,
the excess of the Scotch fall $=$ 70 feet sq. feet.

And the measure of the Scotch acre, in terms of the English

$$\text{statute acre} = \frac{342{\cdot}25}{272{\cdot}25} = 1{\cdot}2571.$$

To bring English Statute Measure into Scotch.

Rule 1.—Reduce the given area into English perches, and then into square feet, by multiplying by **272·25**, the number of square feet in an English statute perch; divide this product by the number of square feet (342·25) there are in a Scotch fall, and you obtain the area in terms of Scotch falls, which bring back to their proper quantities in roods and acres.

Rule 2.—Bring the given area into decimals of an acre; divide this by 1·2571, which is the measure of a Scotch acre, when the English acre is unity, and you obtain the decimals of a Scotch

acre, which multiply by 4 and by 40, to bring to their proper quantities.

EXAMPLE 1.—Reduce 32. 3. 25. (A. R. P.) English statute measure, into Scotch measure.

By the 1st Rule,

$$\begin{array}{r} 32.\ 3.\ 25 \\ 4 \\ \hline 131 \\ 40 \\ \hline 5265 \text{ square perches} \end{array}$$

$272{\cdot}25 \times 5265 =$ 1433396 square feet in the given area

$$\text{sq. ft. in a fall} = \frac{1433396}{342{\cdot}25} = 4188 \text{ sq. falls}$$

$$\begin{array}{r|l} 40 & 4188 \\ \hline 4 & 104{\cdot}28 \\ \hline & 26{\cdot}\ 0{\cdot}\ 28 \end{array}$$

A. R. P.
Ans. 26. 0. 28.

By the 2nd Rule,

$$\begin{array}{r|l} 40 & 25{\cdot}0000 \\ \hline 4 & 3{\cdot}6550 \\ \hline & 32{\cdot}93625 \end{array}$$

$$\text{the measure of a Scotch acre} = \frac{32{\cdot}93625}{1{\cdot}2571} = \begin{array}{r} \text{acres} \\ 26{\cdot}175 \\ 4 \\ \hline {\cdot}704 \\ 40 \\ \hline 25{\cdot}360 \end{array}$$

A. R. P.
Ans. 26. 0. 28. the same as before

EXAMPLE 2.—Bring 20. 3. 39 acres, English, into Scotch measure.

How many Scotch acres are there in 100 English acres, and by how much does the Scotch exceed the English acre ?

THE IRISH MEASURE is the same as the Lancashire; the chain is 84 feet long, and the acres are reckoned in acres, roods, and square perches, as in England.

The perch is 21 feet instead of 16½ feet; the square perch, therefore, contains 441 square feet, and the acre 70,360 square feet or 7840 square yards.

$$\frac{441}{272{\cdot}25} = 1{\cdot}62 \text{ the measure of an Irish acre.}$$

As 85 feet equals 1 chain, 1 link equals ·84 feet. = 10·08 inches.

$$
\begin{array}{rrl}
\text{The Irish chain being equal to} & 84 & \text{feet} \\
\text{and the English only to} & \underline{66} & \\
\text{In each Irish chain there is an excess of} & 18 & \text{feet, multiply by} \\
\text{number of chains in a mile} & 80 & \\
3 & \overline{)1440} & \\
\text{The Irish exceeds the English mile by} & 480 & \text{yards} \\
\text{English mile contains} & \underline{1760} & \text{yards} \\
\text{Irish mile} = & 2240 & \text{yards}
\end{array}
$$

$\frac{2240}{1760} \div 160 = \frac{14}{91}$ or the Irish is to the English mile, as 14 is to 11.

The same rules must be adopted in this case as in that of English and Scotch measure, in the reduction of statute English to customary, and customary to statute.

Example 1.—How many English statute acres are there in 25. 3. 19. Irish acres?

Example 2.—How many Irish acres must be taken to make up a farm of 100 statute acres?

Example 3.—A Gentleman has an estate in a rectangular form, $2\frac{1}{2}$ miles Irish one way, and $\frac{3}{4}$ the other. How many English acres are there in it, and what would be the periphery in Scotch miles of the same sized estate in Scotland?

Example 4.—There is an Irish farm of 120 acres, and a Scotch one of 150 acres, the former worth £10 per acre, the latter 15£. For what quantity of land, at £25 per English acre in the county of Cheshire, could they be exchanged without loss?

GAD MEASURE.

In some places in the Country, measurements are taken by the gad pole, a staff of 8, 9, or 10 feet, indifferently divided; either into feet or into tenths.

The square gad is the space inclosed within the length and breadth of a gad.

Acres are reckoned in square gads and square feet, or in square gads and decimals.

The 8-feet gad square is 64 square feet; the 9-feet gad, 81 feet; and the 10-feet, 100.

To ascertain, in gad, the areas of lands, measured by this method, bring the length and breadth into feet or tenths, multiply these together, the product is either square feet or square tenths, or, using

the 10 as decimals, square gads; the latter is complete, the former is to be divided by the number of square feet in the given gad pole.

To bring areas in gads, and feet of customary measure, into the statute measure of acres, roods, and perches.

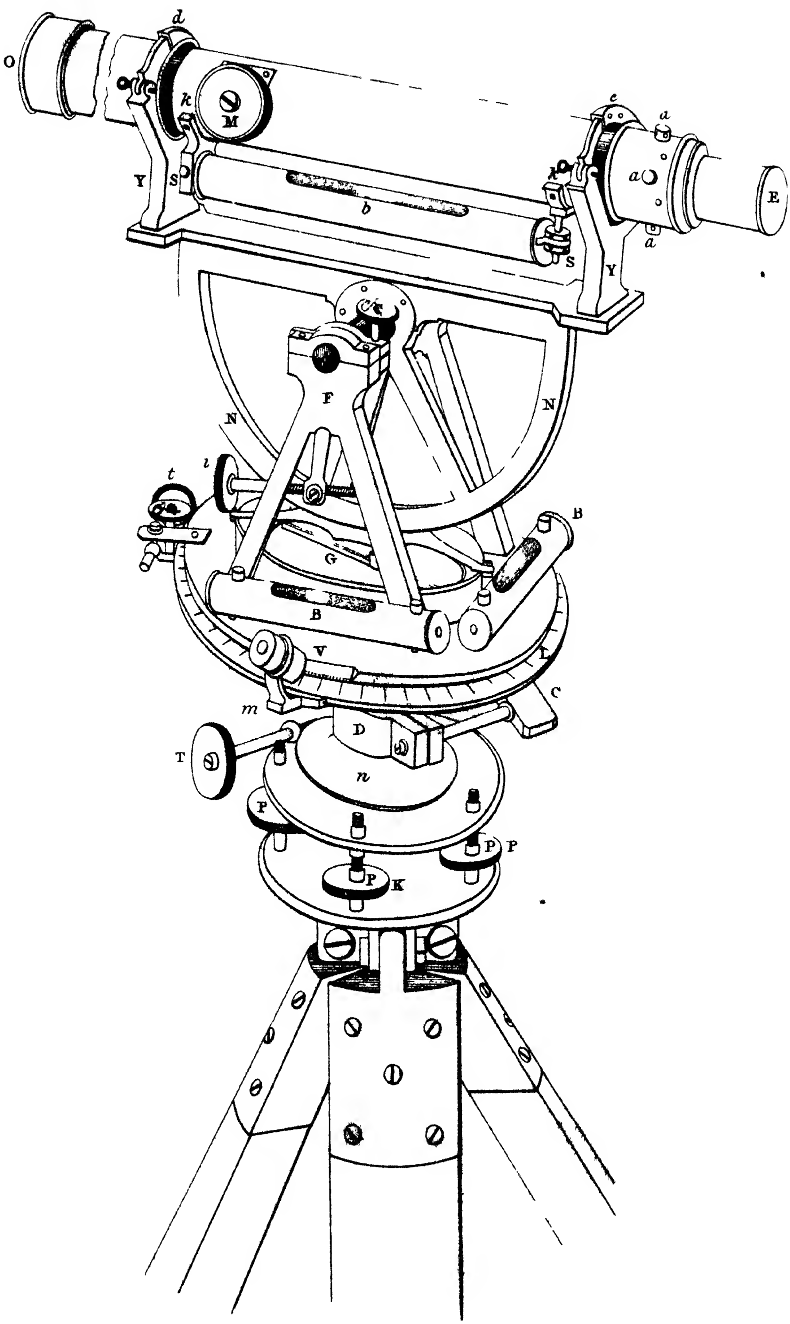
Bring the areas into square customary feet, divide this quantity by the square feet in a statute perch, 272·55, and the quotient is the number of statute perches in the given area, which divide by 40 and by 4, as usual, to bring it into roods and acres.

EXAMPLE 1.—A triangular field measures 28,(10 feet) gads and 5 feet, in its longest side, and in its perpendicular, 19 gads and 7 feet— What is the area in square gads, and also in statute measure?

EXAMPLE 2.—How many square gads (9 feet) are there in a rectangular field, whose length is 10·25 Irish chains, and breadth 6·29.

Simm's 5 Inch Theodolite

Plate 4



LAND SURVEYING.

Part the Second.

THE THEODOLITE.

Its Description, &c.

The Theodolite is the most useful instrument, that has been invented, for taking horizontal and vertical angles, as, by the nature of its construction, it is not necessary, that, in the former, the objects should be in the same horizontal plane; or, in the latter, in the same vertical plane.

This instrument stands upon three legs, and consists of three divisions, and has three motions.

1st.—*The absolute horizontal motion* of the whole instrument moving upon its axis, with clamp-screw (C) to fix it, and tangent-screw (T) for fine adjustment.

2nd.—*The relative motion* (as to the lower) of the *upper* of the two horizontal circles, to which the vernier (V) is attached, with its clamp-screw (*c*), and adjusting-screw (*t*).

These two motions are for taking horizontal angles.

3rd.—The relative motion of the vertical circle, which has also, as well as the other two, its clamp-screw (*c*), and fine adjusting-screw (*i*).

Detail of the first motion, which must be perfectly horizontal.

The lower (K) of the two parallel *Plates* is screwed tightly down to the legs of the instrument. The axis of the whole instrument passes right through to this plate. The centering, at the other end, is fixed to the upper of the two parallel *circles*—the upper one, called the *vernier circle*, from having the vernier attached to it; the lower, the graduated or horizontal limb (L), having its whole circumference graduated into degrees and half degrees. The lower circle has a distinct motion from the upper, working by means of a collar attached to it, upon the centering of the upper. Upon this is again fixed the collar (D) of the large clamp-screw (of the first division), which is attached as well as the tangent-screw to the upper of the parallel plates, which, connected with a ball, works in the socket of the lower plate, and has a double relative motion.

The upper of the parallel *plates* is made *instrumentally* parallel to the lower graduated circle; and the upper circle, (when in correct adjustment), is also parallel to them both.

Upon the upper of these circles are two small spirit levels, B, B, at right angles to each other.

By means of two pairs of conjugate screws, (PPP), which alter the relative position of the plates, the upper one can be always made level, as will be immediately seen, by the two bubbles, in the levels, being in the centre of the tube; and, once set level, the instrument, when in adjustment, will be level in any position.

By tightening the clamp-collar, and using the tangent-screw, the finest adjustment can be obtained.

The second motion.—Unclamp the horizontal circles, and the upper will move independently of the lower, or body of the instrument, with which it is connected. This motion of the upper circle, or vernier plate, as well as that of the two (or virtually the motion of the lower), will be perfectly level, if the instrument be correct and in adjustment; and the bubbles, now, as then, will be, in every position, in the centre of the tubes. This is, however, the weakest part of the instrument. The upper circle is very thin, and having an unequal and constant pressure exerted in one place, the bubbles, which, when the two circles are clamped together, are in perfect adjustment, are often very far from being so, when the upper plate is no longer held down by the clamp-screw (*c*).

The clamp (*c*), and tangent-screw (*t*), are placed along-side these circles, and have the same office as those of the first motion.

The third motion.—In this, which is a vertical motion, a graduated circle (N) is made to move instrumentally at right angles, with the horizontal motion plane of the instrument.

This circle moves upon its axis (A), which, passing through the common centre of the instrument, is supported by two shoulders or supports (FFP), at right angles with the vernier plate, to which this axis is made parallel.

This, like the horizontal circle, is graduated to half degrees, and, like that, by means of a vernier (*v*), supported by the compass box, is capable of being read off to minutes.

Attached to this vertical arc, above it, is placed a telescope, supported on two Ys or arms, in the form

of the letter Y. These Ys are virtually tangents to the tube of the telescope, which is generally furnished with bell-metal collars, ground truly cylindrical, to rest upon them. These are kept in by clips (*d e*), fastened by a pin.

A long spirit-level (*b*) is fixed beneath, to the telescope, parallel to it. The upper part of the inside of the tube is ground truly circular, so that when it is nearly filled with alcohol, or any other spirit, and its axis in a level position, the fluid may recede from the centre, which will then be its upper part.

There is also a compass box (G), with a magnetic needle. This box is generally placed over the two circular plates, and under the vertical arc. It is occasionally used to find the bearing of any line, though it is not correct enough for that purpose. It is really only calculated for ascertaining, *roughly*, the general position, as to the north and south points of the estate.

The Working of the Theodolite.

Adjust, first, the parallel screws (P P), so as to have, as nearly equal lengths of the worm, as possible, above the upper plate.

Extend the three legs, approaching or extending each, until the bubbles in the two levels (B B) are nearly central, and the plummet, suspended from a hook under the body of the instrument, hangs freely above the centre of the station. The better plan is to move only one leg, which is, of itself, capable of a double motion.

Press the legs firmly in the ground, unclamp the

whole instrument by means of the large clamp-screw (C), observing to keep the other motions clamped.

It must now be remembered, that the two levels, on the horizontal plate, are conjugate, *i. e.*, at right angles; and, that the opposite screws, also, are conjugate, each pair of them.

Set one level in a plane, vertically parallel to a plane, passing through any one of the screws; and the other level and the other pair of screws, will be also in parallel vertical planes, conjugate to the former.

If both the bubbles of the levels, thus placed, are not in the centre, take the bubble that is not level, and, loosening one of the corresponding conjugate screws, tighten the other, until the bubble be accurately adjusted. Then loosen and tighten the other pair in the same way, till the same result be obtained. This will probably throw the first out; repeat the process to each, until both bubbles are level.

Having these plates (K k) level, clamp the whole instrument, and, unclamping the parallel circles, set the broad arrow of the vernier, which is in the upper plate, to 360°, or zero, of the larger circle, and clamp it. This must be done by the magnifying or reading glass *(m)*, attached to the horizontal circles.

This large circle is divided into 360, and then again, subdivided into half degrees, which are numbered from left to right, and, by means of the vernier, read off to minutes.

Again, unclamp the large clamp-screw, and turn the whole instrument towards the left of the stations, between which, you are desirous of taking the angle, until you can cut the object as accurately with the intersection of the cross wires of the telescope, as can

be done by the hand. Clamp the screw (C), and slowly turning the milled-head tangent-screw (T), you can obtain all possible degree of accuracy.

Now, as the zero points of both upper and lower circles are together in the present position of the telescope, and as the lower circle is graduated from left to right, by separating the upper circle, and turning it round, till the centre of the cross wires of the telescope, which is attached to it, cut exactly the centre of the object at the second station, you obtain the angle between the two, determined by the position of the vernier and the length of the arc of the circle it has described. This can at once be read off from the plate by the broad arrow of the vernier, which will stand exactly above the number of degrees and minutes of the angle, measured between the two given objects.

When the cross wires, therefore, nearly cover the object, clamp the plates, and use the tangent-screw (*t*); and, with the magnifying glass (*m*), read off the angle, by means of the vernier. (For a description of the vernier, see page 137.)

The angle, thus read off, should always, if great accuracy be required, be read off by each of the two verniers. The common 5-inch theodolite is furnished with two, and the larger instruments have three, equidistant from each other, so that the *mean* of the readings, taken at different points on the circumference, should correct the errors of eccentricity or graduation.

In extensive surveys, other securities are adopted against these errors.

Instead of fixing the broad arrow of the vernier at the zero point of the horizontal limb, at starting,

the telescope is directed to the first station, with the broad arrow indifferently placed upon the lower plate, and its position carefully read off by the several verniers, and the *mean* taken. The difference between this mean, and that of the reading of the second station, is the measure of the required angle.

In incorrect graduation, this is perhaps the best check that can be used.

As an additional check, these angles are often repeated; that is—the angle is not taken again, by separating the upper plate and bringing the vernier back to zero, and then taking it a second time—but, without detaching the two plates after the last observation, turning the whole instrument bodily round to the first station, and, then unclamping the vernier plate, and turning it round to the second station.

The difference between this and the first reading, before starting, will be double the mean angle. Keep the two plates still together, and turn the whole round, repeating the process as before.

The difference between this third reading, and the reading at starting, will be three times the angle required.

It is requisite that the verniers should be separately marked, as A, B, C.

TO TAKE A VERTICAL ANGLE, OR AN ANGLE OF ELEVATION OR DEPRESSION.

First set the whole instrument level, as was explained before, by means of the bubbles on the vernier plate. Then bring the bubble of the telescope level (*b*) to the centre of the tube, observing whether, at the

same time, the zero point of the circular arc coincides with the zero of the vernier. This must be carefully examined by the microscope.

The instrument being thus perfectly level, when the zero point of the circle and the broad arrow are together, raise or depress the telescope, till you distinctly cut the required object with the horizontal wire, or the common intersection of the three wires. The changed relative position of the broad arrow, will give the required angle, which will be an angle of depression, if the broad arrow be found between the zero of the plate and the object-glass of the telescope, and of elevation, if beyond them.

It will be requisite, for particular accuracy, to invert the telescope in its Ys, and read off the same angle from the other end; half the difference of these two will be the angle of error, of the vernier.

ADJUSTMENTS.

THE TELESCOPE.

The accuracy of this instrument, in its application to the purpose of taking angles, depends altogether upon the correctness of the line of collimation. The optical axis of the telescope, which is an imaginary line, joining the centre of the object-glass and eye-glass, should pass through the point of intersection of the cross wires.

These wires are attached to a broad flange of an inner tube, within the tube of the telescope, near the eye-piece, with which it is connected by two pairs of conjugate capstan-headed screws (*a a a*), so as to admit of a double relative motion.

I.—TO ASCERTAIN, WHETHER THE LINE OF COLLIMATION IS IN ADJUSTMENT.

Place the telescope within the Ys, and, having found some point clearly defined, which is cut by the intersection of the cross wires, turn the telescope round on its axis, and observe, whether, during its whole revolution, the centre of the wires remains the same, always covering the same point. If it does, it is in adjustment; if not, turn the telescope round on its axis, and correct for half the error, by means of the small capstan-headed screws (*a a a*), loosening one and tightening the other.

II.—WHETHER THE AXIS OF THE LEVEL IS PARALLEL TO THE AXIS OF THE TELESCOPE.

Place the telescope on the Ys, and unclamping the large clamp-screw, set the telescope over one pair of conjugate screws, and loosen and tighten them till the level, attached to the telescope, is made perfectly level; or it may be made perfectly level by the vertical tangent-screw.

Then reverse the telescope in the Ys, if the level remains the same, this also is in adjustment; if not, correct for one half the error by the capstan-headed adjusting-screw, at the end of the level (S); and the other half by the vertical tangent-screw.

There is also a side adjustment required.

The level may not always be immediately under the telescope, but a little to the right or to the left; this must not affect the position of the bubbles, or a lateral adjustment, similar to the vertical one, is indispensable, by means of the capstan-headed screw, at the other end of the level (S).

III.—HORIZONTAL.

To mak the axis of the bubble on the vernier plate, parallel to that plate.

Let one bubble be over one pair of the circular plate screws, then the other bubble will be over the conjugate pair; make both bubbles level, turn them half round the circumference, and if the bubbles deviate from the centre, correct one half the error, by the small milled-headed screws above the levels: and the other half error, by the circular plate screws; repeat this, till the bubbles are level, in every position, throughout a whole revolution of the circumference.

IV.—HORIZONTAL.

Whether, after having duly corrected for the third adjustment, or made the "axis of the bubbles, on the vernier plate, parallel to that plate," the bubbles will remain perfectly level, during a whole revolution of the instrument upon the common axis.

Clamp the two circular plates, and unclamp the large clamp-screw; set the bubbles perfectly level, as before; when, immediately over each pair of conjugate-screws, reverse them; if they continue level, they are in adjustment; if not, the two circular plates are moving upon different axis, and are not parallel to each other. This imperfection can only be well remedied by an instrument maker.

V.—VERTICAL.

Whether the vertical arc moves in a truly vertical plane.

Set the vertical plate, or upper horizontal circle, perfectly level.

Direct the telescope to some well-defined angle of a building; or, should there be no building convenient, suspend a string, with a plummet attached, from the top of a high pole, and, taking care that the intersection of the wires exactly cut the string, near the plummet, raise the vertical arc, observing whether the cross wires, throughout the whole of the vertical motion of the telescope, cover the vertical string; if it does, this also is in adjustment.

As it is seldom found that two objects, whose horizontal angle is required, are exactly in the same horizontal plane, this adjustment becomes a very important one, and requires great care. Considerable error has resulted from neglect of it.

VI.—VERTICAL.

Whether the vertical vernier is in adjustment, or perfectly central.

Direct the telescope to some point of elevation, and note the angle. Reverse the telescope in its Ys, and raising the telescope to the same object, read off the same angle; if these angles are the same, the vernier is in adjustment; if not, correct the vernier for the error, by means of the small screw, fastening the vernier to the vertical plate, which can be loosened, and half the difference of these two angles will be the angle of error; or, which is better, add this angle of error to every angle of elevation, when you use the end that reads off the smaller angle, and subtract the same, from that of depression, under the same circumstances.

When you read with the larger angle, subtract this angle of error from the angle of elevation, and add it for the angle of depression.

PARALLAX.

Is an error occasioned by the focus of the eye-glass not being at once, with the focus of the object-glass, in the field of the cross wires.

The existence of parallax is determined by moving the eye about, when looking through the telescope, observing whether the cross wires change their position, and are flittering and undefined.

To correct this error, first adjust the eye-glass, by means of the moveable eye-glass tube, till you can perceive the cross wire clearly defined, and sharply marked against any white object.

Then, by moving the milled-head screw (M), at the object-end of the telescope, until you obtain the proper focus, according to the distance of the object, you are enabled at once to see clearly the object, and the intersection of the wires, clearly and sharply defined before it.

The existence of parallax is very inconvenient, and where disregarded, has frequently been productive of serious error. It will not always be found sufficient to set the eye-glass first, and the object-glass afterwards. The setting of the object-glass, by introducing more distinct rays of light, will affect the focus of the eye-glass, and produce parallax or indistinctness of the wires, when there was none before. The eye-piece must, in this case, be adjusted again.

Generally, when once set for the day, there is no occasion for altering the *eye-glass*, but the *object-glass* will, of course, have to be altered at every change of distance of the object.

THE VERNIER.

The vernier is a contrivance for subdividing, to any extent, the smallest division in a graduated scale, varying according to the scale to be subdivided, and the extent of subdivision, but having the same principle in all cases.

The following explanation of that of the 5-inch theodolite, of Simms, which is given in the plate, and which is a very convenient and accurate little instrument, will serve for all.

In this, the lower circle is divided into half degrees or 30 minutes; and the vernier so arranged as to read off to one minute.

Twenty-nine divisions of the graduated scale, which is twenty-nine half degrees of the lower circle, are taken; and are divided, upon the vernier, into thirty divisions: now, as thirty divisions are compressed into the space of twenty-nine, each of these thirty divisions is one-thirtieth less than those of the twenty-nine: or, as the whole arc, in the graduated scale, is equal to 29 half degrees, or 870 minutes, and these are subdivided into 30 divisions, in the vernier scale, each of these subdivisions is equal to 29 minutes—therefore, if the zero point of the limb correspond to the broad arrow of the vernier, the first division line of the vernier is one minute to the right of the corresponding division on the limb; the second division on the vernier, two minutes to the right; the third, three minutes; and, so on, till the thirty divisions of the vernier, exactly coinciding with the twenty-ninth division, is just 30 minutes to the right of the thirtieth, or corresponding division in the limb, and therefore corresponds to the twenty-ninth division.

If the vernier, therefore, be moved, till its first division corresponds to the first division of the limb, the broad arrow of the vernier will be removed one minute from the zero of the limb. If the second division of the vernier be made to correspond with the second division of the limb, the broad arrow of the vernier will be two minutes removed from the zero of the limb. If the third corresponding divisions, coincide, the zero's are three minutes removed, and so on.

Hence, *to set the instrument* at any angle, of any number of minutes, between 0 and 30, and 30 and 60, move the vernier, until the broad arrow becomes in such a position, *within the required half degree*, as, that the number of the line, on the vernier, coinciding with the same numbered line on the limb, shall correspond to the number of minutes required. And, to *ascertain the number of degrees and minutes*, that there are in a given angle, observe where the broad arrow of the vernier is; if, between a full degree and a half degree, so many degrees and as many minutes, as are denoted by the number of the first division line of the vernier, (reading onwards as the degrees number,) that coincides with the corresponding division in the limb; or, if between a half degree and a whole one, so many degrees and 30 minutes, *plus* the broken number of minutes, as denoted by the coincidence of the corresponding lines of the vernier and limb.

The principle of the above subdivision of the vernier of Simms's theodolite, is very simple, and is universally applicable to any subdivision; viz., divide the value of the division, in the graduated scale, by the number of divisions in the vernier, and the quotient will be the value of each of these subdivisions.

Thus, in the theodolite the graduated division is 30 minutes; the number of vernier divisions 30; the instrument (29 being subdivided into 30) reads to one minute.

In the sextant, the graduated division is 10 minutes, or 600 seconds; the vernier subdivisions are 60; the reading of the instrument is to 10 seconds, (59 of these 10 minutes are subdivided into 60.)

In the circumferentor, in this country, the common graduation of the plate is to one degree, or 60 minutes; the vernier subdivisions are 20, making the reading of the instrument only 3 seconds, (19 of these divisions being subdivided into 20); in this case, nine and a half on each side of the zero being divided into 10.

Hence, the value of the graduated division being given, and the extent of subdivision of reading required, to ascertain the number of requisite subdivisions.

By the rule above, where $x+1=$ the required number of the subdivisions in the vernier, and x that of the graduated arc; V, the value of the graduated division; and v, that of the required subdivision, we have

$$\frac{V}{x+1}=v, \text{ or } x=\frac{V-v}{v}$$

thus in the sextant $V=10$ minutes or 600 seconds; $v=10$ seconds

$$x=\frac{600-10}{10}=\frac{590}{10}=59 \text{ divisions.}$$

Again, if, in the circumferentor, $V=1$ degree or 60 minutes; and $v=3$ minutes,

$x=\frac{60-3}{3}=\frac{57}{3}=19$ divisions of the plate, to be subdivided into $x+1$, or 20.

CHAP. II.

THE THEODOLITE.

Its Application.

Having given an explanation of the nature, adjustment, and method of using this Instrument, we will now proceed to show its application to the purposes of surveying.

It is an instrument calculated for extreme accuracy. It is an instrument that should always be used, when, as in the Ordinance Survey, quality and not quantity is the desideratum; when the correctness of the result, and not the rapidity of execution, is the object.

I would here beg to caution my young readers against falling in this profession, as is often the case in many others, into a mere system of exclusively advocating this or that particular system of surveying, whether of the Chain, Theodolite, Sextant, or Circumferentor; they are all useful in their way, and each, under certain circumstances, has the advantage; and there is scarcely any survey, of any extent, but what all of the three may advantageously be brought into use.

In broad and extensive flats, though the triangulation were better carried on by the chain, as the chain is indispensable for determining the cross hedges, yet the long lines of the triangulation must be run in by the theodolite; the common method of ranging by the poles would not be sufficiently accurate.

In broken and hilly countries, where the chaining could only be obtained by an application of the angles, taken by the theodolite, to the determining of the comparative lengths of the hypothenusal to the horizontal lines, this instrument is indispensable.

The correct length of one side of a triangle, together with the measures to minutes, half-minutes, obtained with the accuracy of which a good theodolite is susceptible, of its two adjacent angles, will always more certainly determine the position of a third point, when hills intervene, than the incorrectly measured distances of the two other lines. And, in both the above cases, the circumferentor,* or the sextant, under certain limitations, can be advantageously introduced for the filling in.

There are two methods of using this instrument, generally adopted; the first by the *needle*: the second by the *back angle.*

The first, (*by the needle,*) I will briefly describe—The broad arrow of the vernier, and the zero point of the horizontal limb, are, by means of the adjusting-screw, made carefully to coincide; *always with the magnifying glass;* the needle is then released, and allowed freely to play upon its agate; and the whole instrument, with the two circles, firmly clasped together, turned round until the north end of the needle coincides, *as nearly as the eye can tell,* to the north point or zero *in the graduated circle in the compass box.* The whole is then clamped, and if in clamping, any error has arisen, it is carefully corrected by the large adjusting-screw (*T*). Now, if the two plates be detached, and the vernier plate turned round to the object, the angle read by the vernier, will be the angle made at the station, between the first object and the north end of the needle.

If the vernier plate be again unclamped, and turned round to the second object, the vernier will, in this case, also denote the angle made between this second station and the same north point.

* The third part will be devoted to the uses and application of this instrument.

The difference between the first and second reading will be the measure of the angle required.

This is the method of taking an angle *by the needle*.

It is, however, a method that I consider *very objectionable*. It is after all only an imperfect circumferentor. The graduation of the inner plate of the compass is too contracted, the diameter too small, and the needle too clumsy, to admit of any accuracy in the setting of it in the first instance.

I should certainly advise its being used but sparingly. The only case, when it may be used with advantages, is, in the surveying of roads, as the angle made at the commencement of a line, between the other end and the north point, can always be checked by the angle at the end of the line, made between the commencement and the south point.

The second method, *by the back angle*.—In this case, the instrument is placed at the second station, the bearing between the first and second being assumed as a base line, determined in position, and the angles are all based upon that line; thus, supposing the starting point to be A, the first station B, the base line AB, and the several stations C, D, E, F, &c., then the angle at B is that between A and C; the angle at C, that between B and D, and so on.

The disadvantage of this method is, that, as in the case of the division of a many-sided field into triangles, the error of each angle is not confined to itself, but is increased by the sum of the errors of all the preceding.

While, in the first method, by the *needle*, each angle is confined to itself, being the angle made between each line severally, and certain meridian lines, that are theoretically considered parallel. These meridian or North and South lines, however, depending upon so small a needle, and so confined a graduation as that of the theodolite, can scarcely be made practically parallel. For myself, I should consider, that there would be a greater risk of error in two consecutive readings of the needle, than would ever occur in the reading of the vernier, by the back angles.

And, in the first method, there are double the errors of plotting, to what there would be in the second. In the FIRST, there are the errors of the reading of the angle, which is common to the second method, and then there is the error of the non-parallelism of the needle, which is peculiar to the first.

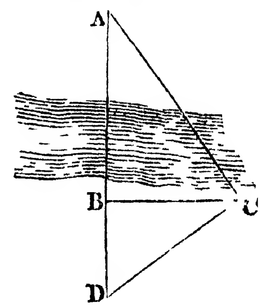
CHAP. III.

HEIGHTS AND DISTANCES.

In measuring a base line across a river, the following problem will be found useful.

PROBLEM I.

Let DA be the direction of the line which has been measured up to the river. It is required to ascertain the distance BA at once upon the ground; so as to continue the measurement of the line.



On the line DA, take any point, B, whence a perpendicular, BC, can be taken, which will be free from obstruction, so that BC can be accurately measured; carry the range on across the river, and at A set up a flag; on BC, take any point, C, whenec A can be seen; and at C erect a perpendicular to AC, intersecting the line AD in D.

Measure BD; then, because ACD is a right angle

$$\frac{BC^2}{BD}=AB, \text{ (Theorem 9, page 13.)}$$

These angles can either be taken by a cross-staff or by the chain, with the distances of 30 links, 40 links, and 50 links; 50 being the hypothenuse of a right angled triangle, when the base and perpendicular are respectively 30 and 40.

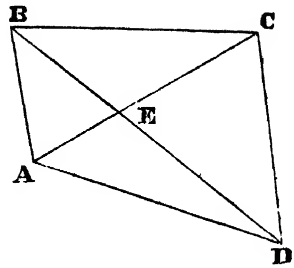
Example.—Was engaged in the measurement of a base line, that unfortunately crossed a river, too wide for the chain—measured up to the river 261 chains 45 links. Sent a man across in a boat, with a flag to carry on the range, and to plant the flag in the line on the other side. At 261 chains, at right angle with the base line on the right, measured 4 chains 50 links to the water's edge, whence only, on account of the trees near the river's side, the flag was visible.

At this point, at right angles with an imaginary line to the flag, measured to a point on the base line, which on trial I found to be 259 chains 25 links: required, the width of the river?

Width of river, 11 chains 57 links.

PROBLEM II.

To determine the area of a quadrilateral figure, whose diagonals are known, and the angle between them.



Let ABCD be the quadrilateral, having the diagonals AC and BD, and the angle AEB given. Let AEB $=\theta$ and diagonals $=$ D and d, viz., AC $=$ D, and BD $=$ d.

Now, the area of ACB $=\frac{D}{2}(\text{EB sin. }\theta)$

and of ACD $=\frac{D}{2}(\text{ED sin. }\theta)$

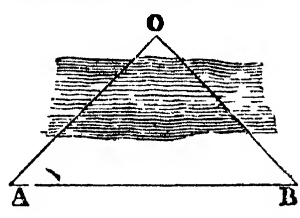
The area of the two therefore $=\frac{\text{D. sin. }\theta}{2}(\text{EB}+\text{ED})=\frac{\text{D. d. sin. }\theta}{2}$.

Example.—Placed, the theodolite in the middle of a field, and taking the angle between its diagonals, 56 deg. 28 min., measured therefrom to the 4 corners, 6·80; 8·27; 5·49; 7.34 chains, what is the area of the field?

	A.	R.	P.
Ans.	7.	3.	39.

PROBLEM III.

To measure the width of a river by a base line alongside of it.



Take at either end of the base line, with a theodolite, the angle made between the base line and a flag placed at the edge on the other side of the river. Compute the length of the sides by the first of the three cases of trigonometry.

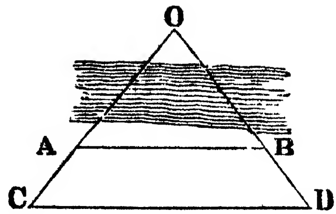
Then AO, nat. sine $\angle$ A $= x$, the perpendicular width of the river.

Example.—Took a base line by the side of a river, 12 chains, and observed the angles at its ends to the flag on the other side, found them 25° and 55°: what is the perpendicular width?

Ans. 4·22. chs.

PROBLEM IV.

To find the distance of one object from another, where a river divides them, without using the theodolite.



Let O and A be the given objects, and AO the distance required. From A draw AB, at any angle to AO, and produce OA to C, measuring AC about $\frac{1}{3}$ the length of AB; from C measure CD, parallel to AB;* and such that OBD may be in one straight line.

Then, because AB and CD are parallel, the triangles are similar, and therefore CD : AB : : CO : AO; and CD—AB : AB :: CO—AO; or :: CA : AO $\quad \therefore$ AO $= \frac{AB.CA,}{CD-AB,}$ the distance required.

Example.—Took a line, 6 chains, alongside a river, and having had a flag placed on the other side, in the direction I was desirous of going, measured in a range with it from one end of the line, 4·50 chains, then took a second line parallel to the former, 8 chains, to such a point that I covered the flag and the second station of the first line. What is the width of the river in the required direction?

Ans. 13 chains 50 links.

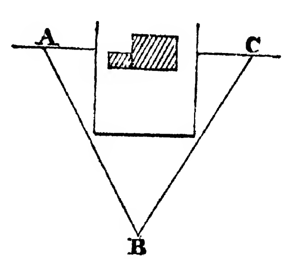
PROBLEM V.

To find the distance between two places, not visible from each other, but visible and accessible from a third point.

Chain the length of the sides; take the included angle by the theodolite. The solution comes under the second of the three cases.

PROBLEM VI.

To continue the measurement and direction of a given line, when any obstacle stands in the way, which cannot be crossed, but can be avoided by going to the right or the left.



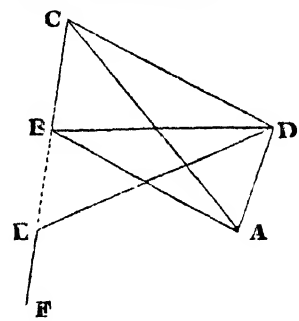
At any point (A), on the given line AC, take an angle with the given line of 120°, if you would turn to the left, or 240°, if to the right, as in this case, and proceed measuring to B, till an angle of 60° made with this line, towards the first line AC, will carry you clear of the obstacle.

* To make CD parallel to AB: at A and B erect equal perpendiculars, which can be done by means of the unit proportion of the sides of a right angled triangle, viz., 3, 4, and 5, or any multiples of them whatever.

Take this angle, ABC, 60 degrees, and measure BC the same distance as AB; the point C shall be in the given line, and AC shall be equal to AB or BC. By taking an angle of 240 degrees with the line BC, the range of the line can be continued.

PROBLEM VII.

It is often desirable to be able to produce a given line BC, which is inaccessible and invisible at the required point of production B.



Take any stations, A and D, whence B and C are visible, and at A and D, take several angles required (see Chap. VI.), *on the determining of the length of a new base line, by angles taken from an old line,*) and determine BC, CD, and the angle BCD : through D, draw DE, at any angle with DC.

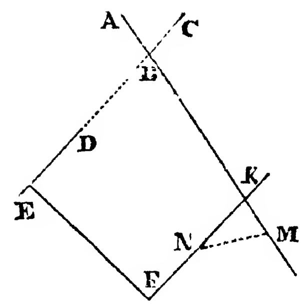
Now, because the line DC and the angle ECD have been computed, and CDE has been assumed, the triangle, DEC, comes under the first case, and the angle CED becomes determinable, as well as the required length of DE; measure this computed length to E; and at E lay off EF, making the angle DEF equal to the supplement of the computed angle CED.

EXAMPLE.—Being desirous of continuing an inaccessible line on the opposite side of a river, I measured a base AD, of 12 chains 50 links, and took the several angles made at each end of it, between this new base and two points that were visible on the given line, viz., at A, 50°, 37°; and at B, 85°, and 126°. I also measured, from the end of the new base line, nearest the river, another line, making an angle of 52°, with the new base. What must be the length of this line, and what the angle between this line and the production of the given line?

PROBLEM VIII.

To measure the angle made between two inaccessible lines, as the angle of a fort, or the salient angle of a bastion.

Let B be the salient angle of the bastion ABC, whose faces are AB and BC. It is required to measure the angle ABC.

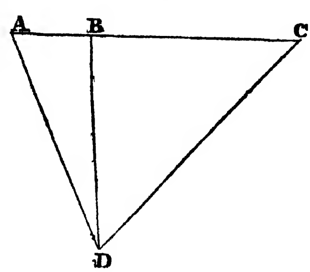


Take any point D, out of musket shot, and produce DE in a line with the face CB. At E, erect EF perpendicular to EC, about equal to ED, and also the perpendicular FK, such that AB, the other face, and K, shall be in the same straight line; produce the direction BK, to any point M. Now, because DEF and KFE are two right angles, EB and FK are parallel, and, therefore, the outward angle FKM is equal to the inward opposite angle EBM, which is equal to the required angle ABC. Measure, therefore, the angle FKM, by the theodolite or the chain, and you obtain the measurement of the required angle.

PROBLEM IX.

It sometimes happens in determining the position of two objects, (visible, but inaccessible from each other,) that both together are also inaccessible from any new station.

The only plan that can, in such cases, be adopted, is to select that station, at which the angle between them can be taken by the theodolite, and whence the most favourable lines, to clear the obstructions, can be drawn on either side, as *new bases*, to determine the lengths of the unmeasurable sides, that include the angle taken.



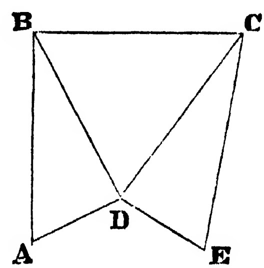
First.—Let B and C be the given objects, whose distance BC is required. Let D be the station, whence B and C can be seen, and the angle BDC measured; but BD and DC cannot be measured.

Now, if the stations B and C are of such a kind, and so situated, that, though AB cannot be measured, *the line of direction of CB can be produced to any point A, whence D can be seen;* by taking the angles at A and D, and measuring AD, you obtain the triangle ABD, and, therefore, BD and the angle ABD, which being the outward angle to the triangle BCD,

is equal to the angles BDC and BCD. BDC being known, BCD, therefore, becomes known, and BD is known; hence, the distances required become determinable, under the first of the three cases of trigonometry.

Example 1.—Wanting to know the distance between two forts, on each side of the entrance to a harbour, I measured a base line, 35 chs. 20 links, along the beach, from a point on the beach in the produced range of the flags of the fort, 8 chains 30 links from the nearest flag, to another point, which was nearly opposite to the centre of the entrance, and found, at this second point, the angles made between this line and the two flags of the fort, to be 8° 29′ 40″, and 45° 11′ 20″.

Secondly.—When CB cannot be produced, it becomes necessary to consider DB and BC, as two new unknown distances required.



Select any points A and E, visible from D, and such, that B is visible from A, and C visible from E.

At D, take the angles ADB, BDC, and CDE; measure DA and DE; and, at A, take the angle BAD, and at E, the angle CED,

The triangles ADB and CDE, have one side and two angles given, and come under the first of the three cases.

Hence, BD and DC are determinable, and the triangle BDC, having now the two sides BD and DC, and the included angle known, comes under the second case.

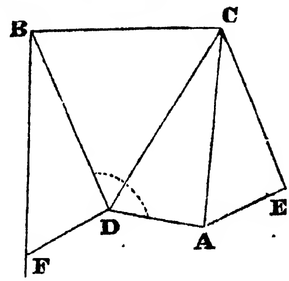
Example 2.—On another occasion, wishing to obtain the distance between two forts, similarly situated, at the mouth of a river, I found that, in consequence of the high ground in the rear, the line of direction could not be produced.

I was, therefore, under the necessity of adopting another plan. Placed my theodolite at the head of the harbour, and took the several angles, made between the flags of the fort and two new stations, whence these flags were also visible, viz., the angle ADB, 58°; BDC, 42°; and CDE, 72°; measured to these stations, DA, 32 chains; DE, 35 chains; and at A, and E, found the angles to be 83° and 86°. *Ans.* 1433 yds.

PROBLEM X.

Sometimes it happens, that no point can be found, whence B and C are both visible.

Some other arrangement, therefore, becomes necessary.



Let B and C be the unknown stations, as before, so placed that only one of them, B, is visible at D. Select any station A, whence D and C are visible; take the angle BDA; measure DA; and take the angle DAC. These data, if AC were known, would give us the length of DC, and the included angle BDC, as BDC would be the difference between the measured angle BDA, and the computed angle CDA; then, by obtaining the length of BD, the triangle BDC would have two sides, and the included angle known as before.

Find, therefore, the lengths of BD and AC, in the manner explained in the last example, considering them as two new unknown lines, by measuring DF and AE, and taking the angles BDF, BFD, and the angles CEA and CAE.

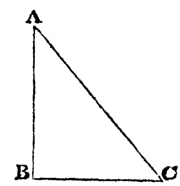
These data give you DB and CA; in the triangle CAD, you have also DA, and the included angle DAC, and DC is determinable, and the angle CDA; but the whole angle BDA was taken, therefore the angle BDC, the included angle of the first triangle, becomes known by computation, which, by the position of the objects, could not be taken by the theodolite.

Example.—On a third occasion, with a similar object, could not, from a number of buildings, find any point at the head of the bay, whence both flags were visible. Took a station D, at the head of the bay, and found the angles, between the flag B and two new stations F and A (whence the flags were visible), FDB, 90°; BDA, 125° 40′; found FD to be 10 chains; DA, 12 chains; at F found the angle DFB to be 54°; and at A, the angles DAC, to the other flag, 79°; and the angle CAE, to another station E, 50°; found AE 8 chains 20 links; and then found the angle AEC to be 93° 30′. What is the distance BC?

HEIGHTS.

PROBLEM I.

To ascertain the height of an inaccessible object.



Measure any distance from the base of the object, as nearly equal as possible to the height, and take the angle of elevation by the theodolite.

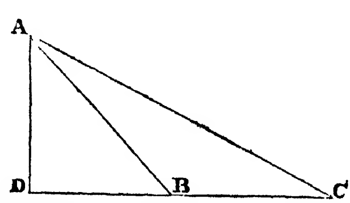
Let BA be the object; measure BC, and take the angle BCA, then BA is determinable by the case of right-angled triangles.

EXAMPLE 1.—What is the height of a tower, whose top, at the distance of 5 chains 75 links, subtends an angle of 33° 17′?

If the object be inaccessible at the base, determine the distance of any favourable station from it, by one of the methods, described in the CHAPTER ON DISTANCES, and take the angle of elevation, as in the preceding example.

Thus, measure a base line, at a convenient distance from the inaccessible object; at each end of this line, take the horizontal angle between the other end of it and the given object, and at some point within the given line, measured from either end, take the angle of elevation.

EXAMPLE 2.—*Another method may, however, be more conveniently adopted, where practicable, viz*:—



From D, take any point C, in a line with DB; measure BC; and at B and C, take the angles of elevation. The angle DBA being the outward angle, is equal to the angle at C, + the vertical angle BAC, and BC is measured; therefore the triangle ABC, in the vertical plane, comes under the first case; and AB, being thus determined, and the angle ABD, at the base, being known, the triangle ABD is

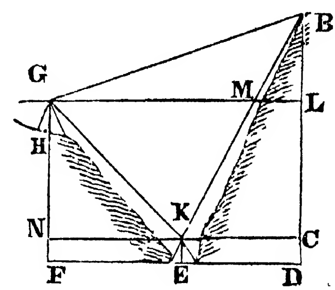
These are merely the same figures in a vertical plane, instead of a horizontal one.

determinable by the case of right-angled triangles, and AD becomes known.

Example.—Wanted the height of a tower, and the width of a moat around it, when the angle subtended by the top of it at the edge of the moat was 64° 20′; and, at 4 chains 50 links off, was 25° 54′.

PROBLEM II.

To ascertain the height of a hill above the level of the Country, wherethe ground is so broken that a horizontal line cannot be measured, and the only angles, that can be taken, are from a point in the rising ground of another hill, which does not admit of measuring the horizontal distance from the object, and from the bottom of the first hill.



Let EBD be the hill, whose vertical height is DB. Let E and H be the two stations, for taking the vertical angles, in the same vertical plane with the object BD.

At E, take the vertical angle BKC, and from E, measure the distance HE along the slope of the hill: at H, take the angle of elevation BGL, and the angle of depression LGK, setting a flag KE, at the point E, equal to the height GH. The heights required are DB, and FG.

Now, in the triangle BKC, because the angle BKC is known, if KC or KB were known also, the triangle were determinable.

But in the triangle BGK, because ∠ BGK is equal to the sum of the angles of elevation and depression, and because GN is parallel to LC, the angle MKC is equal to the angle GMK, which, being the outward angle, is equal to the angles BGM and GBM; that is, the vertical angle GBM is equal to the angle of elevation at K, minus the angle of elevation at G. The angle GBM thus being known, the triangle GBK has the side GK, and two angles, known, and is therefore determinable, and KB becomes known; and, therefore BC and KC and GN, is determinable, because GK is known, and the angle GKN equals the angle of depression, MGK. Add CD, the height of the theodolite, to obtain the height of BD.

The difference between the heights of the two stations, B and H,

is found by subtracting NH from CB, their respective heights above the common *datum* line NC.

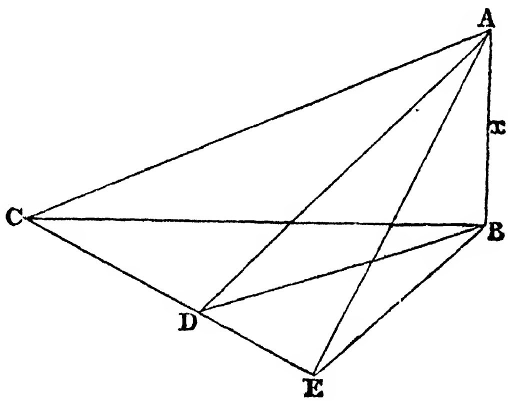
EXAMPLE.—Given at G, the angle of elevation BGL, 12° 15′, and the angle of depression LGK, 42° 29′ ; the hypothenusal distance GK, 8·25 ; and, at K, the angles of elevation NKG, 38° 20′; CKB, 64° 15′. Required the relative heights of the stations G and B.

PROBLEM III.

To determine the height of an inaccessible object by a sextant.

Select three points *in a straight line,* whence a distinct view of the object can be obtained, measure their distance from each other, and take the angles of elevation of the object at these several points.

Let AB = the height = x ; and the angles ACB, ADB, and AEB, or the angles of elevation = β, γ, δ, respectively, and $cd = a$, and DE = b; also let the angle CDB = ϕ.



Now the cos. or angle CDB. $= \dfrac{a^2+DB^2-CB^2}{2\,a.DB}$

$$\left.\begin{array}{l}\text{and—cos. } \phi \\ \text{or the angle BDE}\end{array}\right\} = \frac{b^2+DB^2-BE^2}{2\,b.DB}$$

$$\therefore b\,\underline{(a^2+DB^2-CB^2)} = -a\,(b^2+DB^2-BE^2)$$

$$\text{but } DB^2 = \cot^2\gamma . x^2$$

$$CB^2 = \cot^2\beta . x^2$$

$$BE^2 = \cot^2\delta . x^2$$

$$\therefore b\,(\,a^2+x^2\,(\cot.^2\gamma-\cot.^2\beta) = -a\,(\,b^2+x^2(\cot.^2\gamma-\cot.^2\delta\,)$$

$$x^2\,(\cot.^2\gamma . b - \cot.^2\beta . b+\cot.^2\delta . a - \cot.^2\delta . a) = -(a\,b^2+a^2\,b\,)$$

$$x^2\,(b\ \cot.^2\beta+a.\ \cot.^2\delta-(a+b)\ \cot.^2\gamma) = (a+b)\,ab$$

$$x = \sqrt{\frac{(a+b)\,ab}{b.\cot.^2\beta+a\ \cot.^2\delta-(a+b)\ \cot.^2\gamma}}$$

Where the height equals the root of the product into the sum, divided by the sum of each distance into cot,[2] of its remote angle *minus* the sum of the distances into the cot[2] of the middle angle.

When $a=b$

$$x=a\sqrt{\frac{2}{\cot.^2\beta+\cot.^2\delta-2\cot.^2\gamma}}.$$

EXAMPLE.—Took three stations in the same straight line, at some distance from an object, whose height was required, and, by means of a pocket sextant, took the angles of elevation 12° 45′, 14° 15′, and 18° 7′. The distances between the stations were 12 chains, and 15 chains. What is the height of the object ?

CHAP. IV.

SURVEYING BY THE THEODOLITE.

HAVING given the theories of solution of most of the practical cases, that can occur in the measurement of inaccessible heights and distances, by means of the theodolite, we will now proceed to explain the measurement of a field, by this instrument, and the method of keeping the field notes.

In the survey of a field, by the theodolite, as well as in the survey of roads, as explained in Chap. II., there are likewise two methods adopted: first, by selecting one of the *sides* as the base, and using the back angle; and secondly, by two stations, by making the largest diagonal line the base of a number of triangles, and computing the position of the several corners, considered as vertices of triangles, whose angles at the base are determined by the theodolite, at the two stations.

The annexed field notes refer to both methods.

FIELD NOTES No. 1, (*By the back angle.*)

from △ 25·13 on 25·13	60·47	to 0 on base line 2200
from 13·76 on 25·13	9·83	to 487 on 487
from 487	571	to 1848 on 26·13
from 25·13	487	△ to fence corner
		R
25+0 △	25·13	45+12
R		R
27+9	22·73	43+12
	18·48	△
	13·76	△
R	7 00	3+16
25+53+16	1·00	
between 765 on 793	171°49′	and 25·13
at 1200		
	1200	△
17+38+16	11·90	
R		
18+48+6	10·60	
10+10	260	
	1·50	20+10
between 709 on 790	199°39′	and 1200
at 705 on 793		
	793	18+11
	7·65	△
to paling 25+20	6·90	
	6·50	45 to house
	610	38 to house
2	300	10+45
20	1·00	33
between 697 on 707		and 765 on 793
at 709 on 790	*	
R	7·90	R
12+32 △	7·09	6 R
2	5·00	10+35
14+20	1·50	
R		R
40+30	1·00	11
between 11·11 on 11·11	161°11′	and 709 on 790
at 697 on 7·07		
hedge—	7·07	—×
△	6·97	12 to end of fence
	6·65	6+post+6
12+5	509	30
- 12+13	1·00	20
between 844 on 8·80	158·28	and 697 on 707
at △ 11·11		

12+32	11·11	3
12+8	500	33
R		R
12+14	1·00	22
between 9·79	157·50·30	and 11·11
at 844 on 880		
p R		
12+37	8·80	
	8·44	△8+D
	5·80	22+7 to G. P.
	4·80	20+7 to G. P.
10+27	2·00	6
p R		R
to fence 10+32	1·00	0
between 447 on 447	104·6	and 844 on 880
at △ 979		
hedge—	9·79	—×
road—	9·30	—×
21	9·10	
	9·05	34 to wall
11	8·84	
(12 wide) to style 17	6·93	
gate—	6·90	—× 20 to fence
to parish stone 15	6·76	
	6·40	27 to corner
between 151 on 151	180·22	and 979
at △ 447		
	4·47	
other side—	2·05	—×
fence of enclosure—	1·30	—×
between 2188 on 2200	201·3	and 447 on 447
between 8·72 on 2200	1°58′30″	and 2188
at △ 151 on 151		
	1·51	△
D—	1·45	—×
other side—	1·30	—of hedge
road and hedge—	1·05	—×
paling and road—	62	—×
from 21·88 on 2200		
gate	22·00	post
9	21·88	△
49	18·50	
120	1700	
23	10·00	
to fence 4	8·94	

BASE LINE

No. 2, *(By two Stations.)*

	14·42	to △G.
road—	14·00	—✕
H—	13·73	—✕
to tree blazed 1	13·72	
D—	286	—✕
	264	△
	1·04	to stile
from 16·63		
	23·41	to △G
sign post+		
D+16		
R		
8+30+10	21·63	12✕D
	21·06	21 to G.P.
	2090	20 to G.P.
R	1900	6
D+28×30+20		
	1800	0
	1700	6+D
road+	1600	
road+D—	15·40	—✕
hedge—	15·35	—✕
hedge—	1510	—✕
94	1000	
38	800	
28	700	
D—	5·02	—✕
to G.P. and 25	4·33	
D+22	1·00	
D+21 path—	0·83	—✕
	324°00	and △G
between 16·63	304°38′	and top of St.
at △D		Paul s
	308° 23′	and △C
	129° 10′	and △F
	127° 55′	and △ (G) in Maiden Lane.
	36° 17	and △E
between 2766	21° 10′	and △D
at 16·63 on		
3400		
	350° 48′	and △F
	324° 32′	Ball of St. Paul's
	284°51′	and △E
	242°22′	and △D
between 1663	42° 40	and △C, being
at 27·66		11·97 or 1197

	34·00	
D—	27·80	—✕
	27·66	△
path—	17·90	—✕
D—	16·63	—✕△
D—	10·36	—✕
8	768	0 to gate post
10	600	+gate
30	400	
30	200	
to hedge 100	000	
from 473 on		
473		
producing FL		
	4·73	to a point in FL.
D+5p+25	200	
D+10	1·00	
path—	54	—+
bridge+1	·53	
bridge		
11+5+5	·31	
bank of stream	·30	—✕
from △ B		
producing D B		
between △ B at △ F	36° 43′	and △L
between △ F at △ G	230° 10′	and △L
D+28	1·86	to △G
H—	0·20	—✕
	270° 45′	and △ ranging with AD produce
between △ C at △ F	215° 46′	
D+63	2·69	to △F
D+63	1·00	
between △ D at △ C	275° 14′	

N.B.—The plan of the first portion of these notes is purposely omitted.

The notes, in the first examples, are those of a survey of a road *by the back angle.* In the second example, they are those of a field surveyed *by two stations.*

The student will observe carried out, in these notes, the principle before recommended, of taking the angles always from left to right.

For instance, at station C, between station D, and station F, the angle is 275 degrees 14 minutes; that is, at the station C, the direction of F (CF), is, measuring from left to right, not only 180 degrees, but more than 270 (275°, 14′), extending into the fourth quadrant of the circle. The angle, reading the other way, from right to left, might have been taken, or the instrument might have been set to F, making CF the base line. In the former case, the work would only have to be done in the field, which is now left to be done at home: in the latter, every station, making from left to right a greater angle than 180 degrees, must be made a new station; the entering of the field notes would be more complicated, and errors without number would result.

It is oftentimes requisite to take several angles at the same station, by considering the base line as fixed and constant, and determining the relative directions of the other points to this; the arrangement of the note becomes simple and clear. As, for instance, at 16·63 (see page 156), on line 3400, denotes that all the following angles (*reading upwards*) are taken *at that* station, with the theodolite unmoved. And between 27·66, on 3400, shows that the station, whence the angles are taken, is on the line between which they are measured; and the following lines, in the margin to the left, being left vacant, proves

that all the angles following are taken at the same station, 1663, and measured from the same line.

I have preferred the phraseology of ("between— and") to that of ("from — to"): there is less ambiguity about it; an angle is made *between* two lines, *at* a certain point.

The angles taken at this point are all of them, with the exception of the first, in the third or fourth quadrant. There is no possibility of error in this arrangement, and there is certainly nothing unmathematical in admitting an angle greater than 180 degrees.

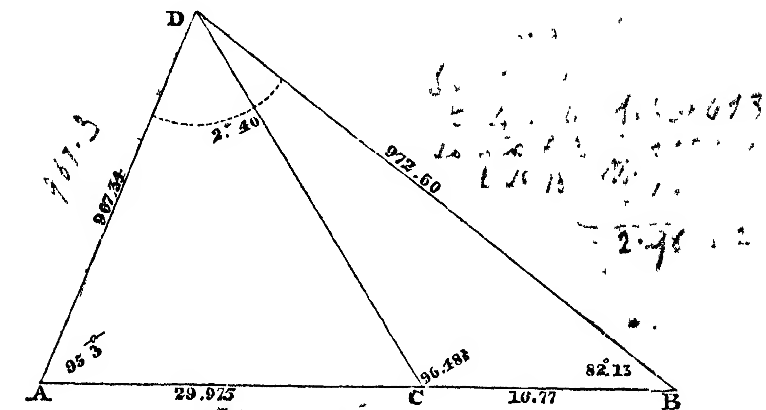
In taking several angles at a station, when it is only desired to measure along the thus ascertained direction of a new line, this angle should be taken last. and placed immediately before the chain distances. Thus, at △ F, the angle between △ C, and △ G, although nearer to the base line FC (215° 46′), is placed after that of △ F (270° 45′), because the following distances, 20 links, 1·86 links, are distances on the line FG (to △ G).

The field notes No. 2, (page 156,) are those of the survey by the theodolite of the same field, which was before surveyed by the chain, the plan of which is given at p. 89. The same points are determined in this case as then; but the offsets have been omitted as unnecessary.

I hope the student will understand, that I have not introduced the example of surveying (by the theodolite) the same field which was before surveyed by the chain, as conveying an opinion, that it is indifferent, which method is adopted under similar circumstances: far from it. I have merely given this, as an example of how it is to be done, should it be necessary. A theodolite would be quite out of place, in so small a survey as this.

FIELD NOTES, No. 8.

Being notes of a survey, actually made at Blackheath, explaining the method of keeping the field book, and the practical arrangement and computation of the angles.



Between A on base line at 16·77 on base line	83° 10′	and Knockholt Beeches.

The base here was so disproportionate to the unknown sides, that it became necessary to take a check-angle to prevent inaccuracy.

vertical angle	0° 22′ 0″	to top of Spire
348°30′ 337°0′	348° 30′	and Charlton Church.
vertical angle	0° 33′	to Summit
162·29	162 29′	small bush right of large
324·58		Tree on Forest Hill.
vertical angle	0° 33′	to top of Spire
108·20		
216·38	108° 19′	and Lee Church.
324·57		
vertical angle	0° 30′ 0″	to Summit
82·12		
164·26	82° 13′	and Knockholt Beeches.
246·39		

9.445780
1.668013

vertical angle	1° 30′	to top of Spire
53·2 106·5	53° 2′ 20″	and Blackheath New Church.
vertical angle	1° 15′	to top of Tower
21·29 42·58 64·28	21° 29′ 20″	and pole at top of Sevendroog Castle.
between base line at △ B		
345·33 123·37	345° 33′	and bush on right of large tree on Forest Hill.
vertical angle	0° 36′	top of Spire of Church.
325·41 83·54 202·6	325° 41′ 20″	and Lee Church.
vertical angle	2° 8′ 0″	to top of Spire
284·46 209·31	284° 45′ 40″	and Blackheath New Church.
vertical angle	0° 34′	to Summit
264·57 169·54	264° 57′	and Knockholt Beeches.
vertical angle	1° 42′	to top of Turrets
207·29 54·54	207° 29′	and Sevendroog Castle.
between base line AB at △ A		
	46·56	to △ A
	49·71	to 14·43 on 14·43
	10·77	△
from △ B the distances were		

Correcting for the 2¼ inches.

This chain had been previously carefully measured at the standard chain length at Somerset House, and was 2¼ inches too long.

	46·70¼	to △ A
	40·83	to 1443 on 1443
	16·82	△
from △		

The chain, which was used the first day, having been mislaid, another chain was used for the second measurement.

End of first day's work.

from 1400 on 3952	30·46	to 5·82 on 47·31
from 585	7·40	to 600 on 3952
	47·51	to 585 on 585
	46·51	△ B

	38·98	△ by side of road
	5·82	to △ 14·43 on 1443
from △ A, by side of Shooter's Hill Road.		
	14·43	△
	1·35	to 39·52
from S. E. corner of park, ranging with easterly wall.		
	38·52	△
side of —	38·57	—road
to wall 37	30·00	
3	20 00	
	1400	△
to park gate 0	13·98	
	600	△
0	1·96	
to wall 4	1·27	
from 65 on 585		along wall of park
△	5·85	
in a line with	0·65	park wall on left
to wall 0	0·62	

from S. W. corner of Greenwich Park, at Blackheath, near the Princess Augusta's.

To find the value of this check-angle by computation.

The base AB being given, 46·56, and the angles at the base 95° 3, and 82° 13′, the other two sides were found to be 967 chains 34 links, and 972 chains 60 links.

Now, in the triangle ADC, we have the sides AD, AC, and the included angle at A; hence, by the second case of trigonometry, the other angles were computed, and that, at the base, ACD, found to be 83° 11′ 12″, the supplement of which is 96° 48′ 48″; the measure of the angle DCB being only 12 seconds less, than the observed angle 96° 49′.

The length DB, being the distance from the Knockholt Beeches to △B, was found to be 972 chains 60 links. To this was added the distance of B, from the S W. corner of the park, viz., $\sqrt{5{\cdot}85^2+1{\cdot}00}$, or 5 chains 93 links, and the total distance, obtained thus, was 978 chains 53 links, or 12 miles, 1 furlong, and 8 chains 53 links. The distance on the ordnance map is somewhat less than 12 miles 2 furlongs.

CHAP. V.

TRIGONOMETRICAL PROBLEMS.

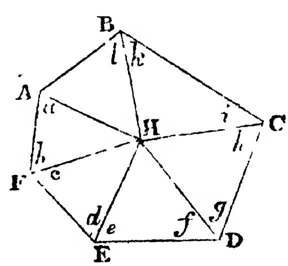
PROBLEM I.

Given the position of two known points, to determine the length of a new base line, by means of angles taken from its extremites with the given base.

(Resolved analytically.)

Before proceeding to the solution of this Problem, it must first be demonstrated, that if any polygon be taken, and lines be drawn from a point, either within or without it, to the several angles of this polygon, the continued produced of the sines of one set of alternate angles, made by these lines, and the sides of the polygon, will be equal to the product of the sines of the other set.

Let H be the point when the lines HA, HB, HC, &c., are drawn to the angles of the Polygon.



Now, in the triangle AHF,

$$\frac{\text{FH}}{\text{AH}}=\frac{a}{b};\ \frac{\text{EH}}{\text{FH}}=\frac{c}{d};\ \frac{\text{DH}}{\text{EH}}=\frac{e}{f},\ \text{\&c.}$$

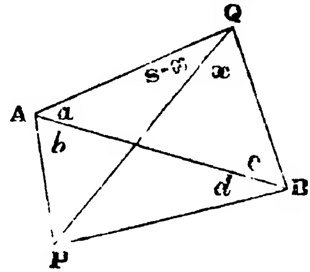
$$\frac{\text{FH}}{\text{AH}}\times\frac{\text{EH}}{\text{FH}}\times\frac{\text{DH}}{\text{EH}}\ \text{\&c.}\times\frac{\text{AH}}{\text{BH}}=\frac{\sin.a}{\sin.b}\times\frac{\sin.c}{\sin.d}\times\frac{\sin.e}{\sin.f}\times\frac{\sin.g}{\sin.h}\ \text{\&c.}$$

and $\therefore \dfrac{\sin.a.\ \sin.c.\ \sin.e.\ \&c.}{\sin.b.\ \sin d.\ \sin.f.} = 1$, or

$$\sin.a.\ \sin.c.\ \sin.e.\ \sin.g. = \sin.b.\ \sin.d.\ \sin.f.\ \&c.,$$

The same result would obtain, where H is *without* the Polygon.

Let QP be a line determined in position; having selected two stations A and B, it is required to find their distance from each other, this distance cannot be measured, but the angles a, b, c, d, can be taken by a theodolite.



Let $s = AQB = (AQP + PQB,)$ or, $180 - (a+c)$ [the other.

$(\sin. - b.\ \sin.\ \overline{s\ \ x}.\ \sin.(c+d) =$ one set; $\sin.(\overline{a+b})\ \sin.x.\ \sin.d =$

$$\sin.b.\ \sin.\ c+d.\ \sin.\ s-x = \sin.d.\ \sin.\ a+b.\ \sin.x.$$

$$\sin.b.\ \sin.(c+d)\ (\sin.s.\ \cos.x - \sin.x.\ \cos.s) =$$

$$\sin.d.\ \sin.\ \overline{a+b}\ \sin.x;$$

dividing by sin. x.] $\sin.b\ \sin.(c+d)\ (\sin.s\ \cot.x - \cos.s) =$

$$\sin.d\ \sin.(a+b);$$

$$\sin.b\ \sin.(c+d).\ \sin.s\ \cot.x - \sin.b\ \sin.(c+d).\ \cos.s =$$

$$\sin.d\ \sin.\ \overline{a+b};$$

$$\therefore \sin.b\ \sin.(c+d)\ \sin.s\ \cot.x =$$

$$\sin.b\ \sin.(c+d)\ \cos.s + \sin.d.\ \sin.\ \overline{a+b}.$$

getting rid of the coefficient of the cot. x; we have

$$\cot.x = \cot.s + \sin.d.\ \sin.\ \overline{a+b}.\ \mathrm{cosec}.b.\ \mathrm{cosec}.\ \overline{c+d}.\ \mathrm{cosec}.s.$$

This will give the value of x.—

Now $\sin.s : AB :: \sin.a : QB$;

$$\therefore AB = \frac{\sin.s}{\sin.a} QB = \mathrm{cosec}.a.\ \sin.s.\ QB;$$

and $\sin.(x+c+d) : QB :: \sin.(c+d) : PQ$;

$$\therefore \sin.\frac{(x+c+d)}{\sin.(c+d)}.\ PQ = BQ.$$

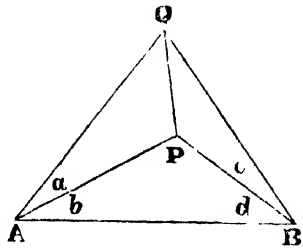
and, therefore, by substituting this value of QB, we have

$$AB = \mathrm{cosec}.a.\ \sin.s.\ \mathrm{cosec}.(c+d).\ \sin.(x+c+d).\ PQ;$$

and $\cot.x = \cot.s + \sin.d.\ \sin.(a+b.)\ \mathrm{cosec}.b.\ \mathrm{cosec}.(c+d).\ \mathrm{cosec}.s.$

Now $(a+b)$ and $(c+d)$ are the angles opposite PQ, therefore,

PROBLEM. II.



If the point, P, should fall within the triangle, and it is required to find AB as before.

We shall have

$$\text{cot. } x = \text{cot. } s + \text{sin. } d. \text{ sin. } a. \text{ cosec. } b. \text{ cosec. } c. \text{ cosec. } s.$$

when $s = 180 - $(the angles QAB+QBA)

otherwise $= 180-(a+b+c+d)$

$$\text{sin. } b. \text{ sin } \overline{s-x} \text{ sin. } c = \text{sin. } d. \text{ sin. } x. \text{ sin. } a.$$

$$\text{sin. } b. \text{ sin. } c. (\text{sin. } s. \text{ cos. } x - \text{sin. } x. \text{ cos. } s.) = \text{sin. } d. \text{ sin. } x. \text{ sin. } a.$$

$$\text{sin. } b. \text{ sin. } c. (\text{sin. } s. \text{ cot. } x - \text{cos. } s) = \text{sin. } d. \text{ sin. } a.$$

$$\text{sin. } b. \text{ sin. } c. \text{ sin. } s. \text{ cot. } x = \text{sin. } d. \text{ sin. } a + \text{sin. } b. \text{ sin. } c. \text{ cos. } s.$$

$$\therefore \text{cot. } x = \frac{\text{sin. } b. \text{ sin. } a.}{\text{sin. } b. \text{ sin. c. sin. } s\cdot} + \frac{\text{cos. } s.}{\text{sin. } s.} (\text{cot. } s\text{-})$$

and $\therefore$ $\text{cot. } x = \text{cot. } s + \text{sin. } d. \text{ sin. } a. \text{ cosec. } b. \text{ cosec. } c. \text{ cosec. } s.$

And the required AB=cosec. $(a+b)$ sin. s. cosec. c. sin. $(x+c)$.= cosec. (opp. ∠ to BQ), sin. s. cosec. (∠ opp. to PQ) sin. $(x+$QBP$)$ PQ

For sin. s : AB :: sin. $(a+b)$: BQ

$\therefore$ AB=sin. s. cosec. $(a+b)$.BQ

and sin. c : PQ :: sin. $(x+c)$: BQ

$\therefore$ BQ=sin. $(x+c)$ cosec. c.PQ

and therefore, by substituting the value of BQ, we have

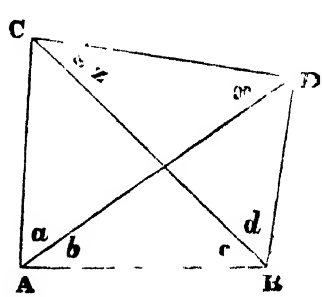
AB=cosec. $(a+b)$ sin. s. cosec. c sin. $(x+c)$ PQ

And, whether P fall within or without the triangle ABQ—

the required distance AB will always be

AB=PQ (cosec. BAQ. sin. AQB. cosec. QBP. sin. QPB.)

PROBLEM III.



Let CD be given in position, it is required to obtain a *correct base* AB, the angles a, b, c, d, being taken by the theodolite.

Now let $b+c=s=$CEA$=$EDC$+$ECD and let EDC$=x$, therefore DCB$=s-x$

Now

$$\text{sin. } x\text{. sin. } c\text{. sin. ADB. sin. } (s-x)=$$
$$\text{sin. } b\text{. sin. } d\text{. sin } x\text{. sin. ACB.}$$

but sin. ADB$=$sin. of supplemental $\angle$ $(b+c+d)$.

and sin. ACB$=$sin. supplemental $\angle$ sin. $(a+b+c)$. (see p. 162)

$$\therefore \text{ sin. } a\text{. sin. } c\text{. sin } (b+c+d)\text{. sin. } (s-x)=$$
$$\text{sin. } b\text{. sin. } d\text{. sin. } x\text{. sin. } (a+b+d).$$

or, sin. b. sin. c. sin. $(b+c+d)$. (sin. s. cos. x—sin. x. cos. s)$=$

$$\text{sin. } b\text{. sin. } d\text{. sin. } x\text{. sin. } (a+b+c).$$

dividing by sin. x) sin. a. sin. c. sin. $(b+c+d)$.(sin. s. cot. x—cos. s)

$$=\text{sin. } b\text{. sin. } d\text{. sin. } (a+b+c).$$

$$\text{or sin. } a\text{. sin. } c\text{. sin. } (b+c+d)\text{. sin. } s\text{. cot. } x=$$

sin. b. sin. d. sin. $(a+b+d)+$sin. a. sin. c. sin. $(b+c+d)$. cos. s

hence, by transposition and getting rid of, as before, the coefficient of cot. x, we have,

$$\text{cot. } x=\text{cot. } s+\text{sin. } b\text{. sin. } d\text{. sin. } (a+b+c)\text{. cosec. } a\text{.}$$
$$\times \text{ cosec. } c\text{. cosec. } (b+c+d)\text{. cosec. } s\text{.}$$

whence x. and $s-x$ are known.

Then as sin. $(a+b+c)$: AB : sin. $(a+b)$: BC

$\therefore$ AB $=$sin. $(a+b+c)$. cosec. $(a+b)$. BC.

again as sin. $(d+s-x)$; BC :: sin. d : DC

$\therefore$ BC$=$sin. $(d+s-x)$. cosec. DC.

AB$=$sin.$(a+b+c)$. cosec. $(a+b)$. sin. $(d+s-x)$. cosec. (d). DC.

$=$cosec. $(a+b)$. sin. $(d+s-x)$. cosec. d. sin. $(a+b+c)$. DC.

$\therefore$ AB$=$cosec. CAB. sin. CDB. cosec. CBD. sin. ACB. DC.

or, by using the former letters of P for D, and A for C,

AB$=$cosec. QAB. sin. QPB. cosec. QBP. sin. AQB. QP

Which is the same result as the two preceding cases, where the relative positions of the given and required bases materially differ from the present.

Hence, if the distance between two objects be correctly known, by previous surveys, and a BASE LINE be required, the actual measurement of which cannot be accurately depended upon, at the same time that perfect accuracy is indispensable in determining it, as it is to become the base of other triangulations, by selecting two stations, from each of which the two given objects and the other station can be seen, whatever may be the relative position of the given and required bases, the length of the *new base* can be always determined.

CHAP. VI.

PROBLEM I.

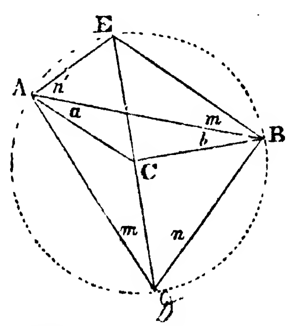
Given the position of three known points, to ascertain the position of a new station, whence those points can be seen.

(Resolved geometrically.)

Having previously determined the position of three different places, A, B, and C, not in the same straight line, and being desirous of connecting with them a new survey, which is at some distance, but from which the three objects can be seen, it is required to determine the position of any point D, within the new survey, by means of angles taken at that point between the three given points.

Let A and B be the extreme stations; join AB, AD, and BD. Now the other given point, C, can either fall on the line AB, or within or without the triangle ABD.

1st. *Let it fall within;* about the triangle ABC; describe a circle, join DC, and produce to E; join AE and EB.



Let the $\angle$ ADE $= m$, and the angle BDE $= n$, (in all the three cases,) then the angles ABE, BAE, will be also respectively equal to m and n, being upon the same segments of a circle, AE and BE.

Now, therefore, there are in the triangle AEB, the base AB, and the angles BAB and ABE, known.

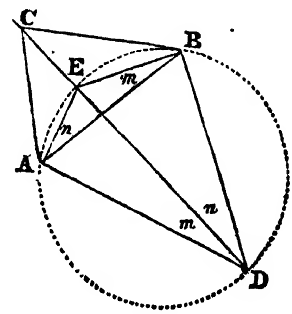
AE and EB are thus determined.

Again, the sides AC, and AE, being known, and the included angle, the angle AEC can be obtained $= d$, which is also an angle of the triangle DAE, whose other angle (m) at D has been taken, and therefore the angle DAE; or, because the angle CAE, $(a+n)$ is

known, their supplement becomes known, and there are, in the triangle DAC, the angles ADC, and DAC, and the side AC, given to determine the sides AD and DC; and in the triangle DCB, you have now DC and CB, and the angle CBD, (n), known.

PROBLEM II.

Now let C fall without the triangle ABD.



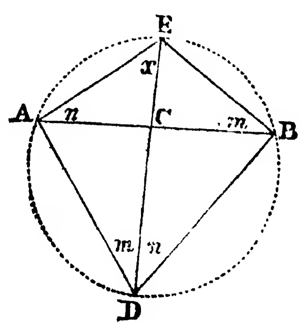
Describe a circle as before; join AE, and BE; then the angle BAE $=n$, and ABE $=m$; and the angle CAE $=a-n$; and the angle EBC $=b-m$.

Determine as before the sides AE, and EB, and, having the sides AB and BC known, you have two sides AC and AE, and their included angle, whence to obtain the angle ACD.

The after process is the same.

PROBLEM III.

Lastly, let A, B, and C, be in the same straight line.

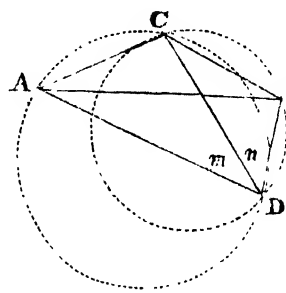


Let $\angle$ ADC $=m$, and CDB $=n$, as before, and describe round ABD a circle; produce DC to E, and join AE and EB.

AE and EB are determined, and the angle E; AC also is given; therefore the two sides CA, AE, and the included angle, DAE, are known, and, therefore, the angle AEC; and in the triangle AED, the angle DAE, the supplement of the other known angles is known also; whence, in the triangle ADC, there are two angles and a side known, and also in DBC. The required distances AD, CD, BD, are thence found.

The above are examples of solution, when the student is not acquainted with analytical trigonometry.

There is an easy method of constructing the figure, and determining upon paper the position of the required station.



Let ABC be the given triangle, and D the given station, whence the angles ADC, and CDB are taken.

To find the point D.

Upon AC describe a segment of a circle, having an angle (m) equal to ADC, and upon CB the segment of a circle, having an angle (m) equal to CDB, the point of intersection of the circles shall be the station D required.

These cases may be proved analytically, if the student should prefer it, in the same way as the Problems in Chapter V.

In the first case, where AB is given, and two angles a and b.

Make $s =$ the difference between the known angles and 180, and s ; and $s—x$, are the unknown angles required.

Equate the product of the sines of the alternate angles, as in Problem--, resolve the sin. $(s—x)$ into its equivalent values of the simple sines.

Arrange the known and unknown values on their proper sides of the equation.

Divide by the coefficient of cot. x, and you obtain the value of one of the unknown angles. $S—x =$ the value of the other. The sides are obtained by the formulæ, for the relative values of the sides, and the sines of their opposite angles.

In the second case, where AC and CB are given, and the included angle, ACB.

Let $s = 360$ degrees—$(ABC+m+n)$; let the angle CBD $= x$, then the angle CAD will be equal to $S—x$; ED is the common base of the two triangles.

Equate the value of ED in the two triangles, thus—

$$\text{As sin. } m : AC : \text{sin. } \overline{s - x} : DC$$
$$\text{as sin. } n : BC : \text{sin. } x : DC$$
$$\therefore \frac{AC, \text{ sin. } S - x}{\text{sin. } m} = \frac{BC, \text{ sin. } x}{\text{sin. } n}$$

$$\text{Or, } AC. \text{ sin. } n. \text{ sin. } (s - x) = BC. \text{ sin. } x. \text{ sin } m.$$
$$AC. \text{ sin. } n. (\text{sin. s. cos. } x - \text{cos. s. sin. } x) = BC \text{ sin. } x. \text{ sin. } m.$$
$$AC. \text{ sin. } n. (\text{sin. s. cot. } x - \text{cos. s}) = BC. \text{ sin. } m.$$
$$AC. \text{ sin. } n. \text{ sin. s. cot. } x = BC. \text{ sin. } m. + AC. \text{ sin. } n. \text{ cos. s.}$$
$$\text{and } \therefore \text{ cot. } x = \frac{BC}{AC} \text{ sin. } m. \text{ cosec. } n. \text{ cosec. s} + \text{cot. s.}$$

CHAP. VII.

TRIGONOMETRICAL SURVEY.

The following field notes are given for examples to the Student. A base line was taken on Hampstead Heath, and angles were taken between it and the Churches of Highgate, Hampstead, and St. Paul's. There was also an angle taken to a Stone Monument, at some distance on the Heath, on the ascent of a hill, whence a second angle could be taken to St. Paul's, which was not visible from both stations on the *base line*.

The distance between St. Paul's and Highgate was subsequently calculated, and this distance being assumed as an *old line*, a new line was taken at Streatham, whence angles, at each end, were taken to this old line, for the purpose of ascertaining the relative position of the new line, preparatory to determining the relative position of the two surveys at Streatham and Hampstead.

FIELD NOTES.

Between △ P & △ Q	86° 50′ 0″ 69° 8′ 27″	and St. Paul's and Highgate Church
betwn. Highgate Church △ P	108° 27′ 0″ 19° 48′ 40″	and △ Q and St. Paul's

	32·10	△ Q
fence—	4·81	—×
side of Reservoir 5	4·42	
side of Reservoir 5	0 61	△ P near new Church, opposite Crown and Sceptre

AT STREATHAM.

∠ elevation	0° 19′	ball of St. Paul's
	96° 2′	and St. Paul's
between △ A	11° 36′ 35″	and Highgate Church
at △ C		
angles of { elevation	0° 18′ 0″	of St. Paul's
angles of { elevation	3° 15′ 5″	of Highgate Church
angles of { depression	3° 15′ 0″	of bottom of hill on B. L.
	306° 3′ 40″	and Stone
	293° 49′	and Hampstead Church
	181° 51′ 40″	and St. Paul's
between △ A	74° 47′	and Highgate
at △ B		
	278° 11′	and Highgate Church
	94° 29′	and Stone
between △ B	81° 0′ 8″	and Hampstead Church
at △ A		
from △ A	24·74 chs.	to △B twice measured

BASE LINE

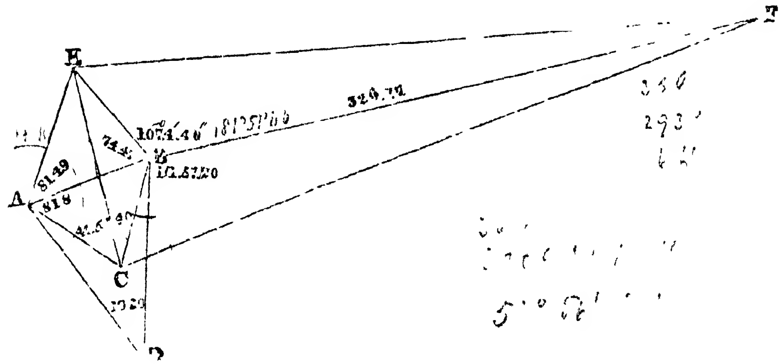
CALCULATIONS OF ANGLES TAKEN AT HAMPSTEAD.

Let AB be the base line, 24 chains 24 links long; E, Highgate Church; D, Hampstead; F, St. Paul's; and C, the Stone Monument by the road side.

At station A the following angles were taken—to the Stone, 81° 8′; to Hampstead Church, 94° 29′; computing, always, from the base line towards the right. The first angle is the angle BAC; the second is the angle BAD; the angle CAD is equal to the difference of these two, or 13° 21′; the other angle is 278° 11′, or an angle in the fourth quadrant; the angle BAE is the difference between this and 360° or 81° 49′.

At station B, the first angle taken, is to Highgate Church, ABE, 74° 47′; the second angle, to St. Paul's, 181° 51′ 40″, being the angle ABE, 74° 47′+the angle EBF, which is therefore 107° 4′ 40″. The next, to Hampstead Church, 293° 49′, making the angle ABD

the difference between this and 360°, or 66° 11′. The next angle is that to the Stone, which is 306° 3′ 40″, making the angle ABC, 53° 56′ 20″.



The angles of elevation were taken, in case of their being wanted.

And, lastly, at the Stone C, the angles were taken to Highgate Church and St. Paul's, 11° 36′ 35″, and 96° 2′.

The distances required, were EF, BF, and CF.

Calculations are annexed, for the sake of example, of a more accurate method of calculation, than is required in this case.

CALCULATIONS OF ANGLES TAKEN AT HAMPSTEAD.

Arrange, systematically, the work to be done, by dividing it into triangles, and apportioning the proper lines to be determined in each.

In the triangle AEB, find AE and BE.
In the triangle ACB, find AC and BC.
In the triangle CAE, find CE.
In the triangle CBE, find CE.
Their equality proves the work.
In the triangle CBF, find CF and BF.
In the triangle EBF, find EF^1.
In the triangle ECF, find EF^2.
EF^1 and EF^2 must agree.

To find BE in the triangle AEB.

As sine ∠ E = 23° 24′	= 9·5989523
is to AB = 24·74 chs.	= 1·3933997
so is sine ∠ A = 81° 49′	= 9·9955552
	11·3889549
	9·9589523
to BE = 61·66 chs.	= 1·7900026

To find AC, in the triangle ACB.

As sine 23° 24′		= 9·5989523
is to AB	= 24·74 chs.	= 1·3933997
so is sine ∠ A = 74° 47′		= 9·9845004
		11·3779001
		9·5989523
to AC = 60·1113 chs.		= 1·7789478

To find BC, in the triangle ACB.

As ∠ C = 44° 55′ 40″	= 9·8489368
is to AB = 24·74 chs.	= 1·3933997
so is sine ∠ A = 81° 8′	= 9·9947788
	11·3881785
	9·8489368
BC = 34·61 chs.	= 1·5392417

Again, to find AC in the triangle ACB.

As sine ∠ C = 44° 55′ 40″	= 9·8489368
is to AB = 24·74	= 1·3933997
so is ∠ B = 53° 56′ 20″	= 9·9076207
	11·3010204
	9·8489358
to AC = 28·3195 chs.	= 1·4520836

In triangle CAE, to find the unknown angles.

As sum of sides (AC + AE), 88·43	= 1·9465996
is to their diff. = AC—AE, 31·79	= 1·5202905
so is tan. of ½ sum unknown ∠ s E and C	
or 8° 31′ 30′	= 9·1757930
	10·6780135
	1·9465996
to tan. of ½ difference, 3° 5′ 4″	= 8·7314839

The angles being known, to find EC.

As cos. of ½ diff. of unknown ∠s E and	
C or, 3° 5 4″	= 9·9993704
is to cos. of ½ sum, 8° 31′ 30″	= 9·9951749
so is sum of sides (AC+AB), 88·4 chs.	= 1·9465996
	11·9417745
	9·9993704
to EC = 87·58 chs.	= 1·9424041

In the triangle ECB, to find the unknown angles, and thence the side EC.

As sum of sides (CB+BE) 96·27 chs.	=	1·9834910
is to difference (CB−BE)=27·05 chs.	=	1·4321673
so is tan. of ½ sum of unknown ∠s E+C = 25° 38′ 20″	=	9·6812000
		11·1133673
		1·9834910
to tan. ½ diff. = 7° 40′ 49″	=	9·1298763
And as cos. ½ diff. of angles = 7° 40′ 49″	=	9·9960862
is to cos. ½ sum, 25° 38′ 20″	=	9·9549845
so is sum of sides 96·27 chs.	=	1·9834910
		11·9384755
		9·9960862
to EC = 87° 58′ as before	=	1·9423893

In triangle CBF, to find CF and BF.

As sine 4° 41′ 40″	=	8·9129747
is to 34·61 chs.	=	1·5392417
so is 51° 6′ 20″	=	9·8911489
		11·4303906
		8·9129749
to BF, 329·17 chs.	=	2·5174157
And, as sine 4° 41′ 40″	=	8·9129747
is to 34·61 chs.	=	1·5392417
so is sine 124° 12′	=	9·9175478
		11·4567895
		8·9129747
to 342·80 chs. CF	=	2·5438146

In triangle EBF, to find angle s.

As sum or sides	=	390·83	=	2 5919879
is to diff.	=	267·51	=	2·4273400
so is tan. ½ sum ∠s	=	36° 27′ 40″	=	9·8685922
				12·2959322
				2·5919879
to ½ diff.	=	26° 49′ 42″	=	9·7039443

∠ BEF = 63°17′22″

∠ BFG = 9°37′58″

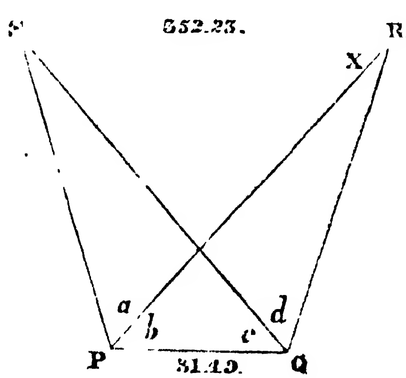
As cos. of diff.	26°49′42″	=	9.9505413
is to cos. ½ sum,	36°27·40	=	9·9053967
so is sum of sides,	390·83 chs.	=	2·5919879
			12·4973846
			9·9505413
to 352·24 chs. = (EF)		=	2·5468433

In triangle ECF, to find EF.

As sine 14° 19′ 38″	=	9·3935204
is to 87 58 chs.	=	1·9424041
so is sine 84° 25′ 29″	=	9·9979409
		11·9403450
		9·3935204
to 352·23 chs. = EF	=	2·5468236

Calculations to determine the length and the relative position with the Hampstead Survey of the base line at Streatham.

As it was doubtful, whether, in the observation taken at station Q, the angle PQS was taken to the proper object, from the extreme fogginess of the morning, which prevented the spire of the Church (Hampstead) from being distinguished from many others in the neighbourhood, though that observation, if to the right object, was, in itself, correctly and carefully taken, it was checked by assuming the distance PQ (taking its measured distance), as known, and *calculating* what the angle should be, between the proper object and the given base line.



To find the angle PQS, being the angle at the point Q, made between the base line and Highgate Church.

First find PR, in the triangle PQR.

As sin. 4° 31′ 40″	=	8·8973097
is to 31·49 chs.	=	1·4981727
so is sin. 86° 50′	=	9·9993364
		11·4975091
		8·8973097
to PR. 398·29 chs.	=	2·6001994

To find the angle PSR, in the triangle PRS.

As 352·23 chs.	=	2·5468246
is to sin. 17° 48′ 40″	=	9·5300976
so is PR 398·29 chs.	=	2·6001994
		12·1302970
		2·5468246
to sin. PSR, 22° 32′ 4″	=	9·5834724

To find PS in the same triangle.

As sin. 19° 48′ 40	=	9·5300976
is to 352·23 chs.	=	2·8283272
so is sin. SRP, 137°·32′·16″ (42°.20′.44″)	=	9·8283972
		12·3752218
		9·5300976
to PS, 700·045 chs.	=	2·8451242
As PS+PQ, 731·53 chs.	=	2·8642311
is to PS−PQ, 668·55 chs.	=	2·8251339
so is tan. ½ s = 35° 46′ 30″	=	9·8576700
		12·6828039
		2·8642311
to tan. ½ d = 33° 21′ 27″	=	9·8185728

the required angle	69° 8′ 27″	which was the same as that observed by the theodolite.
	2° 34′ 33″	
	108° 27′ 00″	
	180° 0′ 0″	

The following calculations were made for the purpose of verifying the correctness of the several angles taken at the two stations P and Q, by comparing the computed with the measured length of PQ.

Let $s=b+c$, where b is the angle RPQ, 88° 38′ 20″, and c is the angle PQS, 69° 8′ 27″; $b+c$ being, together, equal to the unknown angles QSR and SRP.

Let SRP $=x$, then QSR will be equal to S—x.

Now, by Problem III., Chap. V., let $s=b+c$

cot. x=cot. s+sin. b. sin. d. sin. $(a+b+c)$. cosec. a cosec. c. cosec. $(b+c+d)$. cosec. s

sin. b =	88° 38′ 20″	=	9·9998774
sin. d =	17° 41′ 33″	=	9·4827426
sin $(a+b+c)$ =	177° 35′ 27″	=	8·6236120
cosec. a =	19° 48′ 40″	=	10·4699023
cosec. c =	69° 8′ 27″	=	10·0294406
cosec. $(b+c+d)$ =	175° 28′ 20″	=	11·1026903
cosec. =	157° 46′ 47″	=	10·4223198
to rad. (10 Index)		=	70·1305849
			—70
(to rad. 1,) 1.3507831 =			0.1305849

again cot. S=cot. 157° 46′ 47″ and being in the second quadrant =—cot. 22° 13′ 13″.

—(log. cot. 22° 13′ 13″) =—(10·388892) to rad. 10.

subtract 10

(to rad. 1)—(2·4483819)=—(0·388892)

1·3507838 (see above)

—2·4483819

—(42°30′)=natural cot. x =—1·0975971

rad. cot. 42° 31′=1·0977020		5971=given angle
rad. cot. 42° 30′=1·0970609		0609=42°30′
6411		5362
		60
6411	311720	48

=cot.—(42° 30′ 48″)=its supplement 137° 39′ 12″=x

but PQ = cosec. SPR. sin. PSQ. cosec. PQS. sin. SRP (x)

viz. log. SR.	352·23 chs.	=	2 5468246
log. cosec.	19°·78′·40°	=	10·4699023
log. sin.	2°·21′·33″	=	8·6336120
log. cosec.	69°·82′7″	=	10·0294405
log. sin.	137°·39′ 12″	=	9·8284115
the length of PQ=log.	31·4912 chs.	=	1·4981909

CHAP. VIII.

TO REDUCE ANGLES TO THE CENTRES OF STATIONS.

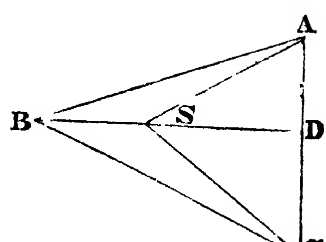
In large trigonometrical surveys, as those objects, which, from their position, are more commanding, and are furthest visible, are generally so situated, as to prevent the theodolite being placed immediately in the centre, it has been found necessary to calculate expeditiously the angle of reduction to the centre, or the difference between the angle, as taken from a station (S) near the centre of this object, and the angle from the centre.

You have, for instance, determined the position of a church and tower, and taken angles to the vane of the church, or the pole of the tower, but on wishing to base upon the line connecting them a further triangulation, you find you cannot place your instrument at the centre of either, you are therefore compelled to take your angles as near to it as possible. This angle, thus taken, will fall either within this new triangle, or without, or upon one of its sides.

If it falls within, it will be greater than the true angle; if without the triangle, but within the circumscribing circle, it is greater; if upon the circumference, equal; if without it, less.

The amount of this excess or deficiency we are now to ascertain.

1st. Let it (S) fall within the triangle ABC; the angle taken will be ASC.



To find its excess, or, to reduce it to ABC, produce BS to D.

$\angle$ ASD $=$ $\angle$ ABS $+$ $\angle$ SAB, $\angle$ CSD $=$ $\angle$ CBS $+$ $\angle$ SCB, and
$\angle$ ASC $=$ $\angle$ ABC $+$ $\angle$ BAS $+$ $\angle$ BCS;
but BC, and AB, and BS are known distances, and the angle BSC is known also.

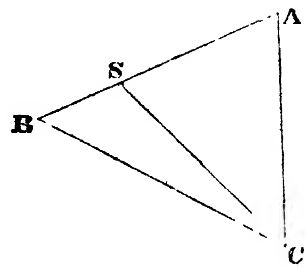
Now, in the triangle BCS,

as BC : sin. BSC :: BS : sin. angle BCS,

therefore $\angle$ BCS $=\frac{BS}{BC}$sin. BSC., and for the same reason,

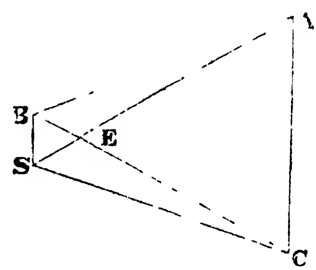
$\angle$ SAB $=\frac{BS.}{BA}$sin. BSA, and, therefore,

$$\angle ABC = \angle ASC - BS\ \left(\frac{\sin. BSC}{BC}+\frac{\sin. BSA}{BA}\right)$$



2nd. Let s fall upon the side AB, then $\angle ABC = \angle ASC - BS\ \left(\frac{\sin. BSC}{BA,}\right)$ $+\left(\frac{\sin. 180^\circ}{BA}=0\right)$

$\therefore\ \angle ABC = \angle ASC - BS\ \left(\frac{\sin. BSC}{BC}\right)$



3rd. Let s fall *without* the triangle ABC, and let E be the point of intersection of AS and BC. Now the $\angle$ AEC $=$ ABC$+$BAS$=$ASC$+$BCS, therefore $\angle$ ABC $= \angle$ ASC $+ \angle$ BCS $- \angle$ BAS; but

$\angle$ BCS $=\frac{BS}{BC}$sin.(BSA$+$ASC)

and $\angle$ BAS $=\frac{BS}{AB}$ sin. (BSA)

$$\therefore\ \angle ABC = \angle ASC + BS\ \left(\frac{\sin. (BSA+ASC)}{BC} - \frac{\sin. BSA}{AB}\right)$$

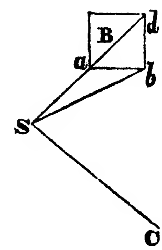
In the case where BC and AB are infinite, each of the expressions, where they occur, will vanish; and $\angle$ ABC becomes equal to $\angle$ ASC, which is the case when the objects are heavenly bodies.

This obtains also, when the station falls upon the circumference of the circumscribing circle.

It is not always possible, however, to measure BC, or take the angle CSB.

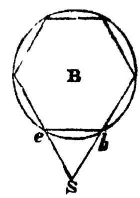
As it is desirable that the instrument should be placed close to the centre of the station, the vane of the steeple, or the flag pole of the tower, may be invisible from it.

When this is the case, if the centre be the vane of a church, as in the diagram, when the tower is squared, select S in a line with the diagonal. Measure $Sa + \frac{a\,d^*}{2} = SB$; and take the angle $CSa = CSB$.

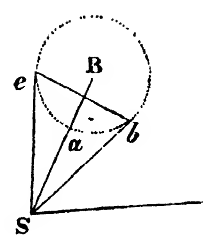


Again, let the tower be a hexagon.

Produce outward two of the sides to S, where place the instruments; (ebS) is an equilateral triangle, and $SB = e\,b\sqrt{3}$ * and $\angle CSB = \angle(C\,S\,b + \frac{bSe}{2})$



If the tower be circular, the $\triangle$ $CSB = \angle CS + \frac{bSe}{2}$, but $\angle SB = bS.\ \text{sec.} \sqrt{\frac{bSe}{2}}$. or if the circumference of the tower can be taken $= S\,a$ + radius.



EXAMPLES OF CALCULATIONS OF THE REDUCTION OF ANGLES TO THE CENTRE.

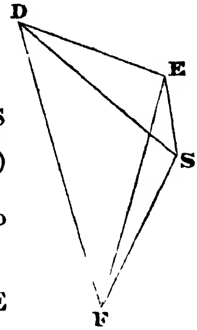
EXAMPLE 1.—Determining the angle made at the centre of the spire of Highgate Church, between the spire of Hampstead Church and St. Paul's, from angles taken in the Church-yard, between St. Paul's and Hampstead, and St. Paul's and Highgate.

---

* The interior angle of a hexagon is 120°, the angle at e 120°, and Be besects it, and the $\angle$ B eb, is 60°, but the angle S eb is also 60°, and therefore BS is bisected by eb, but each of its bisections $=$ tan. 60° $\times \frac{eb.}{2}$

$\therefore$ The whole BS $=$ tan. 60° $\times\, eb$, and, as tan. 60° $= \sqrt{3} = eb.\sqrt{3}$.

from $\triangle$ s	2·56	to Highgate Church Spire
	156° 16′	and Highgate Church
between St. Paul's at $\triangle$ s	85° 25′	and Hampstead Church



Let $x =$ the unknown angle DEF.

Now, by the preceding formula,

$\angle DEF + \angle EDS = \angle EFS + \angle DSF$.

$\therefore \angle DEF$ or $x = \angle DSF + \angle EFS - \angle EDS$

but the sine of the angle $EFS = ES\ (\frac{\sin. \angle ESF}{EF})$

where EF is the distance from St. Paul's to Highgate Church;

and the $\sin. \angle EDS = ES(\frac{\sin. \angle ESD}{DE})$. where DE is the distance from Hampstead to Highgate Church.

Hence the values of the angles EFS, and EDS can be obtained; and, by adding their difference to the observed angle DSF, you obtain the value of the corrected angle DEF, from the centre of the previous point of observation.

To find the distances EF and DE, which are at present unknown, having the data given in the preceding chapter.

In the triangle ADB (see Diagram p. 171), to find the distance DB,

As sine $\angle$ ADB* (19° 20′)	= 9·5199112
is to AB, 24·74 chains	= 1.3933997
so is sine $\angle$ DAB, (94° 29′)	= 9·9986691
	11·3920688
	9·5199112
to DB = 74·50 chains,	= 1·8721576

In the triangle EDB, to find ED.

Having calculated the length of DB, we have two sides and the included angle, therefore—

As sum of sides (DB+BE) 74·50+61·66	= 2·1340495
is to their difference, 74·50—61·66 chs.	= 1·1085650
so is tan. $\frac{(x+y)}{2}$ 19° 41′	= 9·5535477
	10·6621127
	2·1340495
tan. $\frac{x-y}{}$, 1° 30′ 18″	= 8·4194982

* The angle ADB is the supplement of the observed angles DAB, and DBA.

$$\begin{array}{l} 19^\circ\ 41' \\ \ \ 1^\circ\ 30'\ 18'' \\ \hline 21^\circ\ 11'\ 18' = \text{greater angle DEB} \\ 18^\circ\ 10'\ 42'' = \text{smaller angle EDB} \end{array}$$

Again, in the same triangle,

As sin. ∠ EDB, 18° 10′ 42″	= 9·4941207
is to EB, 61·66 chains,	= 1·7900026
so is sin. ∠ EBD, 39° 22′	= 9·8022816
	11 5922842
	9·4941207
to ED, 125·36 chains,	= 2·0981635

Now substitute their proper values in the two equations, viz.,

$$\sin. \angle\ EFS = ES\ \frac{(\sin. \angle\ ESF)}{EF}$$

and $\sin. \angle\ EDS = ES.\ \frac{(\sin. \angle\ ESD)}{DE}$, thus :—

log. sin. ∠ EFS = log. ES + log. sin. ∠ ESF — log. EF,
and log. sin. ∠ EDS = log. ES + log. sin. ∠ ESD — log. DE,

log. ES, 2·56 chs.	= 0·4082400
+log. sin. ∠ ESF, (156° 16′)	= 9·6030166
	10 0112566
—log EF,	= 2·5468246
log. sin EFS (0° 10′ 1″)	= 7·4644320
and log. ES 2·56	= 0·4082400
+log. sin. ∠ ESD (70° 51′)	= 9·9755394
	10·3837794
—log. DE, 125·36 chains	= 2·0981630
log. sin. EDS (1° 6′ 22″)	= 8·2856159

but ∠ DEF or x = ∠ DSF + (∠ EFS — ∠ EDS)

$$\begin{array}{rl} \angle\ EFS = & 0^\circ\ 10'\ 1'' \\ -\angle\ EDS = & 1^\circ\ 6'\ 22'' \\ \hline \text{their difference} = & 56'\ 21'' \end{array}$$

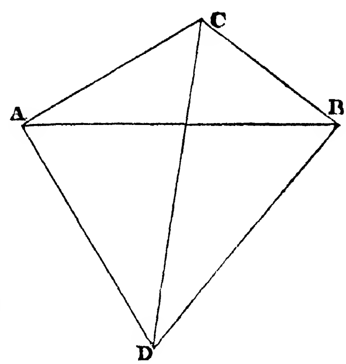
therefore 85° 25′ (∠ DEF) + (—56′ 21″ = (84° 28′ 39″) = ∠ DEF = x which is the angle at the spire of Highgate Church, between Hampstead Church and St. Paul's.

Example 2.—Having measured the distances between two objects AB 2 miles, 6 furlongs, and taken the angles at the base line, to a third object, viz., 59° 25′ 40,″ and 78° 24′ 20″. It was required to *observe* the remaining angle at the third station, as a check upon the other two angles; planted the theodolite at a distance of 3 chains from it, and found one of the angles 80°. What must the other be, so that the previous observations should be correct?

CHAP. IX.

ON TRIANGULATION.

In extensive surveys, carried on by a continued system of triangulation, the most important part is the proper selection of a base line, proportionate to the intended extent of the survey. This base line should be measured on a nearly level surface of country. In hilly countries the plane of a valley is generally selected for that purpose.

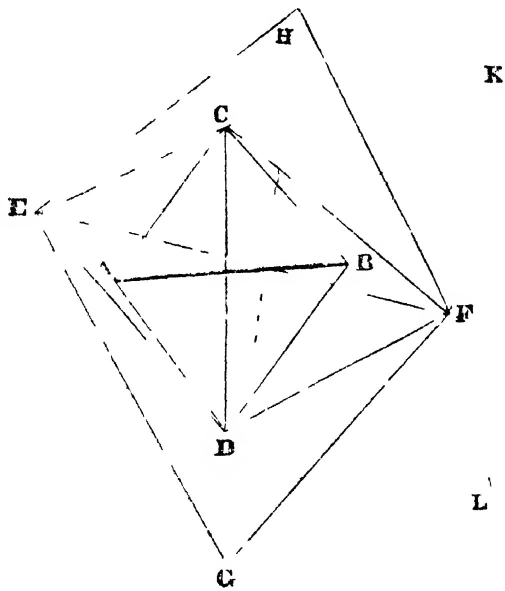


Let AB be the base, measured in a valley, and CD, two prominent objects, on neighbouring hills, which are visible from A and B.

Take at C, the angles ACD, BCD; and at D, the angles ADC, BDC; and the stations, C and D, are determined, by the formulæ given in Chap. V., Prob. I. DC thus becomes a new base line, of longer extent and equal accuracy with AB; or, by taking at A and B, when B is visible from A, the angles between the base, and C and D respectively; C and D can be more directly and more accurately determined, as in either triangle its length will be the same. The three angles should, in all cases, be taken, if practicable, as from their sum, which should be equal to 180 degrees, the accuracy of the work can be tested.

The sides of the triangle should be, as nearly equilateral as possible, or the angle at the new station should not differ, materially, from 90 degrees. All the sides should be calculated and plotted from their determined lengths, and not protracted from their angles, as the smallest error of an angle would be of injurious effect, in determining the position of the new station.

The accompanying diagram will exhibit the method of extending a system of triangulation, and of obtaining, between inaccessible stations, a base line, commensurate with the extent of the survey.



Let GH be the base line required, and inaccessible; AB, the only favourable base line that can be measured.

By determining from AB the position of C and D, CD becomes a new station; from CD determine E and F, EF again becomes a new base for the stations G and H, which, by observing the angles EHF and EGF, and comparing their calculated with their observed values, becomes a base, as much to be depended upon as the first AB.

It is not, of course, necessary that the triangulation should be carried on in the regular manner, exhibited in the figure, as it might be branched off in any direction that might be required; for after having, from AB, determined C and D, CB might be taken as the new base, as correctly as DC, and the triangulation extended towards K; or, after having determined E and F, it might be extended towards L; and by a similar process, in any other direction whatever.

Having carried the triangulation in the direction, and to the extent required, it becomes desirable, for the sake of testing the accuracy of the work, to make one of the lines, of the last triangles, *a base of verification*, by selecting for it a level position, along the slope of a

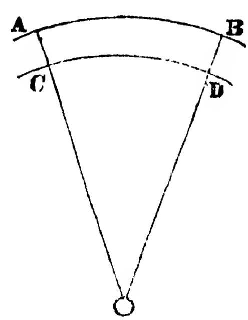
hill, or in the bottom of a valley. This base, as it can be computed trigonometrically, being compared with its actual length by measurement, is a test, not only of its own accuracy, but of all the various triangles that subserve to its determination.

As the triangulation goes on, the sides increase in length, and the angles taken are between objects of some miles distant. It becomes then imperative to call in the aid of science to make the objects distinct. For this purpose, various contrivances have been, at different times, adopted, such as plane mirrors, disks of tin, plane convex lens, parabolic reflectors, to receive the rays of artificial light, that were thrown upon them, by means of Argand lights, balls of burning lime, &c. These latter were introduced by Captain Drummond, who managed to send through a flame of alcohol, a powerful stream of oxygen gas, upon the lime, which was placed in the focus of a parabolic reflector.

By these means, angles have been taken to objects from 40 to 60 miles off.

REDUCTION OF A BASE LINE, TO THE LEVEL OF THE SEA.

Let CD be the length of the line at the level of the sea $= x$, AB, the measured line above it. Let AC be the height of the measured line upon the level of the sea $= h$, and CO be the radius of the earth $= R$. It is required to find the length of CD.



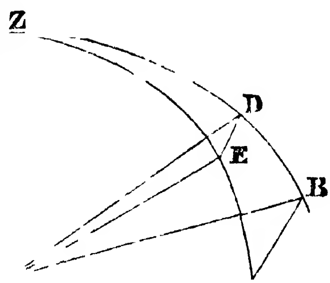
As the arcs of different circles, subtending the same angle, are proportional to their radii, we have,

$$AO : CO :: AB : CD, \text{ or}$$
$$R+h : R : L : x$$

$$\therefore\ x = \frac{R}{R+h} \cdot L; \text{ and } \therefore\ L - x = L - \frac{R}{R+h} \cdot L = \frac{h}{R+h} \cdot L$$

Or the *excess* of any line AB, measured above the level of the sea, is equal to the length of the line, into its height above that level, divided by its distance from the centre of the earth. Now the radius of the earth is 21,008,000 feet, therefore log. L in feet + log. h in feet—log. $(21{,}008{,}000+h)$ in feet = log. excess of measured base above the true base at the level of the sea. This base is the arc CD.

To reduce angles, taken in a plane inclined to the horizon, to angles in the horizontal plane.



Let DAE, the angle taken at the point D, between the objects D and E, be in a plane, inclined to the horizon. Let BAC be the horizontal plane, and Z the zenith of the observer at A, then AZEC, AZDB will be portions of planes, of large circles, passing through the radius AZ, and the horizontal angle will be BAC.

Let CAE be the angle of elevation of the station E, and DAB that of D, of which EC and EB are the measures ; ZE and ZD are the zenith distances, or measures of the complements of these angles of elevation ; and DE is the measure of the angle DAE.

In the triangle ZDE, there are ZE, ZD, and DE given, hence the sine $\frac{1}{2}$ BAC or $\frac{1}{2}$ DZE $= \sqrt{\frac{\text{sin.}\ \frac{1}{2}\ (S-ZE)\ \text{sin.}\ \frac{1}{2}\ (S-ZD)}{\text{sin. ZE. sin. ZD}}}$

S, being the sum of the sides of the triangle ZED ; or sin. $\frac{1}{2}$ Z

$$= \sqrt{\frac{\text{sin.}\ \frac{1}{2}\,(\angle\,\text{DAE} + \angle\,\text{BAD} + \angle\,\text{CAE}).\ \text{sin.}\ \frac{1}{2}\,(\text{DAE} + \angle\,\text{CAE} - \angle\,\text{BAD})}{\text{cosine}\ \angle\ \text{BAD. cosine}\ \angle\ \text{CAE}}}$$

If the angle BAD = angle CAE, or the objects be of the same altitude, then,

$$\text{sin.}\ \tfrac{1}{2}\ Z = \sqrt{\frac{\text{sin}^2\ \frac{1}{2}\ \text{DAE}}{\text{cos.}^2\ \text{CAE}}} = \frac{\text{sin.}\ \frac{1}{2}\ \text{DAE}}{\text{cos. CAE}}$$

In the first case, when the angles of elevation differ slightly, and when each of them is but small, not exceeding 2° or 3°, as the cosines of angles vary slowly, the following formulæ may be safely adopted, viz., sin. $\frac{1}{2}$ Z $= \frac{\text{sin.}\ \frac{1}{2}\ \text{DAE}}{\text{cos.}\ (H+h)}$. when H and h are the respective angles of elevation, which becomes a convenient logarithmic formulæ, viz., log. sin. $\frac{1}{2}$ Z = 10 + log. sin. $\frac{1}{2}$ DAE—log. cos. $\frac{1}{2}$ (H + h) ; but the angle BAC is the measure of the angle Z, or the plane angle made between the two planes.

SPHERICAL EXCESS.

The angles taken between any three points on the surface of the earth, by a theodolite, are, strictly speaking, spherical angles, and their sum must exceed 180° ; and the lines bounding them, are not the chords, as they should be, but the tangents to the earth.

This excess is inappreciable in common cases, but in the larger triangles it becomes necessary to allow for it, and to *diminish* each of the angles of the observed triangle, by one third of the spherical excess.

To calculate this excess.

Divide the area of the triangle in feet, by the radius of the earth in seconds, and the quotient is the excess, viz.,

$$\text{excess} = \frac{\text{area}''}{R^2}. \quad \text{or,}$$

because the radius = as above, 21,008,000 feet, and one second of space = 101·43 feet, then $101{\cdot}43^2$ feet = the area in a square second, and radius = 206·264 seconds of space ∴ excess in seconds = $\frac{\text{area in square feet}}{(101{\cdot}43)^2 \times 206264}$ or, log. excess = log. area —9·3267737.

When two sides and the included angle are given.

$$\text{cot. } \tfrac{1}{2} \text{ excess} = \frac{\text{cot. } \tfrac{1}{2}\, a - \text{cot. } \tfrac{1}{2}\, b + \text{cos, C.}}{\text{cos. C.}}$$

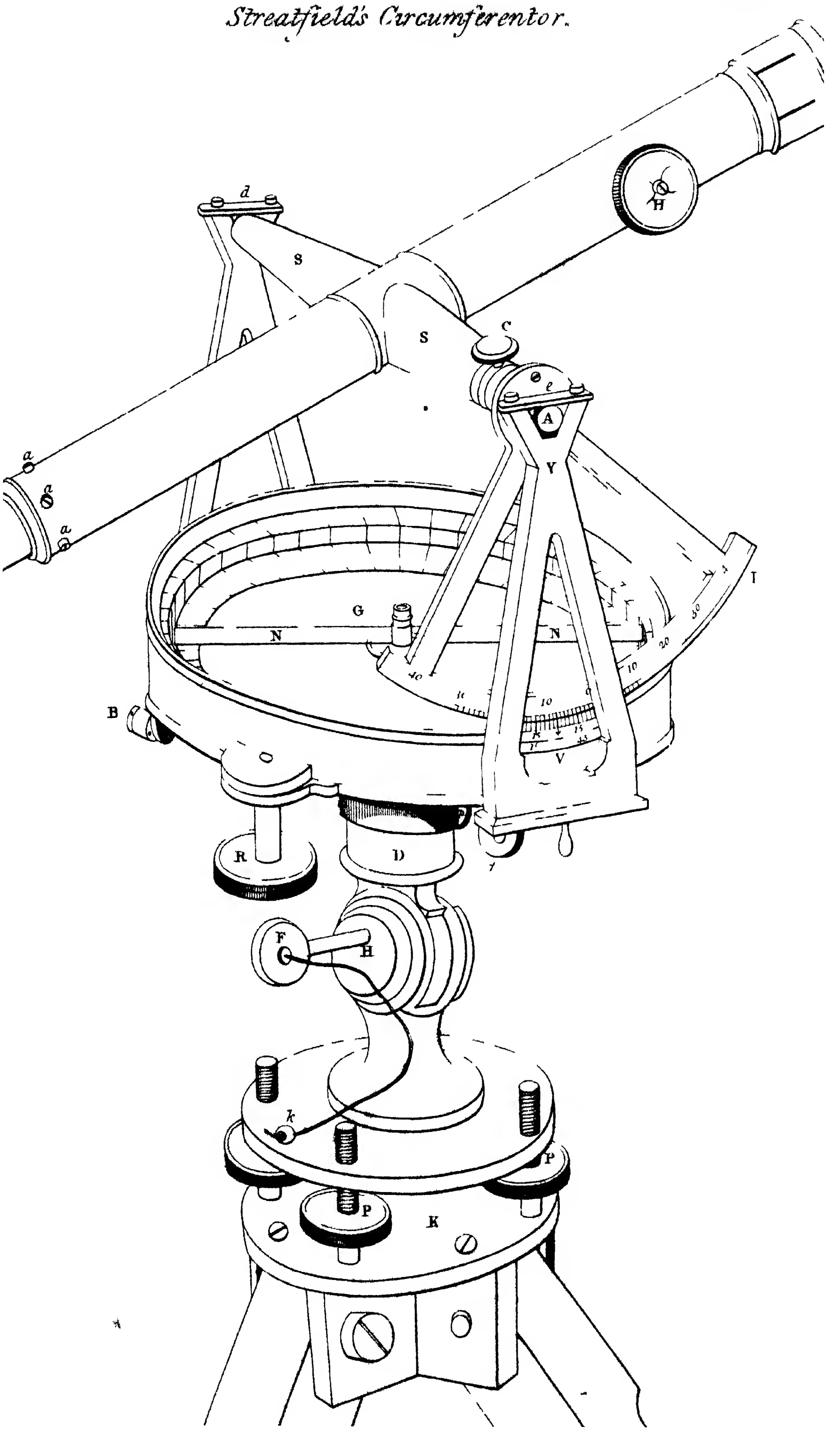
When the three sides are given.

$$\text{tan. } \tfrac{1}{4} \text{ excess} = \left(\frac{\text{tan. } a+b+c}{4}\right) \cdot \left(\frac{\text{tan. } (a+b-c}{4}\right)$$
$$\left(\frac{\text{tan. } a-b+c}{4}\right) \quad \left(\frac{\text{tan. } b+c-a}{4}\right)$$

To allow for this excess.

In any triangle take the three angles, find their sum. Calculate from these angles the spherical excess, by the above rule; the sum of the angles taken should amount to 180°+ this spherical excess ; if not, the difference must be divided among the three observed angles, so as to make them, when thus corrected, equal to 180°+ the excess; then subtract one third this excess from each of the angles, and their sum will be reduced to 180°, the correct measure of a plane triangle.

Streatfield's Circumferentor.



LAND SURVEYING.

Part the Third.

THE CIRCUMFERENTOR.

THE Circumferentor is the mariner's compas differently divided, and furnished with sights, standing upon one or three legs, and capable of a horizontal motion, by means of the usual parallel plates, or a ball and socket. In new countries, where expedition is required, the circumferentor is generally used with a ball and socket, and with one leg or staff, strongly shoed with iron. After taking the angle at any station, the head part is taken off, and is carried by the Surveyor under his arm, the staff is seized in his hand, and the next station is proceeded to. The mould of the woods is soft, and easily penetrated; and as for the purpose of blazing* the trees the general direction alone is wanted, which may be blazed within 10 yards on either side of the line, there is no delay arising from the placing of flags and driving of stakes, the axemen, being

* Blazing, in American phraseology, is taking a slice off the bark of a tree into the wood.

generally in advance or close upon the heels of the Surveyor, and the chain men overtaking him, before he has taken his next sight.

All the LINE trees, as trees are termed, which stand directly in the line, in addition to being *blazed*, are *witnessed* with three notches, and being marked where the line strikes them, become permanent boundary marks—of so permanent and distinctive a character, that the very year almost of their being blazed is distinguishable by an experienced hand. The scar seldom grows over entirely: never without a seam, which, if carefully cut, will, by the number of rings, that every year has added to it, show the exact year of the incision.

The three legged staff, however, is very useful; especially in old countries, or in the surveying of roads in frosty weather, and under various other circumstances, when the single staff cannot penetrate, or when walls intervene, upon which the short legs of the staff can be advantageously placed.

STREATFIELD'S CIRCUMFERENTOR.

This instrument consists of a compass box (G); divided into degrees, and, by means of a vernier, subdivided into three minutes.

It stands upon three legs, and, by means of a pair of parallel plates (K*k*,) is capable of a truly horizontal position, which is determined by a level, placed, so as not to interfere with the reading, under the compass box (the end of it alone can be seen in the accompanying plate at B).

This compass box, (G,) has an absolute horizontal

motion round its centering at D, and is fastened by the clamp screw (a side of which is visible) at *p*.

When this is clamped, by detaching the pin *f*, which passes through the two plates of the compass box, the brass one, which, with the vernier attached, works round the inner one, on which are divided the degrees, is capable of a relative motion, and thus partakes of the character of the theodolite; this motion is communicated to it by a rack and pinion at R.

There is, however, one thing wanting in this instrument, which is indispensable in a new country. There is no means of clamping the vernier, except when it is at zero of the dividing-plate. In running lines, of a given direction, through the woods of a new country, where the bearings are often odd minutes, it would be impossible, at every fresh station, to be altering the vernier.

In the circumferentors that are used in America, they have placed the vernier outside, and by that means have contrived to clamp in to any position; there is also a tangent-screw for fine adjustment, so that being properly adjusted for the odd minutes, or the variation of any kind, and clamped, the bearing of the full number of degrees need alone be referred to throughout the whole line. None but a practical man can be aware of the immense advantages resulting from this arrangement.

YY are two frames, or supports, capable of being taken on and off, on the Y's of which rests the arm (S*s*) of a telescope, so contrived, as to move in a truly vertical position, when the instrument is horizontal. The telescope has its usual adjustments for the object and eye-glasses, and for the line of collimation.

To one end of the cross piece of the telescope is attached a graduated arc of a circle, with a vernier fastened to the supports of the Y, for taking vertical angles. This instrument has also, in addition to the telescope (not shown in the plate), the usual pair of sights, whose position would be, in the meridian line of the instrument, in the same plane as the telescope moves in.

The introduction of the telescope was forced upon me, from the constant inconvenience I experienced in the back woods of America, (where I was engaged in government surveys,) from the almost uselessness of the common sights, in surveying up and down hills, across steep but narrow vallies; all these difficulties were remedied by this contrivance, the advantages of which I have tested practically in the woods. The telescope is capable of a vertical motion either way of 45 degrees, and moving, truly vertically, enables you to carry a line of a given direction either down or up a hill, and from its magnifying power to secure, at the same time, a check position at the top of the ascent beyond; so that, after having descended and ascended, you may prove your correctness.

The instrument, also, has other contrivances; the pin F, when taken out, allows the whole instrument to be turned upon its side, and the spirit level at B, being then in a horizontal position, the instrument is made capable of a vertical motion, reading off to three minutes, by means of the vernier.

This may sometimes become a fair substitute for a sextant on a clear night; though I should myself, in all cases, prefer each instrument being kept to its

especial purposes, as, the more simple an instrument is, the more accurate.

DIVISION OF THE CIRCUMFERENTOR.

The line of sights is made the north and south end of the *instrument*, and from each of these the circular rim is graduated toward the east and west points, from 0° to 90°.

On the right of the north of the instrument, looking to the north point of it, should be lettered west, and on the left should be lettered east; and any point between the north and west point of the instrument, read by the north end of the needle, is read north, so many degrees west.

When the needle is released, and is allowed to play freely, it points toward the magnetic north. The north of the instrument points to the object whose bearing is required, the angle made between these two must necessarily give the relative position of the line of the object, and the magnectic north and south line; and the bearing of the object, by reading off the number of degrees to which the needle points on the graduated circular rim, is thus obtained.

As the needle points to the north, should the object bear to the west, the line of sights would be on the left of the needle, and the north end of the needle would be on the right of the line of sights; if, therefore, on the right of the instrumental north, looking northwards, were marked east, the needle, which should at once, with the number of degrees, read off the bearing, would give east, but the bearing is west. Having been, as was shown above, marked

west, the needle reads west, that is, north so many degrees west. By this arrangement the needle gives at once the degrees and bearing.

As the accuracy of the bearing depends, in a considerable degree, upon the goodness of the needle, great care should be observed in using it, and in marking whether it continues vibrating, or soon settles.

In the latter case there is some radical defect, arising either from the diminution of magnetic power in the needle, or from the wearing away of the centre on which it plays; this must be corrected immediately. A really good needle is actually wearisome in its tardiness to settle.

DEFINITIONS.

Meridian lines are due north or south lines, and strictly considered, are arcs of large circles of the earth meeting at the poles; these arcs, however, are subtended by so small an angle, and are so infinitely small in comparison of the whole circle, as to admit of their being assumed as parallel.

The distance of a line is its horizontal measurement, as a tangent to the earth, not following the surface of the ground, and is usually calculated in chains and links.

The *angle of bearing of any line*, is the angle of bearing made between that line and a meridian line running through the point, where the instrument is placed; and is measured always from north or south, eastward or westward.

Thus a line is said to bear *magnetically* north 16 degrees west, when the needle points to 16 degrees on the graduated circle, and when the direction of the line is to the west or north.

The *reverse bearing* of a line, is merely the bearing taken in a contrary direction.

The reverse bearing of a line, bearing north 38 degrees east, is south 38 degrees west; that of south 75 degrees west, is north 75 degrees

east. The reverse bearing, therefore, is measured by the same angle,* as the direct bearing, only taken in opposite directions: from south bearing northward, from east westward.

Difference of latitude, or *northing* and *southing*, is the distance that the end of the line is further north or south than the beginning.

The *difference of longitude, or departure*, is the distance that the end of the line is east or west from the beginning.

In changing our position from one point on the earth's surface to another, in a direction making any angle with the meridian, we at once change our latitude and longitude—the one is the northing and southing, the other the easting and westing.

To obtain the latitude and departure geometrically, when the length and bearing of a line are given.

Draw a meridian line through either end, and let fall a perpendicular from the other. This perpendicular is the departure; and that portion of the meridian line, intercepted by it, is the difference of latitude.

The latitude,† *departure, and distance*, form the three sides of a right-angled triangle, whose angle at the vertex is the angle of bearing; whose hypothenuse, or radius, is the distance; the latitude, the cosine; and the departure, the sine of the angle of bearing.

The meridian distance of any station, is the distance of that station from the meridian line passing through the first or any other assumed point, and is equal to the difference between the sums of the eastings and westings from that point; and is east or west, as the eastings or westings predominate.

THE TRAVERSE TABLE

Is a table giving the latitudes and departures to any distance and bearing, to the extent of the number of minutes to which it may be calculated.

The tables, which are appended to the present work, are calculated to every three minutes of the quadrant, and to every number from 0

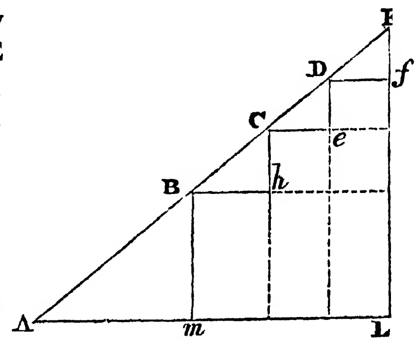
---

* This is not the case where the lines are long, say two or three miles, and where the observations are taken in very high latitudes; for most practical purposes, it is sufficiently correct to assume the direct and reverse bearings the same.

† By the term latitude, here and elsewhere, is meant *difference of latitude*.

to 10, from 10 to 100, &c. They have been prepared upon the following principle :—

The whole northing or southing of a line, of any bearing, is equal to the sum of the northings and southings of any number of lines of the same bearing, when the sum of the several distances is equal to the one distance of the whole line.



Let AE be a line, having any bearing whatever; through E draw the meridian line EL; and, from A, draw AL perpendicular to EL. AL is the departure, and EL is the latitude of the line AE.

Now, divide AE into AB, BC, CD, DE, and through B, C, D, draw the several meridian lines, B*m*, C*h*, D*e*, and B*h*, C*e*, D*f*, at right angles to them.

B*m*, C*h*, D*e*, E*f*, are the latitudes of the several parts of the line AE, and are together equal to EL, that is, the sum of the northings of the parts is equal to the one northing of the whole.

And, in the same way, A*m*, B*h*, C*e*, D*f*, the sum of the eastings are together equal to AL, the one easting of the whole line AE.

Again, if AE be a multiple of AB, (by similar triangles,) AL is the same multiple of A*m*, and EL is the same multiple of B*m*, being but sines and cosines of similar arcs to different radii.

If B*m*, therefore, be the northing of a line, whose distance is AB, or one chain, then the northing of a line AE, which is ten times AB, or 10 chains, is LE, or ten times B*m*.

This being premised, the understanding of the tables of latitudes and departures, or traverse table, becomes very simple.

Example.—Let it be required to find the latitude and departure of a line bearing north 4° 3′ east, and 15 chains 25 links long.

Look in the margin of the pages for the degrees, and down the column on the left hand for the minutes, then on the same line, collaterally with the three minutes, will be found the latitude and departures for any number of links or chains.

Take the latitude first.—For the 15 chains, look out in the tables for the latitude of 1·00 chains	=	·998	
multiply by		10	
and you have			9·98
the latitude of 5·00 chains	=		4·99
For the 25 links, look out in the latter for the latitude of 2·00 chains	=	1·99	
divide by		10	
(because each 10 links are one tenth of a chain)			
and you have			0·19
the latitude of 5·00 chains	=	4·99	
divide by		100	
(each link being one hundredth of the chain)	=		0·05
Add them together, and you obtain, as the one northing of the whole line,			15·21

Or, by supposing the number at the top of the page not to be confined to 1 chain, 2 chains, &c., for it is not necessary that they should be so limited, but to be 1 chain or 10 chains, 2 or 20, 3 or 30 chains, &c.; or 10 links or 1 link, 20 or 2 links, 30 or 3 links, &c., a more expeditious method is used, in obtaining the same result.

To find the latitude of 15·25 *chains to the angle of bearing of* 4 *degrees* 3 *minutes.*

Look in the tables as before, and for one chain you find 0·998, that is some decimal less than 1; instead of 1 chain, let this be 10 chains, then the northing becomes 9·98, or some decimal less than 10 chains; therefore the latitude

of 10 chains	=	9·98
of 5 chains	=	4·98
		14·96

Of 20 links, by looking in the tables under the head of 2, you find 1·99, that is, as before, some decimal less than 2. This may be 20 as well as 2, and 20 chains or 20 links indifferently. The latitude, therefore, of

		14·960
20 links	=	0·199
5 links	=	·050
the total of which is as before		15·21

which is the latitude for 15 chains 25 links, at a bearing of 4 degrees and 3 minutes.

This is north latitude, because the given bearing is north.

To find the Eastings or Westings.

Look in the same place as before, under the head of degrees, and you will find that, at a bearing of 4 degrees 3 minutes, the departure of 1 chain = ·071

and (by the first method) by multiplying by		10
the departure of 10 chains	=	0·710
of 5 chains	=	0·353
of 2 chains = 0·141 dividing by 10	=	0·014
of 5 chains = 0·353 dividing by 100	=	0·003
total, east departure,		1·080

which is the required departure of the whole line.

EXAMPLES.

Required the difference of latitude and departure of a line, which bears south 16 degrees 30 minutes east, 3 chains 47 links.

Ans. 3·33 south lat.; 0·98 east dep.

Given a line, bearing north 13 degrees 30 minutes west, and 6 chains 10 links long, to find the latitude and departure.

Ans. 5·93 north lat.; 1·42 west dep.

What are the latitude and the departure of a line bearing north 41 degrees 9 minutes east, 4 chains 47 links?

Ans. 3·36 north lat.; 2·95 east dep.

A line bears north 22 degrees 45 minutes west, 27 chains 62 links, required its latitude and departure.

Ans. 25·47 north lat.; 10·68 west dep.

These tables are found to be extremely useful; they enable you quickly to test the accuracy of the survey of a large extent of country; for, as the given distances and bearings are all capable of being resolved into their respective latitudes and departures, and as you cannot, from any point, go northwards without (to return to that point) coming back the same distance southward, nor any distance eastward, without remeasuring westward the same distance back, it follows, that, in going completely round a tract of country, the sum of the northings must be equal to the sum of the southings, and the sum of the eastings to that of the westings.

In adding up, however, the northings and southings, and eastings and westings, of bearings and distances, actually taken by the circumferentor, there will be generally found some small discrepancy, which the inaccuracy of the instrument may render inevitable. Within a certain limit this error is allowable. This error should not, however,

exceed one link to every five chains of the sum total of distances. Beyond this, a re-survey becomes necessary. This error must be apportioned among the whole, proportionally to the several distances of each bearing. The method will be explained hereafter.

The method of finding the bearing of a line by the Circumferentor.

Having placed the circumferentor over the point of the station, release the needle, and then, having unclamped the body of the instrument, by means of the parallel plates, as in the theodolite, make the whole instrument level. Now, turn the whole round until the *north* end of it lies towards the object, and looking through the sight, at the *south* end, fix the instrument, so that the fine web-line, in the north sight, exactly covers the object; then, when the needle has perfectly settled, (which should be immediately released on setting the instrument,) read off, by the north end of the needle, the number of degrees that it points to, from the north or south division line of the compass-box, according as the north end of the needle is in the north or south semicircle of the instrument; the angle measured by these degrees is called the angle of bearing.

Should the needle not point exactly to any degree, but lie between two of them, turn the instrument carefully, so as to make the needle point exactly to the next lower, and clamping the whole head of the instrument, detach, by withdrawing the connecting pin, the two horizontal plates. The instrument having been altered to suit the needle, the flag is no longer covered by the web. By using the rack and pinion, and carefully bringing the sights back, which are connected with the same plate as the vernier is, to cover the flag, the difference of the angle, in minutes, is denoted by the distance of the broad arrow in the vernier from the 360°, or the zero point of the other plate, which distance, as in the case of the vernier of the theodolite, is read, by observing which line of division in the vernier, reading to the left or the right (as the broad arrow is to the left or the right of the 360°), first coincides with some line in the graduated circle. In the circumferentor, the broad arrow of the vernier is placed in the centre, and if the distance from the 360° exceed half a degree, it is requisite to carry on the observation, as to which line first coincides, all round the plate, so as to end at the 360°. In some circumferentors the vernier is erroneously marked, as, in taking observations, the 15′ may become 45′, and the 45′ 15′, they should have been marked $\frac{45}{15}$ and $\frac{15}{45}$.

Having taken the angle, the two plates must again be brought to their proper position, by making the broad arrow of the vernier coincide with the 360°, where the connecting pin keeps them, till they are again required to be separated. The needle must be fixed, and the whole instrument clamped. It might not, perhaps, be out of place to observe here, that the kind of surveying required in a new country, as Australia or New Zealand, is directly the reverse of that which must be in use here. Here, a representation is, in most cases, wanted on paper, of actual boundaries of properties in the field: there, various lines, or certain positions, are to be marked down upon the ground. Here, the course of the hedge of a field is required: there, lines have to be run of given courses. The circumferentor, as constructed in this country, answers exceedingly well for the nature of the country it is generally used for here; but, when wanted to run a particular course, the objection, I have before mentioned, holds specially against it.

Having measured the distances to the next station, before taking the forward angle of the second line, take carefully the reverse bearing of the first, this will verify the last forward bearing, as the two should be the same; it also prevents, in a great measure, the probability of the needle being acted upon, without detection, by any magnetic substance in the neighbourhood; and I would especially recommend the young beginner invariably to adopt it. It is anything but a loss of time, and no survey can be depended upon without it.

VARIATION.

Before proceeding, it were better here to observe, that the magnetic meridian may not be the true meridian. The pole-star is not exactly at the pole; the needle seldom points either to the one or the other; two needles seldom point exactly the same way; and the same needle seldom points exactly the same way two years together. There are, therefore, two or three kinds of variation. The variation of the needle, strictly so termed, is the angle made between the magnetic north and the true north, and the variation is called east or west, according as the magnetic north is on the east or west side of the true north.

The measure of this angle, or the variation, is different in different countries, at the same time; and in the same country at different times. The various methods for determining this variation will be hereafter explained.

This general variation is not, however, that variation which principally affects the accuracy of a Surveyor's work; it is not the *general* angle of variation; but it is the *actual* angle made by his own needle and the true meridian line—that is the variation he has more particularly to guard against. This also shall be explained hereafter—observing only, now, that it is but necessary to determine once a true meridian by the method that will be then explained. By determining the true bearing of any fixed line—such as a long wall or fence—once, it becomes a standard of reference afterwards, as, at any time, the difference between its magnetic bearing, taken by an instrument and its true bearing, will determine the positive variation of the instrument required.

No Surveyor should assume that the variation of his instrument agrees exactly with the general *variation of the needle* given. Nor should he attempt to connect a new survey with an old one, after a year's interval, without determining, by the check or standard line, whether any, and what variation had taken place in his needle, and duly allowing for the difference; without this, no new line can be depended upon. In running divisions, or side lines, between lots in a new country, every instrument must be tested by the boundary line between the townships, as a standard line, and the division lines run parallel to it, whatever be the magnetic bearing, or the bearing of the individual instrument. This ever secures the parallelism of the lines.

CHAP. II.

To find the true bearing of a line, the magnetic bearing and the variation being given.

By the variation of the compass, is always meant the variation of the needle, which is the only variable east or west, from the true north, which is ever constant

RULE.—*Mark upon paper*, the relative positions of the given line and the *magnetic* north, which represents the north end of the needle, then observe, whether the variation be easterly or westerly; if easterly, the *true* north will be to the left of the magnetic; and

if westerly, to the right; place this also on paper, in its proper relative position.

The angle made between this last line (of the true north) and the given line, is the angle of bearing; the expression of this angle depending, of course, whether, by the variation, it may or may not have been moved into a different quadrant, being often changed from the magnetic N. W. to the true S. W., or N. E.

EXAMPLES.

1. Let the line AB bear N. 24° E., and the variation be 23½° W.; required the true bearing of the line.

AB, bearing N. 24° E., the needle is on the left of the line, but the needle bears west, the true north is on the right of the needle, therefore the line and the true north, being both on the right of the needle, the line being at the greater angle, the difference of the D angles is the bearing of the line eastwards, N. 0° 30′ E.

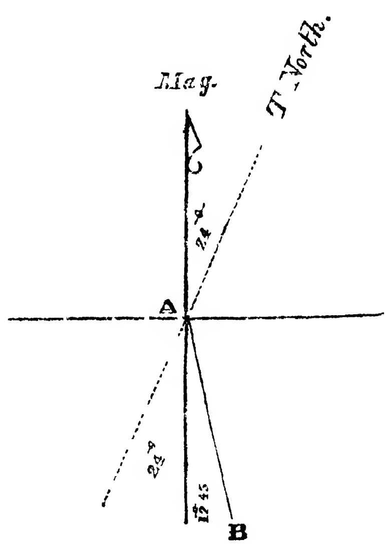
2. Let the reverse bearing of AB be S. 24° W., with the same variation, what is the true bearing?

Ans. S. 0° 30′ W., which is the result of the former.

3. Let AB bear S. 12° 45′ E., and the variation be 24° W.; required the true bearing.

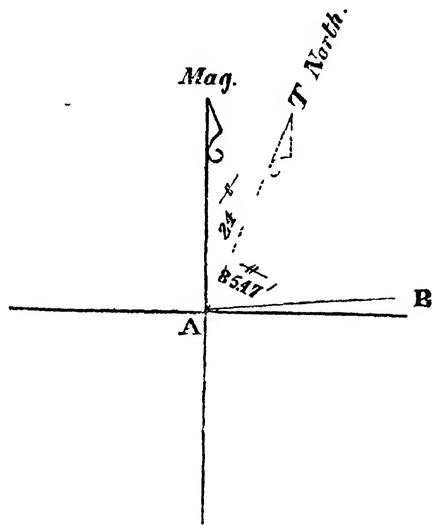
Now the north end of the needle is to the left of the true north, therefore,

$$\begin{array}{l} \quad 24^\circ \\ +12^\circ\ 45' \\ \hline = S.\ 36^\circ\ 45'\ E. \end{array}$$



4. Given AB, N. 85° 47′ E., with the same variation of 24° W.; required the true bearing.

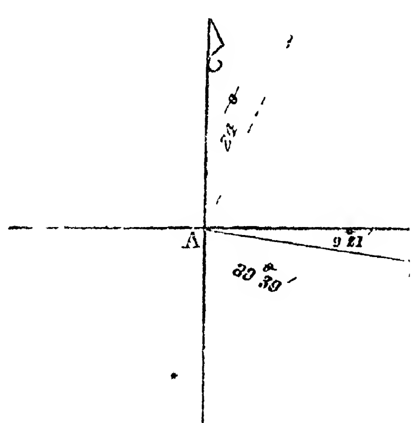
N. 85° 47′
24° 00′
N. 61° 47′ E.



5. Given AB, S. 80°39 E., with same variation; required the true bearing.

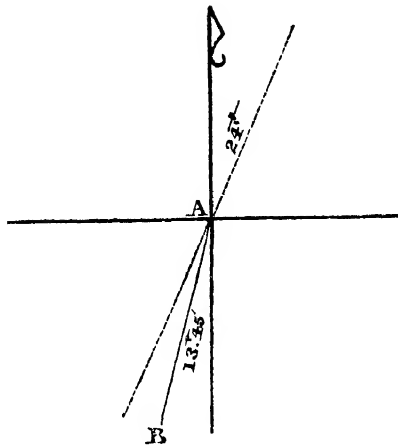
90°
—24
66°

90°
—80° 39′
9° 21′
66°
N. 75° 21′ E.



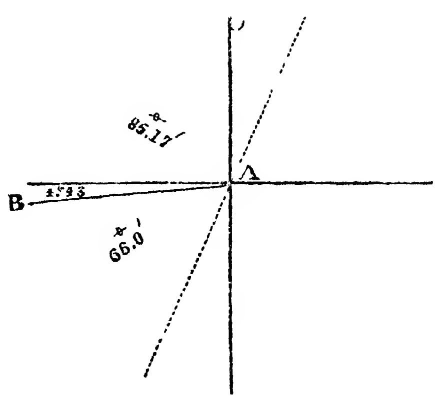
6. Given AB, S. 13° 45′ W., with the same variation; required the true bearing.

$$\begin{array}{r} 24^\circ 00' \\ -13^\circ 45' \\ \hline \text{S. } 10^\circ 15' \text{ E.} \end{array}$$



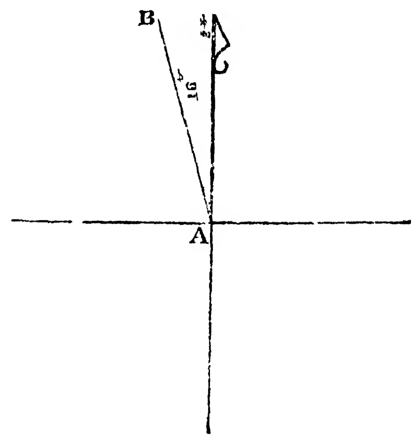
7. Given AB, N 85° 17′ W., variation as before; required the true bearing.

$$\begin{array}{r} 90^\circ \\ 24^\circ \\ \hline 66^\circ \end{array} \qquad \begin{array}{r} 99^\circ \\ -85^\circ\ 17' \\ \hline 4^\circ\ 43' \\ +66^\circ \\ \hline \text{S. } 70^\circ\ 43' \text{ W.} \end{array}$$



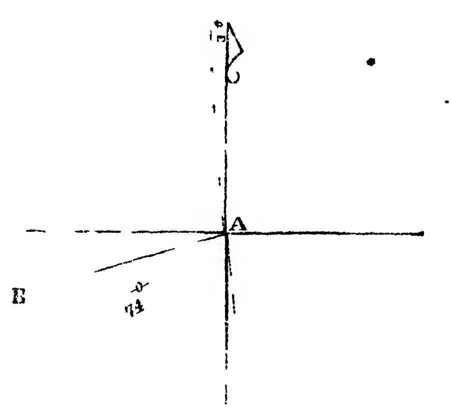
8. Given AB, N. 16° W., variation 3° east; required the true bearing.

16°
—3°
N. 13° W.



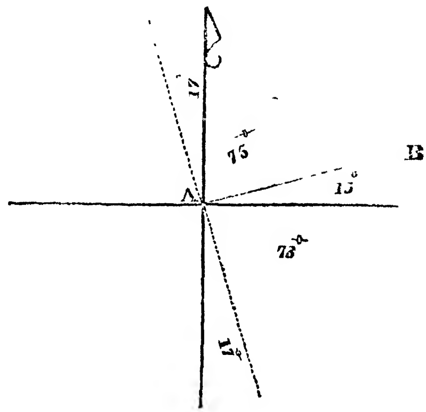
9. Given AB, S. 74° W., variation 3° east; required the true bearing.

74°
+3
S. 77° W.



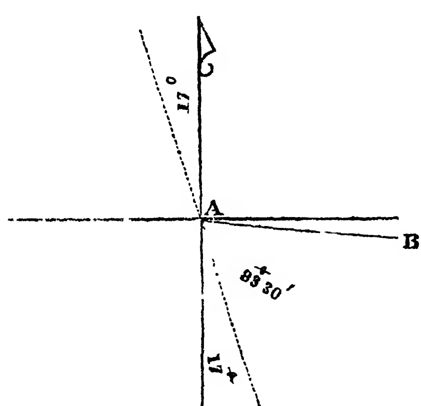
10. Given AB, N. 75° E., and the variation of needle 17° east; required the true bearing.

90°	90°
—75°	—17°
15°	73°
	15°
	S. 88° E.



11. Given AB, S. 83° 30′ E., variation 17° east; required the true bearing.

83° 30′
17°
S. 66 30′E.



CHAP. III.

THE BEARINGS OF TWO LINES BEING GIVEN, TO DETERMINE THE ANGLE BETWEEN THEM.

Rule.—First let both these lines run northwards or southwards. If they run on the same side of north or south, whether eastward or southward, this angle will be the *difference* of their angles of bearing; if on different sides, it will be their sum.

Next, let one run north and the other south.

If they both run on the same side of the meridian, this angle will be the supplement angle of the *sum* of the angles of bearing.

If one be on the east, and the other on the west of the meridian, this angle will be the supplemental angle of the *difference* of their angles of bearing.

In the interior angles of a polygon, as an angle may exceed 180°, the required angle might be the difference between the angle calculated as above, and 360°.

Example 1.—Given AB, N. 16° W., and AC, N. 12° E., to find the angle between them.

$$\begin{array}{l} 12^\circ \\ \underline{16^\circ} \\ 28^\circ = \text{angle between them.} \end{array}$$

EXAMPLE 2.—Given AB, N. 16° W., and AC, S. 12° W., to find the angle between them.

angle = supplement of sum.

12°	180°
16°	21°
28°	152° = angle between them.

EXAMPLE 3.—Given AB, N. 84° 20′ W., and AC, S. 49° 51′ E., to find the angle between them.

here the angle = the supplement of their difference.

84° 20′	180°
49° 51′	34° 29′
34° 29′ = the difference.	145° 31′ = angle required.

EXAMPLE 4.—Given AB, S. 25° E., and AC, S. 16° E.; required the angle between them.

angle = their difference.

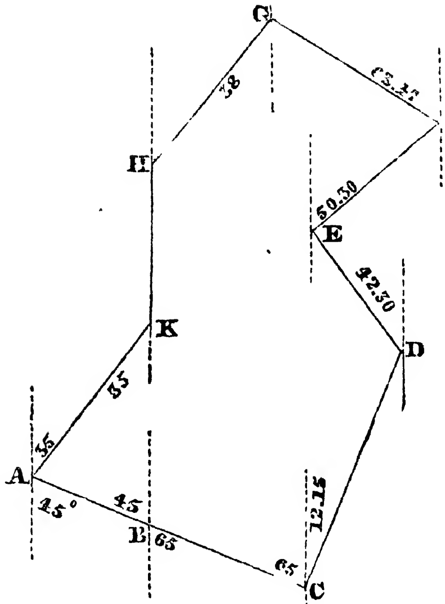
25°
16°
9° = angle required.

For further practice, I adjoin the following Example, where the bearings only are given, and the interior angles (being severally the angles between every two adjacent lines of the whole) are required. This Example will establish an important truth, that the sum of the interior angles of any figure, obtained from the *bearings* of its sides, must, *right* or *wrong*, always amount to four less than twice as many right-angles as the figure has sides, and that, therefore, the sum of these angles, can, in no case, be used as a check upon the correctness of the work.

EXAMPLE 1.—Given a tract of country, with the bearings of its several boundaries, to find the interior angles, and determine their correctness, viz., AB bears S. 45° E.—BC, S. 65° E.—CD, N. 12° 15′ E.—DE, N. 42° 30′ W.—EF, N. 45° 30′ E.—FG, N. 63° 47′ W.—GH, S. 38° W.—HK, due S., and KA (S. 35° W.) Required each of the angles.

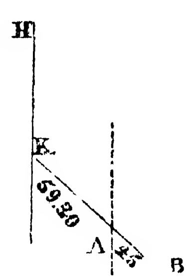
Take first the angle ABC; this is equal to the first angle of bearing, 45°+ the supplement of the second angle, 65°, that is, equal to 45°+115°=160°.

BCD, the second interior angle, is equal to the second angle of bearing, 65°+the third angle 12°15′, or 77°15′.



Thus, ABC being found to be = 160°
and BCD = 77° 15′
CDE = 125° 15′
DEF = 273° 00′
FGH = 65° 43′
FGH = 101° 47′
GHK = 142° 00′ 915° 0′
HKA = 215° 00′
KAB = 100° 00′
90|1260° 00′
14 right angles
+ 4
18 equals twice the number of sides of the polygon.

Now let KA, instead of S. 35° W., bear S. 59° 30′ E., and find the amount of the present interior angles.



All the former angles, but the last two, remain the same; the sum of the constant angles equals 945° 00′

The angle HKA now equals 120° 30′

And the angle KAB 194° 30′

resulting the same total as before, of 1260° 00′

Hence the sum of the interior angles of a polygon, when these angles are calculated from the bearings of the several lines, is no check upon the correctness of the work.

But the equality, or discrepancy of the northings and southings, of the eastings and westings, depending upon the lengths, as well as the bearings of the lines, is a certain test of its accuracy.

The method of applying this test, we will, therefore, now proceed to consider.

Example.—The bearings and distances of the several sides of a polygon being given, it is required to determine their correctness, and to divide the allowable error, proportionally, among the whole.

Rule a schedule, as in the annexed example, dividing it into columns, headed stations, bearings, distances, northings and southings, eastings and westings.

Find, as above shown, the difference of latitude and departure, in the tables for the several bearings and distances, and place the numbers, thus found in the proper column of northings, southings, eastings, or westings. Add them together, separately, find the difference of the northings and southings, and of the eastings and westings; the former is called the error of latitude, and the latter the error of departure. This error, if it does not exceed one link in five chains in the sum total of the distances, must be apportioned among each of the distances by the following proportions, viz.—*As the sum of all the distances is to the whole error, so is each distance to its correction.*

This must be done for the latitudes and departures, and must be placed in a column appropriated to each, called the north and south correction, and the east and west correction; the correction, thus determined, must be placed, collaterally, with the distance to which it refers, without distinguishing as to north, or south, east, or west.

In making this proportion, the links of the distances need not be taken into account, and frequently, as an approximation is sufficient, the apportionment may be made without reference to calculation.

Having found the several corrections for each of the latitudes and departures, add them together, severally, and see whether their total agrees with the whole error; then draw four other columns, heading them, corrected northings, southings, eastings, westings, and proceed to allot the corrections. If the error be an excess of northings, subtract each correction from its collateral northing, or add it to the collateral southing; if an excess of easting, add to the westing; and

subtract from the easting; the respective sums of their corrected latitudes and departures will now be found exactly to agree.

EXAMPLE 1.—Given the bearings and distances as follows, viz.,— S. 16° 30′ E., 3 chains 47 links; and S., 17° E., 3 chains 2 links; S. 26° E., 5 chains 51 links; S. 31° 30′ E., 7 chains 34 links; S. 5° 20′ E., 10 chains 55 links; S. 15° E., 5 chains 9 links; S. 8° W., 4 chains 3 links; S., 3° 30′ E., 4 chains 70 links; S., 45° 30′ W., 6 chains 50 links; S. 64° 45′ W., 7 chains 34 links; and N., 1° 21′ E., 49 chains 18 links, to the place of beginning, to find their northings and southings, eastings and westings, and hence determine their correctness.

Stations	Bearings	Distances	North Lat.	South Lat.	East Long.	West Long.
		ch. lks.				
1	S. 16° 30′ E.	3·47		3·33	0·98	
2	S. 17° 0′ E.	3·02		2·89	0·89	
3	S. 26° 0′ E.	5·51		4·94	2·41	
4	S. 31° 30′ E.	7·34		6·25	3·84	
5	S. 5° 20′ E.	10·55		10·49	0·97	
6	S. 15° 0′ E.	5·09		4·91	1·31	
7	S. 8° 0′ W.	4·03		3·99		0·56
8	S. 3° 30′ E.	4·70		4·60	0·28	
9	S. 45° 30′ W.	6·50		4·56		4·64
10	S. 64° 45′ W.	7·34		3·13		6·64
11	N. 1° 21′ E.	49·18	49·17		1·16	
		106.73	49· 17	49· 17	11·84	11·84

The northings being the same as the southings; the eastings the same as the westings.

It is seldom, however, that they agree so exactly, as has been given in this example. As the sum total of the distances amount to 106 chains 73 links, there would have been an allowable difference of 21 links (*i.e.*, of one link in every five chains) between the northings and southings, and the eastings and westings.

As errors will sometimes, notwithstanding the greatest care, occur in a survey of extent, we subjoin an example of two blocks of land, adjoining each other, having a common line, in which a considerable error was suspected, with the method of determining the locality of the error, and the means of correcting it..

EXAMPLE.—Given the bearings of the one block, S. 49° 30′ E., 7 chains 25 links; S.73° E., 8 chains 82 links; N. 6¼° E., 15 chains 41 links; S. 68° W., 17 chains 84 links, to the place of beginning;

and the bearings of the second block, reversing the bearing of the last line, which is common to the two, N. 68° E., 17 chains 84 links; N. 67¾° E., 6 chains 37 links; N. 47¼° E., 3 chains 86 links; N. 76° W., 14 chains 63 links; S. 32° W., 19 chains 73 links, also returning to the place of beginning. It is required to calculate the northings and southings, eastings and westings, of each of them separately, as it is imagined an error has been somewhere committed, which it is required to discover.

1st. Survey.

(Having a common line with the second.)

Stations	Courses	Distance in chains	North Lat.	South Lat.	East Dep	West Dep
1	S.49°30′E.	7·55		4·91	5·74	
2	S.73° E.	8·82		2·58	8·43	
3	N. 6¼° E.	15·41	15·32		1·68	
4	N.68° W.	17·84		6·69		16·54
			15·32	14·18	15·85	16·54
			14·18			15·85
		Excess of N.	2·14	Excess	of W.	·69

There is some considerable error here.

2nd. Survey.

(Having a common line with the first.)

Stations	Courses	Distances in chains	North Lat.	South Lat.	East Dep.	West Dep.
1	N. 68° E.	17·84	6·58		6·54	
2	N. 67¼° E.	6·37	2·61		5·68	
3	N. 47¼° E.	3·86	2·61		2·84	
4	N. 76° W.	14·63	3·54			14·19
5	S. 32° W.	19·73		16·73		10·46
			15·44	16·73	25·06	24·65
				15·44	24 65	
		Excess	of S.	1·29	·41	Excess of easting.

There is a considerable error here also.

This error is southerly, the former was northerly; but the bearing to this line, in the present example, is the reverse of the former; as this line forms, in itself, in the first survey, the whole of the northings, and, in the second survey, the whole of the southings, and as it has, in each case, an excess of more than a chain, the

error will most probably be found in this line. To aseertain whether this is the case, make one survey of the two, by throwing out of it this common doubtful line, and proceed to test the correctness of the several other lines and bearings by the usual way.

3rd. Survey,

(Being the union of the two, omitting the common line.)

Stations	Courses	Distances	North Lat	South Lat.	East Dep.	West Dep.
1	S. $49\frac{1}{2}°$ E.	7·55		4·91	5·74	
2	S. 73° E.	8·82		2·58	8·43	
3	N. $6\frac{1}{4}°$ E.	15·41	15·32		1·68	
4	N. $67\frac{3}{4}°$ E.	6·37	2·61		5·68	
5	N. $47\frac{1}{2}°$ E.	3·86	2·61		2·84	
6	N. 76° W.	14·63	3·54			14·19
7	S. 32° W.	19·73		16.73		10·46
		76·37	24·08	24·22	24·37	24·65
	5|76			24·08		24·37
	15		Excess of S.	·14	Excess of W.	·28

As 76·37 is the sum of the distances, the allowable error in this case is 15 links, that being one link in every chain of the sum of the distances.

The excess of southing is less than this, being ·14; that of the westings is more (·28), being, however, sufficiently near for our object in proving that there is no material error in any one of the lines that form the present block; and *it may, therefore, be presumed that the bearing and distance, common to the two* polygons, must be incorrect.

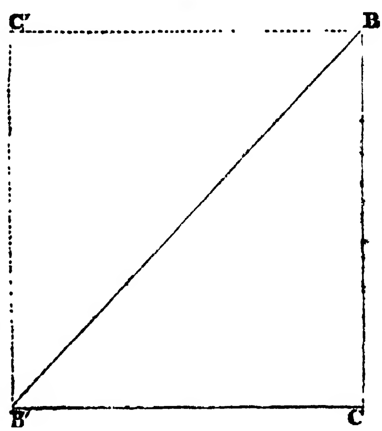
Having found the place of error, we will now consider the best method of correcting it.

CHAP. IV.

ERRORS OR OMISSIONS OF SURVEY.

As it is necessary, for the correction of this common line, when any two of the following data, viz.,—bearing, distance, difference of latitude, and of departure are given, to understand the method of determining the other two; we will first briefly say a few words on the subject.

Any two of the above data being given, to determine the other two.



Let B′B, or BB′, be the distance of the given line; BB′C′, or B′BC, is the angle of bearing; BC, or B′C′, is the difference of latitude, and B′C, or BC′, is the departure; but BC, or B′C′, is the cosine of the angle B′BC, or BB′C′, which is the angle of bearing; and B′C, or BC′, is the sine, also, of the same angle of bearing; therefore, the latitude, departure, and distance, are respectively the cosine, sine, and radius of a circle, whose arc is measured by the angle of bearing. By the rules for the cases of right-angled triangles, given in the Chapter on Trigonometre, in the Introduction, having two terms given, the rest can be found.

EXAMPLE 1.—A line bears N. 45° E., 31 chains, 40 links; required its difference of latitude and departure.

As sine 90°	=	10·00000
to 31·41 chs.	=	1·49693
so is sin. 45	=	9·84949
		11·34642
		10·00000
22 chs. 20 lks. N. lat.	=	1·34642

sine 45° = cos. 45° ∴ dep. = lat.

Example 2.—The difference of latitude of a line was 4 chs. 40 lks. N.; the departure was 25' W.; required the distance of the line.

Ans. 25·57 chs.

Example 3.—A line bears N. 72° W.; its departure is 24 chs. 58 lks.: what is its length?

Ans. 25·84 chs.

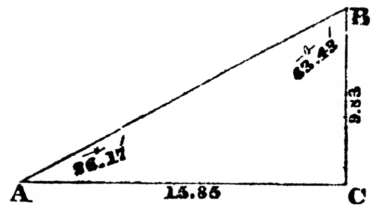
Example 4.—A line is 38 chs. 45 lks. in length, and its departure is 14 chs. W.; what is its angle of bearing, the same being between the north and west?

As 38·45 chs.	=	1·58490
is to rad.	=	10·00000
so is 14 chs.	=	1·14613
		11·14613
sine angle of bearing	=	1·58490
21° 21	=	9·56123

To correct the fourth station, which is common to the two polygons.

Rule.—Add up the northings and southings, and subtract them for the northing or southing of the unknown line; do the same with the eastings or westings to obtain its departure; then, by the preceding note having the departure and latitude of a line given, the distance and bearing can be determined.

Stations	Courses	Distances	North Lat.	South Lat.	East Dep.	West Dep.
		c. l.				
1	S. 49°30′ E.	7 55		4·91	5·74	
2	S. 73° E.	8 82		2·58	8·43	
3	N. 6¼° E.	15 41	15·32		1·68	
4	S. W.					
			15·32	7·49	15·85	
			7·49			
	Defi-	ciency of S·	7·83		15·85	Deficiency of westings.



Now AC, the westing or base of the right-angled triangle, being given 15 chs. 85 lks.; and BC, the southing or perpendicular, being 7 chs. 83 lks.; the angle BAC, or the bearing of the line AB, and its distance, which are required, can be determined by the rules of the bases of right-angled triangles.

Thus—take AC as radius, and describe the circle CD; then CB becomes the tangent to angle BAC, and AB is the secant of the same angle; then say—

As 15·85	= 1·20003
is to radius	= 10·00000
so is 7·83	= 0·89376
	10·89376
	1·20003
to tan. angle BAC, 26° 17′	= 9·69373

Also, as radius	= 10·00000
is to 15·85	= 1·20003
so is secant ∠A, 26° 17′	= 10·04739
to hypothenuse, 17·68	= 1.24742

and the bearing in S. 63° 45′ W.

CORRECTED SURVEY.

Stations	Courses	Distances	N.	S.	E.	W.	
1	N. $63\frac{3}{4}$° E.	17·68	7·83		15·85		
2	N. $67\frac{3}{4}$° E.	6·37	2·61		5·68		
3	N. $47\frac{1}{4}$° E.	3·86	2·61		2·84		
4	N. 76° W.	14·63	3·54			14·19	
5	S. 32° W.	19·73		16·73		10·46	
		62·27	16·59	16·73	24·37	24·65	
				14·59		24·37	
		Excess	of S.	·14		·28	Excess of westings,

which are the same errors as before.

Let us now divide these errors proportionately among the several distances, by the method previously given, viz.,—by saying, as the whole distance is to the whole error, so is each distance to its particular correction.

CORRECTED SURVEY.

Stations	Bearings	Distances	N.	S.	E.	W.	Corrections for Land	Corrections for Sea.	Corrected N.	Corrected S.	Corrected E.	Corrected W.
1	N.63¾° E.	17·68	7·83		15·85		1	5	17·84		15·90	
2	N.67¾° E.	6·37	2·61		5·68		0	2	2·61		5·70	
3	N.47½° E.	3·86	2·61		2·84		0	1	2·61		2·85	
4	N.76° W.	14·63	3·54			14·19	0	4	3·54			14·15
5	S. 32° W.	19·55		16·57		10·36	1	6		16·58		10·30
		62·09	16·59	16·57	24·37	24·55	2	18	16·58	16·58	24·45	24·45
			16·57			24·37						
			dif. S. ·02			dif. E. ·18						

As the whole distance 62 : 18 : 1

17·68	6·37	3·86	14·63	19·55
·3	·3	·3	·3	·3
5·304	1·911	1·058	4·389	5·865

5
2
1
4
6
18 deficiency.

1
62 | 18·000 | ·290 = ·3 links nearly
124
560
558

Examples.

1.—Given the following bearings and distances of an old waggon road, through a new settlement, running from one concession road to another, viz.,—1st. N. 78° E., 2·20 chains; 2nd. N. 45° 30′ E, 14·80 chains; 3rd. N. 16° W., 21·75 chains; 4th. N. 68° 20′ E., 13·90 chains; 5th. N. 10° W., 15·60 chains; 6th. N. 70° E., 8·96 chains. It is required to lay out a new straight road, connecting the two ends of the present road. What will be its bearing and distance?

2.—Given the boundaries of a tract of land, as follows, viz.: —1st. S. 16° 30′ E., 3·47 chains; 2d. S. 17° E., 3·02 chains; 3d. S. 26° E′., 5·51 chains; 4th. S. 31° 30′ E., 7·34 chains; 5th. S. 5° 20′ E., 10·55 chains; 6th. S. 15° E., 5·09 chains; 7th. S. 8° W., 4·03 chains; 8th. S. 3° 30′ E., 4·70 chains; 9th. S. 45° 30′ W., 6·50 chains; 10th. S. 64° 45′ W., 7·34 chains; 11th. to the place of beginning. Required the bearing and distance of this eleventh line.

Ans. N. 1° 21′ E., 49 chs. 18 lks.

When the distance of two sides cannot be obtained.

Rule 1.—Find, by the preceding chapter, the length and bearing of the closing line, connecting the known sides of the survey. This line and the two unknown sides will form a triangle, having a known base, and the bearings known also of all its sides; with these data compute the several angles of the triangle, by the previous rule (page 205), and you have the three angles and one side given; whence the two required sides are determinable by the first case.

Example 1.—The boundaries of a tract of land were taken as follows:—1st. S. 16° 30′ E.; 2nd. S. 17° E.; 3d. S. 26° E., 5·51 chains; 4th. S. 31° 30′ E., 7·34 chains; 5th. S. 5° 20′ E., 10·55 chains; 6th. S. 15° E., 5·09 chains; 7th. S. 8° W., 4·03 chains; 8th. S. 3° 03′ E., 4·70 chains; 9th. S. 45° 30′ W., 6·50 chains; 10th. S. 64°45′ W., 7·34 chains; 11th. N. 1°21′ E., 49·18 chains, to the place of beginning. Required the distances of the two first stations, which, from local obstructions, could not be measured.

Stations	Bearings	Distances	N.	S.	E.	W.
1	S. 16°30′E.					
2	S. 17° E.					
3	S. 26° E.	5·51		494	241	
4	S. 31°30′E.	7·34		6·25	3·84	
5	S. 5° 20′E.	10·55		10·49	0·97	
6	S. 15° E.	5·09		4·91	131	
7	S. 8° W.	4·03		3·99		0·56
8	S. 3°30′ E.	4·70		4·69	0·28	
9	S. 45°30′W.	6·50		4·56		4·64
10	S. 64°45′W.	7·34		3·13		6·64
11	N. 1°21′ E.	49·18	49·17		1·16	
			49·17	42·95	9·97	11·84
			42·95			9·97
		deficiency of	S. 6·22			1·87 deficiency of E.

Therefore,
the southings 6·22 = cosine of angle of bearing }
the eastings 1·87 = sine of angle of bearing } where radius = the distance of the *closing* line.

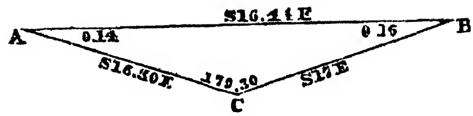
By the case of right-angled triangles.

As 6·22 chs.	= 2·79379
is to rad.	= 10·00000
so is 1·87 chs.	= 2·27184
to tan. angle of bearing 16° 44′	= 9·47805

As cosine of angle of bearing	= 9·98121
is to 6·22 chs.	= 2·79379
so is radius	= 10·00000
to distance required = 6·49 chs.	= 2·81258

The angle of bearing is 16 degrees 44 minutes, which, as the deficiencies are southings and eastings, makes the closing line bear south 16 degrees 44 minutes east, 6 chains 49 links.

This closing line and the two unmeasured lines form a triangle. Draw any line AB, marking upon it the bearing S. 16° 44′ E., and AC, BC, in their respective relative position to AB, viz.,—AC, bearing S. 16° 30′ E., and CB, S. 17° E.



Because AB and AC bear both southwards and eastwards, the angle CAB, is their difference; for the same reason, the angle ABC is the difference of the bearings of AB and CB. The sides AC and CB can now be calculated by the first case of trigonometry.

As sine 179° 30′	=	7·94084
is to 6·49 chs.	=	2·81224
so is sine 0° 16′	=	7·66784
		10·48008
to x 3·46 chs.	=	2·53924
and so is sine 0° 14′	=	7·60985
		10·42209
to y 3° 02′	=	2·48125

being 3·46, instead of 3·47 (see page 221, example), a difference of one link only in the first distance; and being 3·02, the same as the second distance.

Example 2.—The following bearings and distances were taken, viz.:—at station 1, the bearing of 2nd station was, S. 71° 24′ W.; at station 3, the same point bore S. 46° 18′ E,, the distances could not be obtained; at station 3, N. 15° 42′ E., 6·20 chains; at station 4, N. 52° 18′ E., 6·75 chains; at station 5, S. 78° 48′ E., 5·96 chains; at station 6, 5° 51′ E., 4·84 chains; at station 7, S. 49° 15′ W., 4·75 chains; at station 8, S. 4° 57′ E., 3·98 chains, to the place of beginning.

What are the distances of the first and second lines?

Rule 2.—*By means of a changed bearing.*—Suppose the whole tract of land to revolve, until it becomes in such a position, as to have one of the unknown sides in a direct meridian line. This will not change the relative position of the sides, as, whatever angle of variation be taken for this line, the same will be taken for all; but, as in the first instance, there

were but two lines, whose departures were unknown, so now, in consequence of one, from having been made due north and south, having no departure, that, of the only one that now remains unknown, is the measure of the difference of the sums of the eastings or westings.

The changing of the bearing is performed in the same way as correcting for the variation, viz.,—by bringing all the given angles to a *new* meridian, which makes so many degrees east or west, with the first meridian.

Arrange, therefore, the bearings and distances, as in the following columns; change the bearings of each, so as to make the bearing of one of the unknown sides a meridian line. Place these changed bearings in their proper column; find the latitude and departure of these bearings, to their several distances. Find the sum of the eastings and the sum of the westings; their difference will be the departure of the second unknown side, in terms of the deficiency.

Calculate the latitude and distance of this second side, from its bearing and departure, which are known, and place them also in their proper column.

Then add the northings together, and the southings together, and their difference will be the distance of the side, which was made a meridian.

Example.—Solve the preceding Problem, by using a changed bearing.

Make the first bearing S. 16° 30′ E., a meridian, turning all the southerly bearings 16° 30′ to the west, and the northerly bearings 16° 30′ to the east.

Stations	Bearings	Changed	Dis.	N.	S.	E.	W
1	S.16° 30′ E.	South				0·00	0·00
2	S.17° E.	S. 0° 30′ E.			(1·26)	(0·11)	
3	S.26° E.	S. 9° 30′ E.	5·51		5·43	·91	
4	S.31° 30′ E.	S.15°00′ E.	7·34		7·09	1·90	
5	S. 5° 20′ E.	S.11°10′W.	10·55		10·35		2·06
6	S.15° E.	S. 1° 30′W.	5·09		5·09		0·13
7	S. 8° W.	S.24°30′W.	4·03		3·67		1·67
8	S. 3° 30′ E.	S. 13° W.	4·70		4·58		1·06
9	S.45° 30′ W.	S. 62° W.	6·50		3·06		5·74
10	S.64°45′ W.	S.81°15′W.	7·34		1·11		7·26
11	N. 1°21′ E.	N.17°51′ E.	49·18	46·84		15·00	
						17·81	17·92

Eastings	=	17·81
Westings	=	17·92
Deficiency of eastings	=	0·11

Place this in its proper column, against the bearing, S. 0° 30′ E., and find the southing of the line.

Given the bearing of the line, S. 0° 30′ E, and the easting, 11 links, to calculate the distance and the southing.

Ans. The distance and the southing each 1·26 chs.

Now make the second bearing a meridian.

Station	Bearings	Changed Bearing	Distance	N.	S.	E.	W.
1	S. 16° 30′ E.	S. 0°30′W.				(0·04)	
2	S. 17° E.	South.					
3	S. 26° E.	S. 9′ E.	5·51		5·44	1·84	
4	S. 31°30′ E.	S. 14°30′ E.	7·34		7·11		
5	S. 5° 20′ E.	S. 11°40′W.	10·55		10·55		2·15
6	S. 15° E.	S. 2° W.	5·09		5·09		0·17
7	S. 8° W.	S. 25° W.	4.03		3·66		1·70
8	S. 30′ E.	S. 13°30 W.	4·70		4·57		1·09
9	S. 45°30′ W.	S. 62°30′ W.	6·50		3.00		5·76
10	S. 64°45′ W.	S. 81°45′ W.	7·34		1·05		7·27
11	N. 1°21′ E.	N. 18°21′ E.	49·18	46·71		15·40	
		Excess of westings 4 links				18·10	18·14

When the difference above was 11, which is the sine of the angle of bearing, 30 minutes, the distance, or the radius to that angle, was only 12·6, it cannot now, therefore, be more.

Hence, when the two unknown bearings differ so slightly, as to make the changed bearing of one but a few minutes, this method cannot be adopted; the first method, by the closing line, must, therefore, be used.

I subjoin a correct result by the changed bearings, obtained from the same example, in consequence of the angles of the two bearings, whose distances are unknown, differing considerably.

Example.—Make the *seventh* bearing south, and change the other bearings to correspond. The changed bearings have been omitted for practice to the student, but their latitudes and departures are inserted.

Stations	Bearings	Changed Bearing.	Distances	N.	S.	E.	W.
1	S. 16°30′ E.		3·47		3·15	1·43	
2	S. 17° E.		3·03		2·73	1·27	
3	S 26° E.		5·51		4·51	3·08	
4	S. 31°30′ E.		7·34		5·66	4·66	
5	S. 5° 20′ E.		10·55		10·26	2·41	
6	S. 15° E.		{ 5·08 }		(4·67)	(1·98)	
7	S. 8° W.		{ 4·03 }		(4·04)		
8	S. 3° 30′ E.		4·70		4·60	0·96	
9	S. 45°30′ W.		6·50		5·15		3·95
10	S. 64°45′ W.		7·34		4·02		6·13
11	N. 1° 21′ E.		49·18	48·83			5·71
				48·83	44·81	13·81	15·70
				44·11			13·81
	Deficiency of southings			4·02,	deficiency of E.		1·98

As sine 23° = 9·59188
is to 1·98 chs. = 2·29667
so is radius = 10·30000
12·29667
9 59188
to 5·08 chs. = 2·70479

As radius = 10·00000
is to 5·06 chs. = 2·70479
so is sin 670 = 9·97403
12·66882
10·00000
to 4·67 chs. = 2·66882

When the bearings of any two sides of a tract of Survey have been incorrectly taken.

Arrange the given bearings and distances in their proper columns, and find, as before, the difference of the northings and southings, and of the eastings and westings, for the latitude and departure of a closing line. The line, and the distances of the two unknown sides, form a triangle, with sufficient data to determine its inward angles; and thence, as the bearing of the closing line is known, the reqnired bearings of the two other sides are obtained.

CHAP. V.

Given the bearings and distances of a tract of land, to determine its area.

Arrange the several bearings and distances as before, and find the respective latitudes and departures corresponding to each.

Determine, as in the preceding Problems, the several corrections to each, for latitude and departure, and allow for these corrections, placing the amended latitudes and departures in their proper columns.

Now make five more columns, and head them—East Double Departure; West Double Departure; Multipliers; North Areas; and South Areas.

Take the sum or difference of every two consecutive departures; adding them, if of the same kind, and subtracting them, if of different; and place this sum, or excess, in the column to which it belongs; of west, in the column of W.D.D.; of east, in that of E.D.D.

If the sum of the first and last departures, east or west, be found equal to the first corresponding double departure, east or west, the working has been correctly performed.

Commence from any angle whatever of the survey, and assume any line, either close to, or at any distance from it, as a multiplier, and place it in the column of multipliers collateral with that angle, terming it either east or west, according as the principal double departures are east or west. The sum of this assumed multiplier, and that of the double departure, corresponding to the next side, if they are of the same kind, or their difference, if of opposite kinds, in terms of the greater, will be the next multiplier; proceed with the second multiplier, and the following double departure, until multipliers have been found for all the sides.

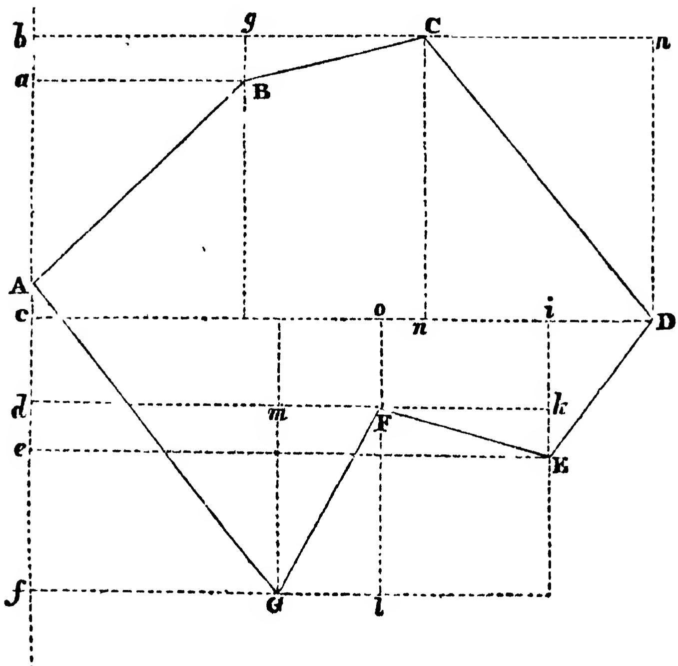
The difference, if of opposite kinds, or the sum, if of the same, of the last multiplier and the first double departure, will be equal to the assumed multiplier, if the work is right.

Multiply each of the corrected differences of latitude by its collateral multiplier, and place the product in the north or south area columns, observing, that whenever the multipliers are of the same name as the assumed, those products are to be placed in the column of areas, which is of the same name as the latitude; if of different names, in the opposite column.

Half the difference, between the sum of the north and south areas will be the area of the survey.

DEMONSTRATION.

Let ABCDEFG be a tract of country whose area is required;



through A draw a meridian line *fb*, and through the several points, B, C, D, E, F, and G, draw B*a*, C*b*, D*c*, E*e*, F*d*, and G*f*, perpendicular to this meridian, and also *g*B, C*n*, D*h*, and C, parallel to it.

Let the assumed multiplier be AB.

Now the area of the figure, *b*CDEFG*f*, is equal to the figure *b*CB AG*f*+the required area of the tract. And the difference of latitude and departure of the several sides will be found as in the following table. Thus A*a* is the northing of AB, and *a*B is its easting; *ab* is the northing of BC, and *g*C its easting, and so on.

Also, according to the rule, *a*B+*g*C or *b*C, is the east north departure of BC, as they are of the same kind, and *f*G—*a*B, is the W.D.D. of *a*B.

Having obtained the double departures for the several sides, assume the multiplier *a*B, and find, by the rule, the several other multipliers *ab*+*bc*, *bl*, observing carefully, whether these multipliers

are east or west. And place the areas of the product of the e multipliers into their collateral northings or southings, in the columns of north or south areas, according to the proper relations between them; multipliers and northings is given in the rule. These areas are double the required areas—being each of them double the areas of the several trapezoids into which the figure has been divided.

Half the difference, therefore, of the north and south areas, will be the required area.

	N.	S.	E.	W.	E. D. D.	W. D. D.	Multiplier	N. Areas	S. Areas
AB	A*a*		*a*B			*fg*—*a*B	*a* B E		
BC	*ab*		*g*C		*b*C		B *bc* E	2 *ab*CB	
CD		*bc*	C*h*		*gh*		*b*C+*c*D E		2 *b*CD*c*
DE		*ce*		D*i*	*hi*		*c*D+*ci* (*c*E) E		2 *c*DE*e*
EF	*ed*			*k*F		D*o*	*e*E+*d*F E	*d*FE*e*	
FG		*df*		F*m*		*km*	*d*F+*dm* (*fg*) E		2 *d*FG*f*
GA	A*f*			G*f*		*co*	*dm* E	AG*f*	

Now "*dm*" the multiplier last found—the first W. D. D., *i. e.* *dm* or *fg*—(*fg*—*a*B) = *a*B, the assumed multiplier.

Example I.—Given the bearings and distances of a tract of country, to find the area, viz.:—1st, S. 16° 30′ E., 3·47 chains; 2nd, S. 17° E., 3·02 chains; 3rd, S. 26° E., 5·51 chains; 4th, S. 31° 30′ E., 7·34 chains; 5th, S. 5° 20′ E., 10·55 chains; 6th, S. 15° E., 5·09 chains; 7th, S. 8° W., 4·03 chains; 8th, S. 3° 30′ E., 4·70 chains; 9th, S. 45° 30′ W., 6·50 chains; 10th, S. 64° 45′ W., 7·34 chains; 11th, S. 60° 15′ W., 4·98 chains; 12th, N. 22° 45′ W., 27·62 chains; 13th, N. 67° E., 2·40 chains; 14th, N. 13° 30′ W., 6·10 chains; 15th, S. 62° 30′ W., 4·16 chains; 16th, N. 27° 30′ W., 8·24 chains; 17th, N. 41° 10′ E., 4·47 chains; 18th, N. 4° W., 16·40 chains; 19th, N. 84° E., 4·03 chains; 20th, S. 69° 30′ E., 18·21 chains—to the place of beginning.

Station.	Bearings	Dist.	N. Lat.	S. Lat.	E. Long.	W. Long.	Corrections of Latitude	Cor. N.	Cor. S	Cor. E.	Cor. W.	E D. D	W. D. D.	Multipliers		N. Areas	S. Areas
1	S. 16°30′ E.	3·47		3·33	0·98		—		3·33	0·98		18·04		00·00	E.		000
2	S. 17° E.	3·02		2·89	0·89		—		2·89	0·89		1·87	—	1·87	E.		5·4043
3	S. 26° E.	5·51		4·95	2·41		1		4·94	2·41		3·30	—	5·17	E.		25·5398
4	S. 31°30′ E.	7·34		6·26	3·84		1		6·25	3·84		6·25	—	11·42	E.		71·3750
5	S. 5°20′ E.	10·55		10·51	0·97		2		10·49	0·97		4·87	—	16·23	E.		170·2527
6	S. 15° E.	5·09		4·92	1·31		1		4·91	1·31		2·28	—	18·51	E.		90·8841
7	S. 8° W.	4·03		3·99	—	0·56	—		3·99	—	0·56	0·75	—	19·26	E.		70·8474
8	S. 3°30′ E.	4·70		4·67	0·28	—	—		4·69	0·28	—	—	0·28	18·98	E.		89·0162
9	S. 45°30′ W.	6·50		4·56	—	4·64	1		4·55	—	4·64	—	4·36	14·62	E.		66·5210
10	S. 64°45′ W.	7·34		3·14	—	6·64	1		3·13	—	6·64	—	11·28	3·34	E.		10·4542
11	S. 60°15′ W.	4·98		2·47	—	4·33	1		2·46	—	4·33	—	10·97	7·63	W.	18·7698	
12	N. 22°45′ W.	27·62	25·47	—	—	10·68	5	25·52	—	—	10·68	—	15·01	22·64	W.		577·7728
13	N. 67° E.	2·40	0·94	—	—	—	—	0 94	—	2·21	—	—	8·47	31·11	W.		29·2434
14	N. 13°30′ W.	6·10	5·93	—	—	1·42	1	5·94	—	—	1·42	0·79	—	30·32	W.		180·1008
15	S. 62°30′ W.	4·16	—	1·92	—	3·69	—	—	1·92	—	3·69	—	5·11	35·43	W.	6·80256	
16	N. 27°30′ W.	8·24	7·31	—	—	3·80	1	7·32	—	—	3·80	—	7·4	42·92	W.		314·1744
17	N. 41°10′ E.	4·47	3·36	—	2·95	—	—	3·56	—	2·95	—	—	0·85	43·77	W.		147·0672
18	N. 4° W.	16·40	16·36	—	—	1·15	3	16·39	—	—	1·15	1·80	—	41·97	W		687·8883
19	N. 84° E.	4·03	0·41	—	4·01	—	—	0·41	—	4·01	—	2·86	—	39·11	W.		16·0351
20	S. 69°30′ E.	18·21	—	6·37	17·06	—	4	—	6·33	17·06	—	21·07	—	18·04	W.	114·1932	
		154·16	59·78	60·00	36 91	36 91	22	59·88	59·88	36·91	36·91					200·9880	2558·5761
				59·78													200·9880

22 deficiency of northings.

2|2357·5881

5|154·16

31 links of allowable error.

10|1178·7944

A. R. P.

Ans. = 117 3 20

117·8794

Required the area of a tract of land, whose bearings and distances are as follows, viz. :—1st, N. 15° 42′ E., 6·20 chains; 2d, N. 52° 18′ E., 6·75 chains; 3d, S. 78° 48′ E., 5·96 chains; 4th, S. 5° 51′ E., 4·84 chains; 5th, S. 49° 15′ W., 4·75 chains; 6th, S. 4° 57′ E., 3·98 chains; 7th, S. 71° 24′ W., chains; and 8th, N. 46° 18′ W., to the place of beginning.

Given the following bearings and distances to find the area, viz.:—1st, N. 10° 21′ W., 4·50 chains; 2d, N. 9° 48′ E., 5·20 chains; 3d, N. 75° W., 3·00 chains; 4th, N. 20° 3′ E., 4·86 chains; 5th, S. 45° E., 5·20 chains; 6th, N. 3° 18′ W., 2·50 chains; 7th, E. 700 chains; 8th, S. 12° W., 3.94 chains; 9th, S. 43° E., 4·15 chs.; 10th, S. 46° 57′ W., 8·20 chains; 11th, S. 29° E., 9·15 chains; 12th, S. 48° W., 4·56 chains; 13th, N. 19° W., 3·42 chains; 14th, S. 28° W., 8·54 chains; 15th, N. 53° W., 4·60 chains; and 16th, —,,—, to the place of beginning.

CHAP. VI.

DIVISION OF LAND.

PROBLEM I.

To lay out a given quantity of land in a square form.

Bring the given quantity into square chains, and take the root which will be in chains.

Example 1.—It is required to lay out 400 acres of land, in a square form.

400 acres = 4000 square chains.

$\sqrt{4000} =$ 63·24 chs. the length of the side required.

Example 2.—What will be the side of a square, which contains 496 acres, 2 roods, and 32 perches?

PROBLEM II.

To lay out a given area in a rectangular form, having the length to the breadth in a given ratio.

Let A $=$ the area in square chains, and $x =$ the common unit measure of the sides, and m and n the ratio ; then, $mx =$ one side, and $nx =$ the other, and $mnx^2 = A$, and $x = \sqrt{\frac{A}{mn}}$ or the unit of measure of the sides $=$ the root of the given area divided by the product of the ratios.

EXAMPLE 1.—There are 2000 acres of land to be laid out in a rectangular form, whose sides are to each other, as 4 is to 5 ; what will their lengths be ?

$$m = 4, \ n = 5, \text{ and } mn = 20. \qquad A = 20{,}000 \text{ sq. chains}$$

$$\therefore x = \sqrt{\frac{20000}{20}} = \sqrt{1000} = 31{\cdot}62 \text{ chs.}$$

$$4x = 31{\cdot}62 \times 4 = 126{\cdot}48 \text{ chs.} = \text{one side}$$

$$5x = 31{\cdot}62 \times 5 = 158{\cdot}10 \text{ chs.} = \text{other side}$$

EXAMPLE 2.—One side of a rectangular field is double the other, what are the sides, when the area is 20·0·18 perches.

Ans. 10·03 chs. and 20·06 chs.

EXAMPLE 3.—A man has a farm of 150 acres, of a rectangular form, the depth of the farm is 50 chains. He is desirous of adding 50 acres to it, having the same depth, to make up 200. What will be the length of frontage of his farm, after the addition?

Ans. Half a mile.

PROBLEM III.

When one of the sides is a certain length longer than the other.

Let x be the unit of measure as before, and m the excess of the one side, then x and $x+m$ will be the two sides, and their product, or x^2+mx, will be the given arc (A) in square chains: completing the equation we have $x = \sqrt{A + \overline{\frac{m}{2}}|^2} - \frac{m}{2}$; or the following rule,

To obtain the smaller side (x), square half the difference of the two sides, add it to the given area, and taking the root of their sum, from this root subtract half the difference.

EXAMPLE.—Given 464 acres, it is required to lay it out in a rectangular form, the one side being 6 chains longer than the other.

Let $x =$ the one side, $x+6 =$ the other

$$\begin{aligned} x^2+6x &= 4640 \text{ square chains} \\ x^2+6x+9 &= 4649 \\ x+3 &= 68{\cdot}18 \\ x &= 65{\cdot}18 \\ x+6 &= 71{\cdot}18 \end{aligned}$$

Ans. One side $= 65{\cdot}18$ chs.; the other $= 71{\cdot}18$ chs.

PROBLEM IV.

From a given block of land, with parallel sides, the angle of inclination of the front and sides being given, to cut off a given area, by a line parallel to the sides.

RULE.—Find first the perpendicular depth of the given block, (which is the nat. sin. of the angle of inclination × the length of the side), then, as all parallelograms upon equal bases, and between the same parallels, are equal, divide the given area by this depth for the frontage required.

EXAMPLE 1.—The concession road, of a certain township, bears N. 74° E., while the side lines bear N. 9° W. The length of the side lines is 66·66 chains, and the fronts of the lots are 30 chains. Required the frontage that must be taken, to cut off 100 acres, by a line parallel to the side lines.

The side lines, bearing N. 9° W., and the fronts, N. 74° E., the included angle, or angle of inclination, of the front to the sides, is $9°+74°$ or $83°$.

∴ the depth of the lots equals nat. sin. 83° into 66·66 chs.

$$\begin{aligned} \text{log. sin. } 83° &= 9{\cdot}99675 \\ +\text{log. } 66{\cdot}66 &= \underline{1{\cdot}82387} \\ & \quad 11{\cdot}82062 \\ -\text{log. radius} &= \underline{-10{\cdot}} \\ \text{log. perpend. depth } 66{\cdot}66 &= 1{\cdot}82062 \end{aligned}$$

100 acres $=$ 1000 chains

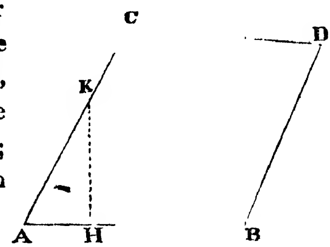
$$\frac{1000}{66{\cdot}16} = 15{\cdot}12 = \text{the frontage required.}$$

EXAMPLE 2.—When the fronts of the lots bear S. 80° W., and the side lines N. 15° E., the side lines being 101·50 chains long, and the concession road frontage 20·00 chains, what will be the frontage required, to divide the whole lot between A and B, giving A 50 acres more than B, and what quantity will each have ?

PROBLEM V.

With the same data as the preceding, except that the dividing line is not to be parallel to the sides, but at right angles to the concession roads.

Let ACDB be the given lot; from C draw CE, at right angles to its front AB ; CE is the perpendicular depth of the lot; find this, as in the preceding Problem, and determine AE, which is the cosine of the given angle of inclination, to which CE is the sine ; find the area of the triangle AEC, which is equal to $\frac{AC}{2}$ sin. θ. cos. θ.



If this be greater than the required area, then the dividing line will fall within the triangle AEC as Hk, and the triangle AHK will be similar to AEC ; but similar triangles are as the squares of their similar sides, (see Theor. 19, p. 15,) and, therefore, making A$k = x$, we have the following proportion to determine it, viz.:—

As the whole area AEC : the given area :: $AE^2 : x^2$

EXAMPLE 1.—The data of the preceding example being asssmed, it is required to cut off six acres, by a line at right angles to the front of the lot.

We found in the last example CE = 66·16

log. cos. CAE	=	9·08589
+log. AC	=	1·82387
log. 8·12	=	0·90976

area of AEC $= \frac{66{\cdot}16 \times 8{\cdot}12}{2} =$ 268·61 sq. chs.

By proportion,—As log. 268·61 chs.	=	2·42911
is to log. 60 chs.	=	1·77815
so is 2 (log. 8·12)	=	1·81912
		3·59727
		2·42911
	2 |	1·16816
AK = log. 3·63 chs. frontage	=	0·58408

EXAMPLE 2.—It is required, with the same data, to find the length of the side HK, and determine the actual area of AKH.

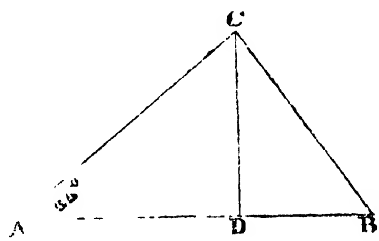
A. R. P.

Ans. HK = 31·28 chs.; and the area of AEC = 5 3 39

PROBLEM VI.

To lay out a given area in the form of a triangle, when the base, and the angle at the base, are given.

Let ABC = the given triangle, having the base AB, and the angle BAC, given; and let the area of the triangle = A; it is required to find, first, AC, and thence AD, and DC.



By the nature of triangles,

$$A = \frac{AB}{2} . DC = \frac{AB}{2R} . \sin.\angle A . AC$$

$$\therefore \frac{AC}{2} = \frac{\text{Area} \times R}{AP \sin.\angle A}$$

$$\text{then } DC = AC \sin.\angle A$$

$$\text{and } AD = AC \cos.\angle A$$

EXAMPLE 1.—There are 46 acres of land to be laid out in the form of a triangle, whose base is 15 chains, and the angle 35 degrees; required the lengths of the other two sides.

Here AB = 15, and the angle CAD = 35°,

$$\text{now } \frac{AC}{2} = \frac{\text{Area}}{AB \sin.\angle A}$$

$$\begin{array}{lrr}
\text{log. area (460 square chains)} & & = 2{\cdot}66276 \\
+\text{log. rad.} & & = \underline{10{\cdot}} \\
 & & 12{\cdot}66276 \\
\text{log. AB (15 chs.)} = 1{\cdot}17609 & & \\
+\text{log. sin.} \angle A\ (35^\circ) = \underline{9{\cdot}75859} & & \\
\phantom{+\text{log. sin.} \angle A\ (35^\circ) =\ } 10{\cdot}93468 & & = -\underline{10{\cdot}93468} \\
\frac{AC}{2} & = \text{log. } 53{\cdot}47 & = 1{\cdot}72808 \\
 & \underline{2} & \\
 & \text{chs. } 106{\cdot}94 = AC &
\end{array}$$

EXAMPLE 2.—Having the same data, to find the length of BC, in the previous example, and also DC; and thence to determine, whether, with these lengths, the triangle will contain the given area.

PROBLEM VII.

From a given triangle, to cut off any given area, by a line drawn from the vertex to the base.

Triangles, of equal altitudes, are proportional to their bases (see Theorem 17, page 15); therefore, making A, the given area of the triangle; a, the part to be cut off; and x, the required portion of the base, we have

$$x = \frac{a.\text{base}}{A}$$

EXAMPLE 1.—There is a Gore of land between two townships, whose area is 425 acres, and base is 85 chains. It is required to cut off 400 acres by a line drawn from the vertex.

As 435 : 400 : 85 chains.

$$\begin{array}{r} 400 \\ \hline 425)34000(80 \text{ chains} \\ 34000 \\ \hline \end{array}$$

length of new base = 80 chains

EXAMPLE 2.—From a Gore of land, having a base of 40 chs., containing 125 acres, to cut off 50, by a line from the vertex, required the base.

EXAMPLE 3.—From a triangle, with a base of 74·54 chs., containing 35 acres, to cut off 860 square yards, required the base.

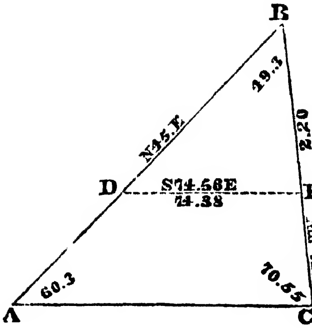
I have annexed a few questions of a more practical nature, and worked them out fully, so that the reader may make himself acquainted with the means of ensuring that practical accuracy, which is indispensable for success, in a profession to which so much responsibility is attached, and where the consequences of inaccuracy are so serious and lasting as in the survey of a new country. In Canada, where I was engaged for some time, one third (I believe I am speaking within compass) of the common law cases were either disputed surveys, or originated in the bad feeling engendered from encroachments, or supposed encroachments upon each other's property.

PRACTICAL EXAMPLES.

1st. There is a Gore of land between two townships, the boundary line of one being N. 45° E., and that of the other N. 4° 3′ W.; the length of the first line, 240·00 chs., that of the second 220 chs.

It is required to cut off, at the vertex end, 300 acres, by a line parallel to the base of the Gore.

Because the one line bears N. 45° E., and the other N. 4° 3′ W., the angle between them is 49° 3′.



$$\frac{240 + 220 + \text{sine } 49^\circ\ 3'}{2\ \text{R}} = \text{area.}$$

log. $\frac{240}{2}$ = 120 chains	=	2·07918125
log. 220 chains	=	2·34242268
log. sin. 49° 3′	=	9·87810900
		14·29971293
	—	10
1994 acres = 1994000 sq. chains	=	4·29971293

Now, as similar figures are proportional to the squares of their homologous sides in the whole, the whole area is to the given area as the square of one of the given sides is to the square of that portion of the side, that is to be cut off: that is,

As 19940 sq. ch. : 3000 sq. ch. : : $\overline{240}^2 : x^2$

when x^2, = length to be cut off.

As log. 19940	=	4·29971293
is to log. 3000	=	3·47712130
so is log. 240^2	=	4·76042240
	=	8·23754370
	=	4·29971293
to log. 93 chs. 09 lks.	= 2	3·93783077
		196891538

The other side BE will by the same process be found to be 85·33 chs.

Having obtained the lengths of BD and BE, the two sides of the piece to be cut off, that will contain 300 acres, together with the included angle 49° 3′, we can calculate the angles at the base by the second case of trigonometry.

These angles will be found to be as follows, viz.:—70° 55′ = greater angle, and 60° 3′ = the less.

To prove the calculation.

$$\text{Because the area} = \frac{\text{DB. BE. sin. } 49^\circ\ 3'}{2\ \text{R}}$$

$$300 \text{ area} = \frac{85{\cdot}33 \times 93{\cdot}09 \times \text{sin. } 49^\circ\ 3'}{2\ \text{R}}$$

	log. 42·66 chs.	= 1·630029
+	log. 93·09 chs.	= 1·968915
+	log. sin. 49° 3′	= 9 878109
		13·477053
		10
	3000 square chains	= 3·477053

Having verified the correctness of the calculated sides, we proceed to obtain the length and bearing of the base.

As the sin. 60° 03′	= 9·9377492
is to 85·83 chs.	= 1·9311260
so is sin. 49° 03′	= 9·8781090
	11·8092350
	9·9377492
DE = 74·38 ch.	= 1·8714858

To obtain the bearing.

AB bears N. 45° 3′
add 60° 3′

105° 6′ is the angle made with the north, because it is greater than a right angle; subtract it from 180, and its supplement, 74° 54′, is the angle made with the south point of the needle, being, therefore, S. 74° 54′ E.

In order, therefore, to cut off 300 acres from the Gore ABC, measure from B to A, 93·08 chs., and at that point run a line, bearing S. 74° 54′ E, which will be parallel to the base of the Gore, and should be 74·38 chs. in length.

The same data being given, it is required to cut off 300 acres towards the township, whose boundary line bears N. 45° E, by a line drawn from the vertex to the base.

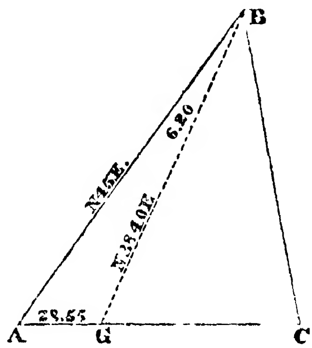
Using whatever calculations may have been made above, that may be useful to our purpose, first ascertain the base of the Gore.

As sin. 60° 3′	= 9·9377492
is to 220 chs.	= 2·3424227
so is sin. 49° 3′	= 9·8781090
	12·2205317
	9·9377492
to 191·17 chs. = AC length of base of Gore	= 2·2827825

Now triangles of equal altitudes are to each other as their bases; therefore, the whole Gore is to the piece to be cut off, as the whole base is to the base of the piece cut off.

As 19940 sq. chs.	=	4·27991293
is to 3000 sq. chs.	=	3·47712130
so is 191·77 chs.	=	2·28272250
		5 75990380
		4·29971297
to AG = 28·85 ch.	=	1·46019087

Next find the angles of the triangle ABG, by the rule for the second case of trigonometry, so as to determine the length and bearing of BG.



These angles will be found as follows, viz.:—113° 36′=larger angle AGB, and 6° 20′ = the smaller angle ABG.

BA is S. 45° W.		45° 0′
	subtract	6° 20′
		38° 40′

the bearing, therefore, is S. 38° 40′ W.

To find the length of the line BG.

As sine 6° 20′	=	7·04262490
is to 2885 chs.	=	1·46019087
so is sin. 66° 3′	=	9·93774920
		11·39794007
		9·04262490
to 226·65 chs. = BG	=	2·35531517

To lay off the line BG, cutting off 300 acres on the side AB, proceed to the vertex B, and run BG south 38·40 west, till you come to the base, and measure the distance GA, which should be 28·85 chs.; if the actual distance be found to be the same within a few links, the work is finished, but if not (*let it be* 29·85 chs. *instead*—

that is, one chain too much to the east), it becomes necessary to correct the bearing of the line by the compass, and run it over again.

To find the angle of correction.

As 226·65 chs.
is to sin. 90° or radius
so is 1·00 chs.
to the sin. opposite ∠

As 226·65 : 57° 3′ : : 1·00 : x when $x =$ length of the arc in degrees, and 57° 3′ is the angle, whose length of arc equals radius.*

57·3		
1·00	present bearing	S. 38° 40′ W.
57·3	add	15′
60	corrected bearing	S. 38° 55′ W.
226)343·80(15′		

PROBLEM VIII.

The bearings of three sides of a triangle being given, to cut off a given area by a line parallel to the base.

Rule 1st.—These three lines form a triangle, whose angles are all known, make one side = unity, and calculate the proportionate lengths of the other sides, making this side $= x$, and multiplying by the other two sides; all the sides are known in terms of x; find the area, which will be equal to the given area; the solution of the equation gives the answer: or,

Rule 2nd.—Say, As the rectangle of the sines of the angles at the base : the rectangle of radius into the sine of the opposite angle : : twice the area : square of the side required. The other sides may be found in the same way.

Example 1.—Let one of the boundary lines of a Gore, between two townships, bear N. 38° W., and the other N. 16° 30′ E.; it is required to cut off 325 acres from the Gore end, by a line, bearing N. 88° 15′ E.

* $2\,\pi$ rad. = circumference = 360 degrees.

π rad. = 180 deg. : rad. $= \frac{180°}{\pi} = 57° 3'$.

RULE 1st.—First, find the interior angles, and let AB $=1$, or unity.

To find BC or AC

As sine 53° 45	$=9{\cdot}90657$
is to 1°	$=0{\cdot}00000$
so is sine 71° 45′	$=9{\cdot}97759$
to 1·178 chs. BC	$=0{\cdot}07102$

As sine 53° 45′	$=9{\cdot}90657$
is to 1°	$=0{\cdot}00000$
so is sine 54° 30′	$=9{\cdot}91069$
to AC $=$ 1·01	$=0{\cdot}00412$

$$AB = 1{\cdot}000$$
$$AC = 1{\cdot}010$$
$$BC = 1{\cdot}178$$

Let fall a perpendicular BD.

$$\text{Area} = \frac{AC.AB}{2} \cdot \frac{\sin. \angle CAB}{R}$$

log. AB, or 1·000	$=0{\cdot}00000$
log. AC, or 1·010	$=0{\cdot}00412$
sin. $\angle$ CAB	$=9{\cdot}97757$
—log. rad.	$=10{\cdot}00000$
2 | ·9588	$-\ 1{\cdot}98171$
·4794 square chains.	

Because similar figures are as the squares of their homologous sides.

As ·4794 sq. chs.	$=1{\cdot}68070$
is to 3250 sq. chs.	$=3{\cdot}51188$
so is 1^2	$=0{\cdot}00000$
to x^2	$=2 \mid 3{\cdot}83118$
82·34 $=$ AB	$1{\cdot}91559$

which is the measure of the side taken as the unit side of the three.

RULE 2nd.—

$$\begin{array}{lll}
\text{As sine A. sine B} & \left\{\begin{array}{r}9{\cdot}97759\\9{\cdot}91069\end{array}\right\} & = 19{\cdot}88828 \\
\text{is to rad. sine C.} & \left\{\begin{array}{r}9{\cdot}90657\\10{\cdot}00000\end{array}\right\} & = 19{\cdot}90657 \\
\text{so is twice the area (6500 sq. chs.)} & & = 3{\cdot}81291 \\
 & & \ 23{\cdot}71948 \\
 & & \ 19{\cdot}88828 \\
 & \text{to } x^2 = & 2 \,|\, 3{\cdot}83120 \\
82{\cdot}34 = \text{AB.} & = x = & \ 1{\cdot}91560
\end{array}$$

the same as before.

To find AC and BC, the other two sides (by the first rule); multiply each of the sides of the unit triangle by this common unit of measurement, or marking A = the actual area, and a, the unit area, and x, as before, the length of any other side, in the unit triangle a, which was taken; then we have, by similar triangles,

$$a : \text{A} : : x^2 : y^2.$$

i. e. $\frac{\text{A}x^2}{a} = y^2$ where y is the actual length of the same side.

$$x\sqrt{\frac{\text{A}}{a}} = y \text{ or } \log. x + (\log. \sqrt{\frac{\text{A}}{a}} = 1{\cdot}91560) = \log. y.$$

where $\sqrt{\frac{\text{A}}{a}}$ or 1·91560 is the actual length of the unit side (AE).

$$\begin{array}{ll}
\text{Let } x\ (= 1{\cdot}010) \text{ then log. } x & = 0{\cdot}00412 \\
\log.\ \sqrt{\frac{\text{A}}{a}} & = 1{\cdot}91559 \\
\text{AC} = 83{\cdot}12 \text{ chs.} & = 1{\cdot}91971
\end{array}$$

Again,

$$\begin{array}{ll}
\text{Let } x = 1{\cdot}178 & = 0{\cdot}07102 \\
\text{log. unit of measurement} & = 1{\cdot}91559 \\
\text{BC} = 96 \text{ chs. } 96 \text{ lks.} & = 1{\cdot}98661
\end{array}$$

$$\begin{array}{l}
\text{AB} = 82{\cdot}34 \text{ chs.} \\
\text{BC} = 96{\cdot}96 \text{ chs.} \\
\text{AC} = 83{\cdot}12 \text{ chs.}
\end{array}$$

To find the side AC (by the second rule).

$$\begin{array}{lll}
\text{As the sin. } \angle \text{ A sin. } \angle \text{ C.} & \left.\begin{array}{r}9{\cdot}97759\\9{\cdot}90657\end{array}\right\} & = 19{\cdot}88419 \\
\text{is to radius sin. } \angle \text{ B.} & \left.\begin{array}{r}9{\cdot}910692\\10{\cdot}000000\end{array}\right\} & = 19{\cdot}91069
\end{array}$$

so is twice the area (6500)	3·81291
	23·72360
	19·88416
	to $x^2 =$ 2|3·83994
AC = 83·12 chs.	$= x =$ 1·91972

To find the side BC.

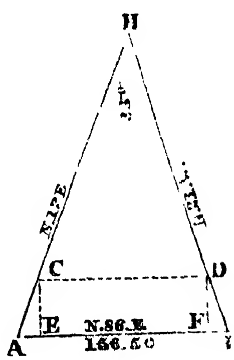
As sin. B sin. C	9·91069 } 9·90657 }	= 19·81724
is to rad. sin. A	9·97759 } 10·00000 }	= 19·97759
so is twice the area (6500) sq. chs.		= 3·81291
		23·79050
		19·81724
		to $x^2 =$ 2|3·97324
BC = 96 chs. 96 lks.		$= x =$ 1·98662

the same as before.

Example 2.—Given the bearing of a Gore of land, north 12 deg. east, and north 21 deg. west, and the base 156 chains, 50 links, bearing north 86 deg. east.

It is required to cut off 100 acres from the frustrum end of the Gore.

As sin. 33°	= 9·73611
is to 156·50 chs.	= 2·19451
so is sin. 73°	= 9·98060
	12·17511
	9·73611
to AE =	2·43900



perpendicular of whole triangle = AE. sin. A,

= 9·98284

= 2·43900

log. 264·20 = 2·42184

log. 264·20 = 2·42184
+ log. 156·50 = 2·19451
2|41330 = 4·61635

Area of triangle = 20665 square chains.

Subtract 1000 sq. chs., the area of the frustrum to be cut off.
there results 19665 sq. chs. the portion to be left at the vertex end.

As 20665 square chains = 4·31523
is to 19665 square chains = 4·29368
so is 274·80|² 4·87800
9·17168
4·31523
2|4·85645
= 2·42823

274·80
278·05 chains
6·75 chains = AC

As sin. 74° = BD, sin. 73°

$$BD = \frac{AC, \sin. 74}{\sin. 73^\circ}$$

log. 6·75 = 0·82930
log. sin. 74° = 9·98284
10·81214
—log. sin. 73 = —9·98060
BD, 6·785 chs. = 0·83154

To prove this work,

cos. 74°. AC = AE
cos. 73°. BD = BF

log. AC, 6·75 chs. = 0·82930
log. cos. 74 = 9·44034
log. AE, 1·8605 = 0·26964

and EC = sin. 74°. AC
log. sin. 74° = 9·98284
log. 675 = 0·82930
log. 6·488 = 0·81214

log. BD, 6·78 chs. = 0·83154
log. cos. 73° = 9·46594
0·29748

1·8605
1·9837
2|3·8442
1·9221

156·50
1·92
log. 154·58 = 2·18915
log. 6·48 = 0·81214

A. R. P.
100 1 8 = sq. chs. 1003 = 3·00129
R. P.
or 1 8 of excess in 100 acres.

sufficiently near for practical purposes, as if the perpendicular EC had been 6 chains 47, instead of 6 chains 48, the result would have been exactly 100 acres.

CHAP. VII.

POLE STAR.

THE α of Ursa Minor, or as it is more commonly termed Polaris, is about $1\frac{1}{2}°$ from the true pole, and revolves round it in 23 hours 56 minutes. When it is at its greatest distance, east or west, it is said to be at its eastern, or western elongation.

The true bearing of the pole star, that is, the angle made at the centre of the earth, between the true pole and the pole star, is called the polar azimuth; this, which should be taken at the time of its greatest elongation, depends upon the latitude of the place, and the distance of the star from the pole. This distance is called the polar distance, it is subject to a small annual diminution, called precession, which is 19·3 seconds annually. In the year 1830 this distance was $1° 35' 50''$; by multiplying the number of years since by 19·3 seconds, and deducting the product, the actual polar distance can be obtained. Now the azimuth, or angle of variation of the pole star, can be always determined by the following proportion:—

As radius : sin. lat. :: sin. polar dist. : azimuth.

Having obtained the polar distance to any day and place, in order to ascertain the angle of variation, or polar azimuth, find the time in the Nautical Ephemeris, when the star is about its greatest eastern or western elongation, and, with a theodolite or circumferentor which is furnished with a telescope, observe, carefully,

when the star, having ceased to move in its first direction, begins to retrograde; fix the telescope carefully in that place, and direct an assistant to take a stake, with a lighted candle upon it, and put it down in the same line, at some 8 or 10 chains distance. This is the line of elongation; then say,—as radius is to the whole distance, which must be carefully measured, so is the tangent of the angle of variation, to the actual distance in feet, measured at right angles to the former; the line connecting this new point, and the place where the instrument was, is the meridian line required. Take the bearing of this line, and the angle, found between this and the magnetic north, becomes the angle of variation of the compass; should the needle point to the east of this meridian line, the variation is easterly; should it point to the west, it is westerly.

CHAP. VIII.

ON LOCAL ATTRACTION.

THE needle being often, from various causes, diverted from its polarity, it becomes requisite, in running a line, to try the *backward* bearing at every station, and to see if it corresponds with the forward: if it does not, try the last forward station again, to see if any error may have been committed in taking it, should there be none, as the previous backward bearing, by assumption, must correspond with the preceding forward station, there can be no attraction there, and the attraction must be at the one where the backward bearing differed. Allow for the error or angle of attraction at the next station,

and proceed until this error be compensated. During the continuance of this angle of error, there is local attraction in the neighbourhood.

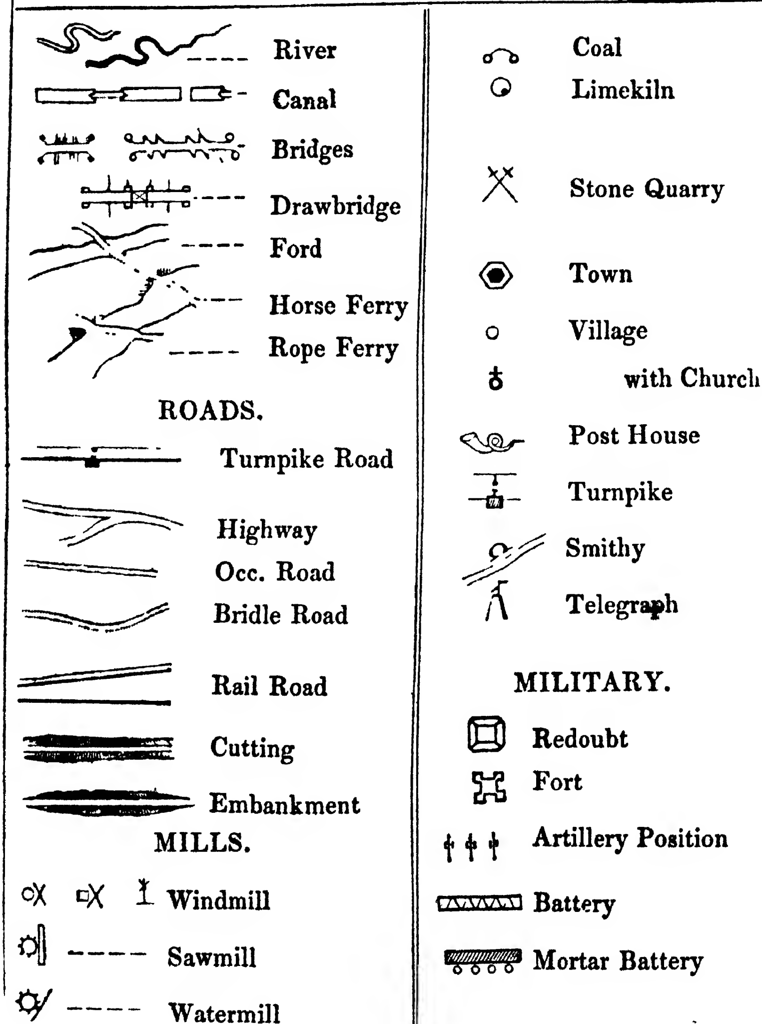
As at the first station, in starting, there is no backward bearing to prove the non-existence of attraction, it is impossible to say whether the error be at the first or second station, should the backward bearing of the second station not correspond to the forward bearing of the first; by taking, however, a third station, and taking therefrom and thereto, the bearings of both the first and second stations, the error will be discoverable immediately, as the backward and forward bearings of (from 2 to 3 and from 1 to 3) cannot both agree. Where the disagreement is, there is the local attraction.

Attraction may, however, commence with the first station, and not be discovered until some stations afterwards; should this be suspected, it becomes necessary to test the first bearing relatively, by a line making any angle with it, *(which angle has been measured by the chain,)* to such a distance, as may be considered beyond the sphere of the supposed attraction. There are many other *local* checks beside this; lines are seldom run in space unconnectedly in the woods, and, by measuring the distance from the next base line, its correctness may soon be determined.

CHAP. IX.

THIS page contains a small collection of those conventional signs that are most in use; the method of representation is such as is generally adopted.

CONVENTIONAL SIGNS.



MINES.		BOUNDARIES.	
☉	Gold	— — — — —	of a County
☽	Silver	—·—·—·—·—	of a Parish
♃	Tin	- - - - - - - -	of a Township
♀	Copper		
♄	Lead	△	Station Point
☿	Quicksilver		

CHAP. X.

ON PLOTTING, SCALING, &c.

Before commencing to plot, it is always requisite to consider carefully the shape of the plan to be plotted, its size and character, and the most desirable position to place it upon the paper, so as to admit of the best vacant space for the insertion of the heading or title, with the usual specification, that should accompany it. It has generally been considered indispensable to place the plan, so as to have the north side of the plan on the top of the paper fronting you; but, I would recommend, that the position of the meridian line should be but a secondary consideration, and should, in every case, depend upon the size and shape of the plan; where these two desiderata can be combined, it is better, of course, to do so; though I certainly cannot deem it a matter of so much moment as it is often made.

Before commencing your plan, take care also to have the paper properly stretched upon a drawing board, if the size of the plan will admit of it, and finish the whole plan before taking the paper off the board. At the bottom of the paper, make a scale of the required proportion, carefully dividing it into tenths and hundredths as the case may be, and let all *long* lines upon the plan be measured off this scale. The *short* ones, that is, lines of offsets; lines of distances, less than tenths, may be taken off the ivory scale, from which the scale upon the paper was first obtained.

Paper contracts very much, so much so, that there is frequently a difference in the length of the same line between one year and another of two hundredths, when the line is 10 or 11 tenths long; a line one year may measure 20 or 30 chs., and the next be only 19 chs. 80 lks., or 29 chs. 75 lks., being a loss of a link or two in each chain. In distances, therefore, of less than a chain's length, there can be no preceptible error. These distances might always be taken by the common ivory scale of equal parts, and, in fact, should be, as these smaller subdivisions could never be so well divided upon a paper scale; and the divisions upon paper would, from constant using in the making of the plan, soon wear through one into the other. These remarks may appear superfluous to a novice, but experience will soon show him the value of them. The neglect of these considerations has been the source of many a day's loss of work to a beginner.

Having made the scale, lay down your base line very accurately, and draw it carefully in with lake, marking the various stations upon it and its total length. Then take, with the compasses, the various lengths of the

sides of the several triangles, of which the survey is composed, and lay off the different points of intersection, testing rigorously, as you proceed, the constructed, with the measured lengths of the respective *check-lines.*

Do all this *before* an offset is put in, unless the offset be afterwards used as the point of a more convenient base line for another triangle.

When this part of the plotting is found correct, draw the lines in very plainly with lake.

In marking off the several distances on the base line use one of the long scales, and, placing it close against the given line, prick off, with a fine needle, the proper distances, and round the points, as centres, draw a small circle, but on no account use any black lead pencil, however finely sharpened, in stations of this kind. Do this also, in determining the point of inclination of the sides of the triangle. And in cases where great accuracy is required, I should prefer striking the radii, not with a bow pen, but with the finely pointed compasses themselves.

This can be done so slightly, as accurately to show the intersecting arcs, without at all injuring the paper. The mere injury of the paper, however, should never be a subject of any consideration, in comparison with the accuracy of the drawing, as, in all cases, a correct and accurate office drawing must be made, as a plan of reference, in which accuracy is the only desideratum,—accuracy in the outline,—accuracy in the determining of the areas. In a finished plan it is totally different; a finished plan is seldom made, in fact, it ought never to be made, an authority for the extension or compilation of surveys, as accuracy of detail, and beauty of colouring, are seldom, and not easily, combined.

Having finished these subsidiary lines, as they may

with propriety be termed, proceed to the laying off the offset points.

The best method of doing this, is to place the long scale, above referred to, close against the given line, having the zero points of each coinciding, and get an assistant to read off the several distances thereon, whence offsets are taken, first going through the right offsets, then the left. An offset scale is now necessary, which is a small scale of about 3 inches, divided in the same way as the long one, but the zero points being either edge of the scale. This is placed against the long scale, and the lengths on the measured line are determined by the long line, while the distance of any point, or offset therefrom, is determined by the offset scale; this latter point is alone marked. Practice and care will ensure considerable rapidity, as well as accuracy, in this plan.

When the scale is 2 or 4 chains to the inch, any offset, less than 10 links, must be done by the hand.

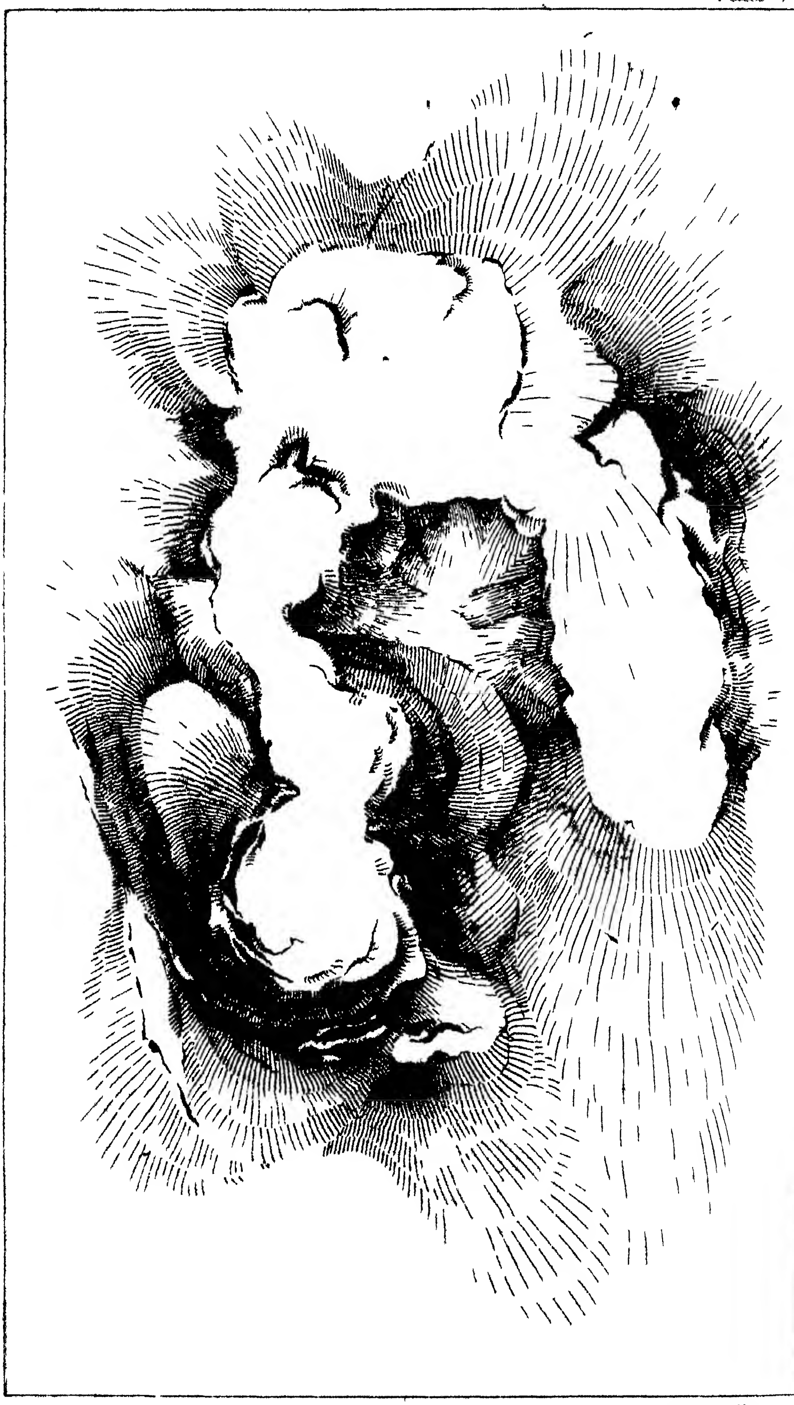
With reference to the division of the scales—the scales used for horizontal delineations are generally 2 or 4 chains to the inch, or 20 chains, or 40 chains. The usual scale used in the plotting of the tithe commutation surveys, is that of 3 chains to the inch.

The division of the common and vertical sections, is that of 5 or 50 feet, 10 or 100 feet to the inch.

I have one caution to give beginners, in the purchasing of scales, never to purchase a scale having different divisions upon it, as it is productive of considerable and serious error.

The additional expense when you are purchasing—of purchasing one of a set—is soon made up by the saving of the frequent losses of time, that a difference of divisions on the same scale must inevitably occasion.

Plate 7



F Mansell sc

CHAP. XI.

TOPOGRAPHICAL SURVEYING.

In surveying a tract of country, it is frequently not only necessary that a correct delineation should be obtained of the various outline of hill and dale, of river and forest, but that some method should be adopted of conveying upon the same plan a pretty fairly correct representation of the relative heights of its different parts. The positions of the various boundaries in the plan are, or always should be, the horizontal spaces they would occupy on the earth's surface, totally independent of their height; this height must, therefore, be obtained in some other way; straight lines drawn from the summit, diverging to the bottom, more or less close, as the hill is steep or otherwise, are used in the one case: this is called the vertical method; in the other, the irregularities of the earth are represented by waving horizontal lines, approaching or receding in proportion to the steepness of the ground, or its graduated ascent. The two plates, Nos. 6 and 7, are examples of the first method. There is a boldness of style, and a faithfulness of representation, which I am disposed to consider must give it the preference. This style of drawing is called military drawing, probably from the special demand for these

topographical features of country for military purposes. By this, an officer is enabled, at a glance, to ascertain whether a commanding point, he is desirous of occupying, is accessible to cavalry, or unapproachable by infantry; and he is thereby enabled to decide, whether he can venture to attempt the dislodgement of an enemy from one hill, or can permanently occupy another, in spite of odds that may be sent against him.

This kind of drawing has its advantages even to the civil Surveyor; if well done, the introduction of the features of the country is a great improvement to the plan, abstractedly; and, to the proprietor of an estate, the topographical details are often useful in assisting him in the laying of it out.

I need scarcely refer to the great assistance, that the engineers of the present day have derived from the faithfulness of the topographical delineations of this country in the ORDNANCE MAPS, in the selection of the best lines of routes for railways; every professional man must have experienced this himself.

Plate [illegible]

F. Mansell sc.

LEVELLING,

ITS NATURE AND OBJECTS.

CHAP. I.

LEVELLING is the art of representing the inequalities of the earth's surface, and of determining the relative heights of any number of points above or below a line, equidistant, at every point, from the centre of the earth. This line is what is understood by the term—a level line; it is that line which is assumed by water when at rest.

The instrument, used for the purpose of levelling, is called a spirit level.

THE Y SPIRIT LEVEL.

Its Description.

This instrument is merely one portion of almost every other instrument, carried out to its greatest practical perfection. The bubble, which in most instruments forms only a subordinate part in the construction, is in this the chief, the only object of the instrument being to obtain a practical tangent to the earth's surface, or to place

the line of collimation of the telescope in a truly horizontal line. Hence it is termed THE SPIRIT LEVEL—*par excellence.*

This instrument consists, like the others, of its parallel plates, with their two pairs of conjugate screws, of its telescopes, and its spirit level beneath. The telescope stands upon Y's, (I have selected the Y level in preference to the rest for description,) as in the case of the theodolite; and has, also, like that instrument, its milled-head adjusting-screw for the object-glass; and the moveable eye piece for neutralizing the parallax. The cross wires, however, are not always arranged the same way as in the theodolite; sometimes one horizontal and two perpendicular wires are used instead. By the latter contrivance you are enabled to secure the perpendicularity of the staff, in one direction, which you have no means of doing by the former.

The spirit level here, also, is furnished with its capstan-headed screws, for making it parallel to the axis of the telescope, vertically and laterally.

But in this instrument there is one contrivance which the other instruments do not possess—of raising or depressing one of the Y's, or supports, on which the telescope rests, so as to have the axis of the telescope always at right angles to the axis of the instrument.

ADJUSTMENTS.

I need only repeat, that, for the line of collimation,* the telescope must be turned round on its axis, that the intersection of its wires may always intersect one point, this adjustment will be found fully explained in

* The two first adjustments are similar to those of the theodolite.

the description of the theodolite. The adjustment of the second liability to error, in the non-parallelism of the level and the telescope, is obtained as before, by reversing the telescope in its Y's. These two adjustments must be completed first. Afterwards,

To make the axis of the telescope always at right angles to the axis of the instrument; in other words, to secure the line of collimation being perfectly level in any portion of a complete revolution of the instrument.

Set the telescope over any pair of conjugate screws, and make the bubble level; turn the instrument, till the telescope be over the conjugate pair: level it in this position; then turn it back to the first pair, and correct for any error that may have arisen from the last levelling, and continue till the bubble be central over the two pairs of conjugate screws; then turn the instrument one half revolution round, and, if the bubble still remain in the centre, the instrument is in adjustment; if not, the error can only be occasioned by the axis of the bubble, or, which is the same thing, the axis of the telescope not being truly perpendicular to the centering of the instrument.

To correct this error, raise or depress the moveable (Y) support by the milled-headed screw beneath, until the bubble be brought half-way to its proper position, and correct for the other half by the parallel screws. By repeating the correction two or three times, the greatest accuracy will be obtained.

It is necessary to examine the adjustment every morning before starting, and it should be seen to at every observation, though it will scarcely require re-adjusting the same day. I should observe, that there is, or ought to be, a cap over this adjusting (Y) screw which should be carefully kept on.

This level is a very delicate instrument, and soon liable to get out of adjustment without great care.

There are several other kinds of levels—Troughton's, Gravatt's, &c.: all good of their kind, and each, perhaps, more fitted than the rest for some peculiar kind of levelling.

In trial and check-levels, I would recommend Gravatt's or Troughton's, being calculated by their lightness and non-tendency of disarrangement, to get rapidly over the ground.

For the main sections, at every two chains, I should prefer the Y level; and for the putting down the rails, the formation of roads, and all work where accuracy, and not expedition, are required, I should decidedly give it the preference.

There is one fault I have found with most levels, that the tube of the telescope is not long enough to admit of reading off the staff within short distances, few reading within half a chain. Having had placed, for myself, in addition to the extending tube at the object end, another at the eye-glass to remedy this defect, I have been enabled to read within three yards; to this inner tube, of course, was attached the diaphragm of the cross wires and the lengthening eye-piece.

LEVELLING STAVES.

These are generally made twelve feet high, divided into feet, and again into tenths of a foot, and subdivided for facility of computation, into hundredths. The method of arranging this subdivision, constitutes the difference between the several staves in use. Some persons prefer one, some another, and I must leave the reader to select for himself. There are

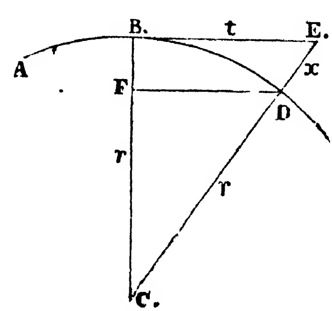
Sopswith's, Gravatt's, Cowper's, Stephenson's, &c. The one I myself prefer is, what I believe goes by the name of Stephenson's: and was first used on the London and Birmingham; the hundredths are obtained in the same way as in the common ivory protractor; the tenths of a foot through the whole length of the staff are bisected, making the two divisions "twentieths;" and these division lines extend the whole breadth across the staff; the opposite ends of these lines are connected by diagonal lines, each one with its preceding, viz.,—the left of No. 1 with the right of No. 2, the right of No. 2 with the left of No. 3, and so on. And five vertical lines are drawn along the whole of the staff, which thus divides each of these diagonal lines into five equal parts, each being the fifth part of the twentieth, or the hundredth of a foot.

The feet are distinguished by large red figures; the tenths in black, with a large full point at every ·5.

The lines obtained by this means are only level lines for very short distances, being *tangents* to the earth at the several points of observation, and not their corresponding *arcs*, which are the true levels.

The value of this error and the practical methods adopted had better be at once explained.

Let ABD be a portion of the earth's circumference, whose centre is C. Let BE be any level distance of the instrument (t), the true level line will be BD, and the error between the apparent and true level will be the versed sine BF. When the distance BE is great, this versed sine must be calculated by trigonometry.



nometry. But, for the usual distances of observation by the spirit level, ED, or sec.—rad., can be safely taken instead.

Now, CBC is a right-angled triangle, and therefore

$$\overline{r+x}^2 = r^2+t^2$$
$$r^2+2rx+x^2 = r^2+t^2$$
$$2rx+x^2 = t^2$$

throwing x^2 away, as indefinitely small, in relation to $2rx$, we have

$$2rx = t^2$$

and $$x = \frac{t^2}{2r}, \text{ or}$$

the error $x =$ the square of the tangent, divided by twice the radius; and, as this divisor is a constant quantity, this error is proportioned to the squares of the distances.

The mean diameter of the earth is 7,916 miles; for one mile distance, therefore, we shall have $x = \frac{1}{7916}$ miles, or 8,004 inches; for two miles distance, four times that quantity; for three miles, nine times; throwing away the $\frac{4}{1000}$ of an inch as immaterial, the error of one mile's distance is 8 inches or $\frac{2}{3}$ of a foot; for two miles, $\frac{2}{3}{\cdot}4$ feet; for three miles, $\frac{2}{3}{\cdot}9$ feet, &c.; or x in feet $= \frac{2}{3}{\cdot}4$ (distance in miles)^2; which formula may be easily remembered. This value of x would not be sufficiently correct for many miles. Referring to the equation $\overline{r+x}^2 = r^2+t^2$,

the value would then be $x = \sqrt{r^2+t^2}-r$.

Example of applying this correction.

Placed a spirit level at any point B, (see fig. page 255,) on the earth's surface, and found the point E, at 3 miles off, to be on an apparent level with the point B. What is the comparative height of the object E?

Now, BE is the apparent level, and BD the true level, B and D being points equidistant from the earth's centre. DE (x) is the height of E above B, which, in feet, equals the distance (DE, in miles) $3^2 \times \frac{2}{3} = 9 \times \frac{2}{3} = 6$ feet, the height of the object E above B.

The apparent height, therefore, of every distant object, as observed by a spirit level from any point, is always *less* than the true height, by this value of x, for the curvature of the earth.

CHAP. II.

REFRACTION.

THERE is also, in long observations, another correction necessary, arising from the effects of the density of the atmosphere, in refracting the rays from the object, which makes the apparent greater than the true height. The correction, therefore, must be subtracted from its apparent height.

Refraction increases the distance at which objects can be seen (cæteris paribus), in a proportion of 14 to 13, and raises the apparent height one-seventh of its correction for curvature.

Thus, in the last example, the object observed at a distance of three miles was apparently level with the instrument, but the correction for curvature being six feet, and the height (independently of the subsequent correction for refraction) was 6 feet, $\frac{1}{7}$ of $\frac{6}{1} = \frac{6}{7}$ feet $=$ 10 $\frac{2}{7}$ inches, and therefore 5 feet $1\frac{5}{7}$ inches $=$ the true height of the object, allowing for both corrections.

EXAMPLES.

1st.—The observed heights of three objects, at a distance of 4, 6, and 8 miles, (calculated from observations taken by the theodolite,) were found to be respectively, 24, 25, and 28 feet. What are their true heights?

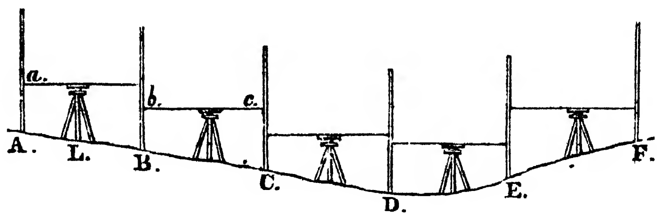
2nd.—Found the angle of elevation of the spire of a church, which was 420 chains 75 links off, to be 1° 10′. What is its real height above the point of observation?

CHAP. III.

To find the difference of levels between several points, or to trace a sectional line of the inequalities of the earth's surface.

Let ABCDE be the line to be traced. Set the level (L) between the object, and read off the height A*a* and that of B*b*, the difference between A*a* and B*b* will be the number of feet that B is higher or lower than A*a*; if B*b* be greater than A*a*, the point B will be

lower (by this difference) than A*a*; for the height, read off by the level staff, is the number of feet that each point observed is *beneath* the level of the line of collimation of the telescope—hence, where there is a number of points beneath the same level line, the greater the reading of the staff, the lower this point must be.



Then, because, in the first observation, the height at B (read by the level staff) is greater than that at A, the point B is lower than the point A. Again, in the second example, where, it must be observed, that another line of collimation is taken, because the height by the staff at C is greater than at B, the point C is lower than B. In the third observation, also, D is lower than C, and C being lower than B, and B than A, the ground falls thus far. At the fourth observation, however, because the height at D is greater than that at E, the point D is lower than E, and, therefore, E being higher than D, the ground rises to E, and as the reading at E is greater than at F, it goes on rising to F. The relative heights of the two ends of the line, at A and F, depend upon whether the ground falls, more or less, from A to D, than it rises from D to F.

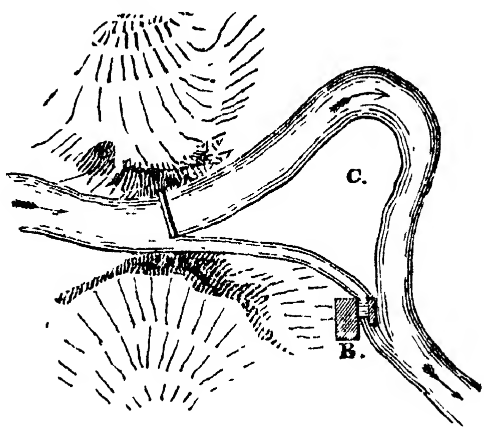
Now, the difference between the reading at B and A, in the first observation, added to the difference of readings at C and B, in the second observation, plus this difference between D and C, in the third, as there is a continued descent to the point D, will give the actual fall from A to D, or the number of feet, that the point D is lower than A. In the same way, the sum of the difference of readings of D and E, and of E and F, in their respective observations, will be the number of feet F is higher than D; if, therefore, the fall from A to D, be greater than the ascent from D to F, the difference will be the actual fall from A to F, or the number of feet that the point A is higher than the point F.

This is the principal object of levelling. It is very simple in theory, but in the carrying out of the practical operations, great care is

necessary. In this, as in most things which are of a simple character and which do not admit of checks in the course of the work, errors are very like to creep in imperceptibly. There are certain mechanical contrivances to guard against them, which will be subsequently explained.

This taking of several stations is called compound levelling. Simple levelling is merely the determining, by one observation, the relative heights of two given points.

Thus, supposing that it it is required to ascertain, in the accompanying diagram, whether there was sufficient fall of water between that part of the river at A, and that, at B, to turn a grist and saw-mill at B. Placed the level at C, between the two stations, and found the first or back reading 0·45 feet, and the forward 11·52 feet; the back distance was 2 chains, and the forward distance 20 chains; required the fall.



The back reading is 0·45 feet, at the short distance of 2 chains, which will require no correction for curvation or refraction.

The forward reading, being at a distance of 18·50 chains, must be corrected.

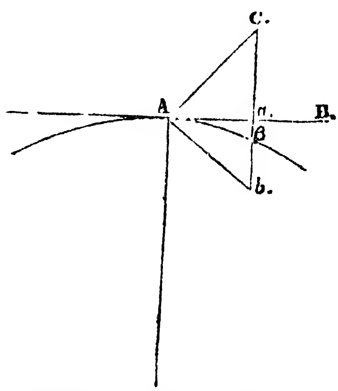
Now, x (for curvature) $= \frac{20^2}{7916} = 5$ inches $= \cdot 416$ feet. This must be *subtracted* from the observed reading, and you obtain the true reading (independent of the refraction),

11·52 feet, observed reading
0·416

11·104 true distance beneath the first point.

Add, for refraction, $\frac{1}{7}$ of ·416, or ·059, and the true height of the water at A, above that at B, is 11·163 feet.

As in this case, to obtain the true reading, the correction for the curvation has been subtracted, and that for refraction added, which is the reverse of the preceding rule at page 257, we will investigate the subject a little more fully.



Let AB be a tangent line to the earth, at the point A, being the apparent level line; let Aa be the true level, and β the corresponding point to a. Let C be any object *above* the apparent level line, and b any object *below* the same level line. The object above would be observed by the theodolite, those beneath (for short distances and for the common purpose of levelling) by the level staves. Let aC be the distance above the positive height ($+h$), and ab the distance beneath, which will then be ($-h$). Now aC is the height above the ine AB, βaC is the true height, therefore $\beta a+a$C is the true height of the point C, above A.

On the other hand, ab, the reading by the level staff, is the distance beneath the apparent level line AB, but the true line is Aβ, and $b\beta$ is the true difference of levels between A and b, which is, therefore, equal to ($-h+b$), or the difference between the two.

Again, for the refraction.

As refraction raises the apparent above the true height of an object, its correction must always lower the apparent height (it is therefore negative $-$R).

If the object be above the apparent level, the error for refraction must be subtracted from the apparent height, thus ($h+\text{C}-r$), where $+$C is the correction, for curvature, and r, is that for refraction, when these values are positive.

But if the object be below the apparent level, we have ($-h+c-r$), that is, the curvature (which is the only positive value) must be subtracted, and the refraction must be added, to the negative value of h, which still remains negative, or beneath the apparent level.

Generally, therefore, let the reader bear in mind, that the correction for the curvature raises the object, and that for refraction depresses it, reducing or increasing its distance from the apparent level, according as the object is above or below it, *i.e.*, has been observed by the theodolite, or read off by the level staves.

As the necessary calculations for curvature and refraction would be exceedingly tedious, in extensive operations, the following method renders them altogether unnecessary.

Set the level in the centre of the object, as nearly as *the eye can tell*, and these corrections for both objects become equal and opposite, and therefore neutralize each other. The apparent level, being always parallel to the chord, that connects two objects, equidistant from the place of observation, must, therefore, have the same versed sine.

Before proceeding to the main level, we will briefly go over the method of conducting the trial and check levels, as they are termed, before taking the final level.

CHAP. IV.

TRIAL LEVELS.

Having determined upon the general line of route, the line is marked down upon the Ordnance map, and the several points, where the roads are crossed, are carefully measured from the scale, and determined upon the ground.

The portions of the line between these points are roughly picketed, *if time will permit*, or, if not, their general direction, by the compass, is ascertained, on the plan. The levels are then taken, as near as possible to this direction—the error of deviation being always confined to the intervals between the roads—the relative heights of these points of the roads being always ascertained, and bench-marks taken near. The inclination of the ground, on the right and the left of the line, is also, in the first trial level, carefully marked, so that the engineer may know on which side of the

levelled line to deviate, when he is in want of a piece of cutting, or an embankment.

The trial level, however, is, after all, but very rough work, and serves only as a general check upon the correctness of the subsequent levels.

The heights of the several points upon the roads are useful, as checking their heights by the next level, but the intervals are generally all *out of line*, and never to be depended upon; certainly not, when the direction of the line is not picketed. The only use, then, of this first trial level is, in addition to its furnishing the general face of the country, in a certain direction, its acting as a check upon the height of the roads, and upon the relative heights of the two ends of the line, so as to prevent any serious error in the final Section.

Such being the nature of the first trial level, it may easily be understood how easily errors may creep in in the parliamentary plans, when these level trials and the ORDNANCE MAP are often the only data, that the indecision of companies and the pressure of time, place within reach of the engineer.

CHECK LINES

Are levels taken for the purpose of determining, a second time, the heights between two distant points, which, agreeing with the first, becomes a strong proof of the correctness of the work in detail.

METHOD OF LEVELLING ON A LINE OF RAILWAY.

Having determined upon the point of commencement, select some fixed point, as near to it as possible, whose height being taken, becomes a mark of reference for any subsequent level. This is called a *bench mark*. Place your level about half way between this point and the next onward station, and fixing the legs

firmly in the ground, set the instrument level, and observe the height read off by the staff at the back station: this observation is called a back-sight, or back-set. Turn the instrument round to the bench mark and read off the height there. This is an intermediate, as not being a connecting-link in the consecutive series of backsets and foresets. Lastly, read off the height at the forward station, which is termed the foreset, taking care, before each of these observations, to see that the bubble is duly in the centre of the tube. Now take up the level, and place it, as before, between the next two stations, and so on, observing the back and forward readings in every case, and taking, in the way, such bench marks as may appear desirable for the purposes of reference hereafter. These bench marks should be of a permanent character, *near* to the line, and in conspicuous places.

These observations should, in all cases, be carefully taken, and due attention paid, on the part of the Surveyor, to the non-existence of parallax.

The staff-holder has, also, considerable responsibility reposed in him, as all the care on the Surveyor's part would be neutralized by inattention in the placing or moving of the staff. At every station the staff has to be read off twice, in opposite directions, and great care is requisite in turning it round. A peg, or board, or a penny, is sometimes used by the staff-holder for that purpose, so as to keep the staff always in the same point. Much error also frequently arises from the staff not being held perfectly upright. There is some difficulty in keeping it so, and men are apt to become tired and careless. The wind too will often disturb it.

Personally, I have found the greatest difficulty in getting men to keep it upright; they do not understand that it can be out of the perpendicular in two ways; they will often keep it straight, as to it not being too much to their left or right, while, frontwards, it will be considerably out.

This inconvenience is avcided in some level staves by a small plummet in the side of it, which enables the st ff-holder to see whether

it is perpendicular in one direction; and its lateral perpendicularity the Surveyor can judge of for himself.

Method of keeping the Field Book.

Divide the page into the several columns, as in the example, page 268, which are the field notes of a portion of a line of railway from Wandsworth to Croydon, commencing near the Wandsworth station, at a point which is carefully described, and can at any time be easily ascertained, being on the surface of the rail at $\frac{1}{4}$ mile, or 20 chains, *below* the second bridge from the Wandsworth station.

This point, which was in a cutting, was assumed to be 140 feet above an imaginary level, to which the heights, at the several points on the line, bear reference, which is called a Datum Line.

This point might have been considered zero, but as whenever any other points were below this, their height would be negative, the constant changing of, from positive to negative, would be productive of considerable trouble and probable error, especially to those of my readers who might not be quite *au fait* at Algebraical calculation.

The height of the point of starting is put in the column of reduced levels, 140, and the 0·00, in the column of distance, shows that it is the starting point. The instrument is now placed between this point and the next favourable station (not exceeding, in any case, eight or ten chains from the level to the station) in this, from the deepness of the cutting, only 50 links between the *two* stations, and the back reading was found to be 11·04 feet, and the forward reading 1·49. These are placed in their respective columns. Now, as the foreset reads less than the backset, the ground

rises, and the difference between the two readings (9·55) is placed in the column of *Rises.* The height of the ground, where the instrument was placed, of course is not known. The instrument was then removed, to between the second and third station, and the staff turned round, when a second reading of each was taken, and placed in their proper columns. The last reading being less, the ground still rises, and the difference (11·47) is again placed in the column of Rises, and its distance *(always from the starting point)* is marked down, viz.:—86 links. Thus we proceed till, at 1 chain 66 links, we get to the top of the cutting, always rising.

Before continuing, we will see how the reduced levels are obtained. These reduced levels are the actual height of each point respectively above the assumed datum or standard line of height

The rises and falls are the differences at each point of its own and the preceding height, and denote whether the ground has risen or fallen between any two consecutive points. By adding, therefore, each rise to the preceding actual height, we obtain the actual height or reduced level of the point, and by deducting this difference where there is a fall, we also obtain the same result.

At the next station the ground falls, the foreset being greater than the intermediate, and 2·61 is placed in the column of falls, which, subtracted from 182·52, the reduced level at the preceding point, gives 179·91, the reduced level at this point. The height being that of the field immediately beyond the ditch on the top of the cutting, the distance was not wanted again, and it was, therefore, made an intermediate, the foreset at

the same station being at a distance of 10·89 from the starting point.

It must be observed generally that there should be no *foreset* without its distance. All B. M. of every kind—the heights of water,—in rivers and drains,—in fields,—on banks,—and every height taken for the purposes of illustration, should be made intermediate.

Having filled the page with the observed backsets, intermediates and foresets, add up the foresets and backsets, their difference will be the difference of height between the starting point and the last point on the page, and it will be equal to the difference between the sums of the rises and falls; it should be also equal to the difference between the assumed datum line and the last reduced height.

Having these three checks, there is no possibility of error when they all agree, and *in all cases every page must be so tested.*

Before calculating the reduced levels, as any error in the rises and falls must induce the same error in the reduced levels, observe first, whether the difference of the rises and falls is the same as that of the sums of the backsets and foresets, you can then safely proceed to the calculations of the reduced levels.

Observe, in placing the different readings, that, in the first line in the column of reduced levels, must be placed the assumed height of the starting point above the datum line; collateral with this must be 0·00 of distance, accompanied with a full and clear description of the exact locality of the starting point. (Consider that you are describing the place to a perfect stranger, who has never been on the spot before.)

On the next lower line, the first back reading must be placed, and, collaterally with it, *in the same line*, the next reading in the *intermediate* column, if it be that of a *bench mark* without a given distance, or of a station, which is not intended to be the backward station to the next observation. If it is, place it in the column of foresets, collaterally with the backset, in the same line. Observe, that every line in the reduced level must have a given height, and that to each of these heights, in the column of distances, there must be a distance given, or an observation of individuality.

The reason of placing the foreset in the same line as the backset is, that the height of each backset point is the height of the preceding foreset, and that it is the difference between the actual backset and foreset upon the same line, that gives the height of the latter.

In calculating the rises and falls, take the difference between the

two readings in the same line, when the one is a back reading, and the other, intermediate or foreset, and place the difference in its proper place, in the same line. Then, if the next reading be an intermediate, it will be placed under the preceding intermediate ; the difference between these two intermediates must be placed in the same line as the latter, and so on throughout any number of intermediates. After these intermediates must, of course, come a foreset point, before the instrument can be removed, which foreset becomes a backset point of observation at the next placing of the level. The difference must be taken between this foreset and last preceding intermediate, and placed collaterally in the same line with the foreset.

This closes one portion of the line of section, and another is recommenced that would be totally unconnected with the former were it not that the last point in that becomes the first point in this,

Collaterally with the reduced levels, if these levels refer to bench-marks, write B. M. in large characters in the column of distances, and the nature of it under the head of remarks. If the reduced levels refer to points taken out of the line, which is frequently the case in trial levels to avoid obstacles, write "out of line;" also, in the column of distances, carry the distances on always from the starting point to 80 chains, and, if an observation be taken at that point, which should be done if possible, write 1 mile against it, and begin the chaining afresh,—continue to another 80 chains, marking this point 2 miles ; begin here again also, and proceed as before.

CHAP. V.

THE first page of the following notes has been completed, the rises and falls calculated, and the reduced levels put in. The other pages contain only the several readings at the different stations, and the respective distances or observations to each ; the rest must be filled in by the student.

TRIAL LEVELS

On a proposed line of Railway between Wandsworth and Croydon.

Backset	Inter.	Foreset	Rise	Fall	Reductn. Levels	Dist.	Remarks
					140·00	0·00	Datum line being 140 ft. below the level of the sea.
11·04		1·49	9·55		149·55	·50	Commencing at the surface of the
11·60		0·23	11·37		160·92	·86	rails opposite the centre of the
11 50		0·39	11·11		172·03	1·23	watch box, ¼ mile below the second bridge, from the Wandsworth station.
11·61		1·12	10·49		182·52	1·66	top of cutting of railway
6·04	8·65			2·61	179·91		in field
		4·79	3·86		183·77	10·89	
5·18	7·05			1·87	181·90		within field
		8.20		1·15	180·75	18·80	in field beyond
3·31		11·97		8·66	172·09	26·08	
1·23		9·95		8·72	163·37	29·84	
2·92	7·28			4·36	159·01		in field
		9·84		2·56	156·45	35·68	in field beyond
0·42		10·55		10·13	146·32	43·91	
0·55		6·94		6·39	139·93	52·39	
4·40		5·10		0·70	139·23	63.31	
3·66		2·97	0·69		139·92	B M	being hook of lower hinge of gate in lane
2·53	5·40			2·87	137·05	63·83	centre of road No. 1 BURNTWOOD LANE
		2·42	2·98		140·03	64·33	in field beyond hedge and ditch 64·10 and 64·20
4·90	8·73			3·83	136·20		in gravel pit
		4·69	4·04		140·24	71·70	71·60 path crosses, 71·76 hedge crosses
5·40		4·84	1·56		141·80	0·15	1 MILE. 76·40 & 79·70 hedges cross
4·82		3·72	1·10		142·90	11·23	520 hedge, 11·03 ditch
1·00	2·35			1·35	141·55	B.M.	on rail by tree near shed
		6·27		3·92	137.63	18·70	18·78 and 21·74 hedge and fence
4·77		9·54		4.77	132·86	26·10	30·08 and 30·70 garden fence

TRIAL LEVELS CONTINUED.

Backset	Inter.	Foreset	Dist.	Remarks
4·11		2·09	31·02	By brick wall, Occupation-road, same level
4·95	3·15		B. M.	on lower hinge of gate of blue rails
		3·22		out of line
5·20		2·45		out of line
7·37		4·18		out of line
3·48	6·03		47·53	Centre of road No.2, London and Tooting road
		4·50	B. M.	nut of lower hinge of white gate
5·06		2·51	57·48	52·53 ditch crosses
7·67		0·45		out of line 5914, and 6296, fence & ditch cross
8·83		0·68	75·00	69·70, and 76·30, ditch and fence cross
9·95		0·70	19·75	2 Miles
9·86		0·60	2·80	325 and 380, fences of road cross
8·97	11·78			Centre of road No. 3 Church-lane
		0·55	5.80	in field beyond
10·32		0·96	9·30	within field near fence of lane
2·01	5·92		9·80	Centre of road No. 4, Back-lane
		0·00	B. M.	on tree in Back-lane blazed (with nail)
8·73	0·21		B. M.	on top of post near drain on left of line & road
		0·25		out of line
4·34		1·60		out of line
0·40		11·77		out of line
0·40		9·65		out of line
2·72	6·10		B. M.	on top of post, on right, near corner of wall
		8·25		out of line
5·50		4·96		out of line
5·40		3·70		out of line
5·09	2·81		B. M.	on stump near gate at culvert
		2·15		out of line
5·00		1·00		out of line
8·05		7·02		out of line
1·30		11·35		out of line
1·86	2·08		B. M.	on top of further side of gate, other side of road
		6·85	48·30	Centre of road No. 5
5·56	1·48		48·12	4812 and 48·48 fences of road cross
	4·52		48·52	in field beyond
		6·55	49·82	
1·36		10·93	57·58	59·30, ditch crosses
1·57	6·41		59·63	60·70, ditch crosses
		6·19	61·03	
10·65	5·53		64 00	
		4·15	71·25	6900 fence, 71·86 ditch
5·70		4·03	79·18	
5·80	3·72		3·78	3 Miles, 390 and 430 fences
	4.30			Centre of road No. 6
	4·00		B.M.	top of near post of black gate on left of line

CHAP. VI.

PLOTTING OF THE MAIN SECTION LINE.

Draw first the datum line in ink, carefully and finely; then lay off the several miles upon it, by means of a 12-foot ivory rule. The usual scale for sections of railways, deposited for Parliament, is 20 chains to the inch, or 4 inches to the mile. These distances must be carefully gone over once or twice to guard against error, and the number of miles must be marked against each. The method of chaining on a line from the beginning, and dividing the length into miles on the ground, confines every error of plotting within that distance, as the distances are always taken from the last mile point—so many miles + (plus) so many chains, till the next mile mark, when it begins again.

At every third mile, as well as at the beginning and ending, erect perpendiculars to the line, geometrically by the compasses, to about twice the height of the datum line, and draw a line through the distance parallel to the datum line; measure off upon this line the intermediate miles, and connect them with the corresponding points in the datum line.

Now, lay off, upon the datum line, the several distances on the column of distance for one mile, and draw very fine lines therefrom, parallel to the perpendiculars. These parallels, or supposed perpendicular lines, being confined within every mile, cannot be far wrong. In fact, the correctness of the section line depends upon their being truly perpendicular, or the distances in the section line, though correctly measured on the datum line, might be incorrectly projected.

Upon these perpendiculars measure carefully off, with a fine needle, on a scale of 100 feet to an inch, the several heights in the reduced levels corresponding to the distances, carefully distinguishing the roads and rivers, the position of hedges, &c., as in the accompanying plan, and connect their several heights; *put this in ink at once*, while the plotting is fresh in the memory, marking off in pencil the "reduced level" of the centre, against every road, the surface of the water of rivers, &c., and also the height of the ground at every half mile.

One mile being now finished, and not before, proceed to the next; finish that before you commence the following, and so on throughout.

CHAP. VII.

CROSS LEVELS.

Cross Section on line of London and Southampton Railway.

Backset	Inter.	Foreset	Dist.	Remarks
7·80			0·00	
	7·24		3·00	Commenced at surface of rails opposite the
	6·98		4·00	centre of the watch box, ¼ mile below the
	6·85		6·00	2nd bridge, from Wandsworth Station.
	6·97		8·00	
		7·40	10·00	
4·74	5·03		12·00	
	5·30		13·00	
	5·49		16·00	
	5·75		18·00	
		6·05	20·00	Ground level with rail line hereabouts.
3·57	3·78		22·00	Being surface of rail at the chain oppo-
	3·92		24·00	site the intersection of a drain, with
		4·18	26·00	B. M. railway on the right side, downwards.

Cross Section No, 1, Burntwood Lane

Backset	Inter.	Foreset	Dist.
0·83			0 00
	1·84		1·00
	2·87		2·00
	3·27		3·00
	3'79		4·00
	4·82		6·00
	5·35		7·00
	6·04		8·00
	6·66		9·00
		7·55	*10·00
0·00	1·43		11·00
	2·60		12·00
	3·74		13·00
	5·74		15·00
	7·32		16·00
	9·30		17·00
	10·87		18·00
		12·00	19·00

* Line crosses.
Reduced level, 137 05.

Cross Section No. 2, London to Tooting.

Backset	Inter	Foreset	Dist.
0·15			0·00
	1·04		1·00
	1·86		2·00
	2·79		3·00
	5·87		4·00
	6·00		6·00
	6·83		7·00
	7·54		8·00
	8·15		9·00
	8·50		†10·00
		6·90	‡B. M.
2·14	3·54		11 00
	4 64		12·00
	4·50		13·00
	5·12		14·00
	6·13		16·00
	6·16		17·00
	6·42		18·00
	6·50		19·00
		5·92	20·00

† Line crosses. Reduced level, 130·93.

‡ On nut of lower hinge of white gate.

CROSS SECTIONS CONTINUED.

No. 3, Church Lane.

Backset	Inter.	Foreset	Dist.
0·56			0·00
	2·31		1·00
	7·23		3·00
		10·53	4·00
0·14	7·45		6·00
		11·45	7·00
0·72		10·51	9·00
0·43	4·36		* 9·82
		9·97	11·00
0·12	7·68		13·00
		10·37	14·00
0·07	2·32		15·09
	6·27		17·00
	8·02		18·00
	9·22		19·90
		10·05	20·00

* Line crosses 32 down hill from fence on left of line. Reduced level, 175·35

No. 4, Back Lane.

Backset	Inter.	Foreset	Dist.
1·07			0·00
	1·81		1·00
	2·80		2·00
	6·58		4·00
	8·76		5·00
0·91		10·55	6·00
	7·61		8·00
2·75		10·67	9·00
	6·51		10·00
	7·22		†10·25
	1·30		‡ B.M.
0·53		9·48	11·00
0·41		8·24	13·00
0·56		8·32	15·00
1·53		7·84	17·00
	6·59		19·00
		7·95	20·00

† Line crosses. Reduced level, 192·30.

‡ On blazed tree

Cross Section No. 5.

Backset	Inter.	Foreset	Dist.
6·04			0·00
	5·70		1·00
	4·20		3·00
	3·10		4·00
		2·34	5·00
5·15	0·71		B. M.
	3·85		7·00
		4·21	8·00
4·95	0·42		B. M.
	6·80		11·00
	8·47		12·00
		10·27	13·00
1·21	5·77		14·00
		8·13	15·00
4·75	1·50		§B. M.
		7·79	17·00
0·64		8·15	19·00
0·47		8·27	21·00
0·69		7·39	23·00
2·76		5·75	25·00

§ Line crosses at 16·63 chs. Reduced level, 184·39.

CHAP. VIII.

LEVELLING AND PLOTTING OF THE CROSS SECTIONS.

Having, in the course of proceeding with the main section, carefully marked upon the ground the exact spot in the centre of the road, where the line crosses, and taken the height of the nearest bench mark to it, about ten chains on either side of this spot are measured, in chains' lengths, along the road, for the purpose of obtaining the height of the road at every chain.

In taking the cross levels, be careful always to level one way, *from left to right*, that is, measure always from the left side of theline, (looking in the direction the main section has been levelled, or towards the termination of the line,) to the centre, where the line crosses, onwards to the right side of the line. This will prevent error. Unless the number of chains levelled on either side, of where the line crosses, be always the same, which, depending upon the inequality of the section line of the road, is seldom the case, the consequences of neglecting this might be serious, as the line would cross in its wrong place, and the calculations for lowering or raising the surface, for the bridge across it, would be altogether incorrect.

Call these chains' lengths, therefore, in your level book (*beginning* as far, on the left of the line, as from the nature of the ground may be necessary, and numbering to the centre), 1 chain, 2 chains, and so on.

Place your level opposite to the centre stake, between 0 chains and where the line crosses, and take the several readings at every chain's full length, and also at the spot where the line crosses, and should the bench mark, before alluded to, be on that side, take that also; then remove your level to midway between the centre, where the line crosses, and the extreme point you intend levelling to, and take the several readings.

Enter these in your book, according to the examples given, making the reading at 0 chains your first back set, at 1 chain your first intermediate, placing it in the next lower line, so that you may be enabled to ascertain the height of the starting point, 0·00 in the column of distances. Mark all the subsequent readings, taken at the same place, *intermediate*, excepting that where the line crosses, which should be first *foreset*, saying in every case, in the column of remarks, line ×. The B. M. will be an intermediate, as usual.

Having now obtained all the field data necessary, calculate carefully the rises and falls, using the ordinary checks to ensure accuracy. Then turn to the notes of your main section, and observe there the height of the B.M., and of that part of the road where the line crosses; enter these in the column of reduced levels, as the reduced levels of the same points, taken in the cross section, and, determining the subsequent portion of the section as usual, *reverse the calculations of the upper portion*. By this means, you obtain the several heights of the road, relatively to the whole section, and as the height of where the line crosses, obtained by the section levels, should be the same as that by the main section, you have a certain check upon the general accuracy of the levelling, as well as a special proof that the *correct* bench mark, and point of crossing of the road, have been taken.

Now plot the sections of the several cross roads, numbering them, carefully, No. 1, No. 2, &c., to agree to the corresponding roads in the main section, marking where the line crosses.

The scale, generally used for these cross sections, is 5 chains horizontal, and 50 feet vertical.

The section being now completed, the field book is no longer wanted, and the subsequent proceedings depend upon the skill and judgment of the engineer, to overcome, in the selection of his gradients, such local difficulties of hills or hollows—of roads and rivers—as the face of the section may present.

In determining the lines of gradients, practice and experience are the best masters, and I refer my readers to them.

CHAP. IX.

Method of determining the heights, that the several roads should be raised *or* lowered, *and whether they should be* passed over *or* under.

When a road approach is *passed over* by the line, the usual distance allowed, from the surface of the road to the surface of the rails, is 20 feet; when the approach is passed under, the allowance is 19 feet.

This being premised, we will proceed to the calculations of roads, Nos. 1, 2, and 3, in the given section, and leave the remainder to the reader for practice.

The heights of the several gradients selected, were, 140 feet at starting, which running level for a quarter of a mile, then ascended at a rate of 1 in 354, to the centre of road No. 2, (at a distance of 1 mile, 47 chs. 53 links, from the starting point,) where it is 160 feet above the *datum* line; the gradient then changes into one of 1 in 413 to the 3 mile, where its height is 178 feet. The gradient line in the plan, it must be remembered, is *the line of the surface of the rails,* and not what is called the *balance* line, or ground line of the ballasting, which is so called, from its being the line adopted in the calculations, of both the cuttings and embankments; the earth, in the former, being dug out, and in the latter, filled up, as far as A; upon this, afterwards, is placed the ballasting, which, in the calculations of earthwork, is reckoned at 2 feet.

The distance of the first road (cross section No. 1), by turning to the main field book, will be found to be 63·83 chains, it falls, therefore, in the second division of gradients, being 140 feet at one end, and 160 at the other. Find the length of this gradient, which begins at 20 chains, and ends at 127·53 chains, being, therefore, 107·53 chains long. Knowing the length and excess of height at the end, the excess of height, at any point upon it, can be calculated by similar triangles, thus—

As the whole distance, 107·53 chains : 20 feet : : 23·83 chains,

the distance of the road No. 1, from the commencement of the gradient: its corresponding excess at that point, viz., 5·45 feet, which added to 140 feet, its height at starting, gives 145·45, the height of the surface *of the rails*, where the line crosses the centre of the road No. 1. By turning again to the field book (to the reduced levels), the height of the surface *of the road* is 137·05 feet, making the rail line 8·45 feet higher than the road line, or surface of approach.

If the road can be conveniently lowered, lower it, so as to make it 20 feet beneath the rail line; to effect this, it must be lowered 20 feet—8·15 feet or 8 feet 15 inches, and passed over.

In the same way, it may be found, that at road No. 3, the surface of rails is 165·75 feet, while that of the approach is 175·35, making the approach 9 feet 7 inches *higher* than the rail. The road, therefore, must be *raised* 19 feet—9 feet 7 inches, or 9 feet 5 inches, and passed *under*.

When, as in the case of road No. 2, the surface of the rails crosses a road, 20 feet, or more than 20 feet above it, the road remains the same, *the level is unaltered.*

EXAMPLES.

The reader is now recommended to calculate for himself, roads Nos. 2, 4, and 5, and see if he makes them agree with the plan.

At No. 2, the line is 20 feet higher than the road; at No. 4, the road is 25¼ feet above the line; and at No. 5, the road is to be raised 8 feet 10 inches, and passed under.

CHAP. X.

Calculations of the superficial areas, occupied by the railroad, and of the solid contents of the cuttings and embankments.

SUPERFICIAL AREAS.

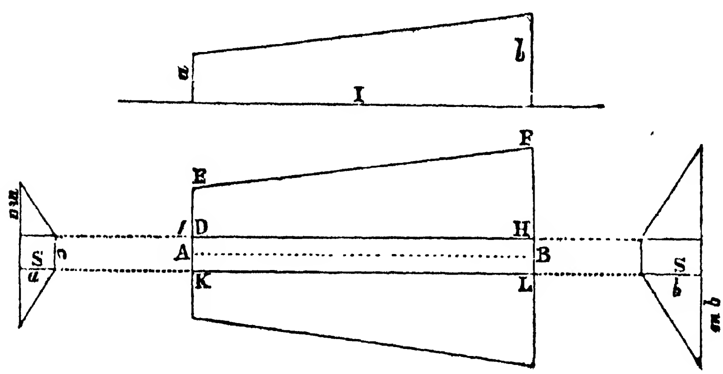
HAVING determined the effects of the line in the altering of the approaches, the next thing to be ascertained, is the superficial area that it will occupy, so as to estimate the expence of purchasing the ground. This will, of course, depend upon the heights or depths of the cuttings and embankments, upon the width of the line, and the proportion of the slopes.

In making these calculations, much depends upon the object in view and the pressure of time. In preparing for deposit, the difference of heights between the section line and the *balance* line are taken

by the compasses; while, in preparing for the contracts, it is requisite they should be calculated from the field book, and the rates of inclination in the section. Again, for deposit, these differences are only taken at any apparent change of inclination in the ground line, while for the contracts they must be made for every two chains.

Being able to obtain these differences of heights, whether by compasses (from the section), or by calculations (from the field notes), as the case may require, we will take some general case, and explain the best method of calculating the areas of the several blocks.

Let a, b, be the several heights; let c be the width of the railway; and let the slopes be as m to 1 (by the term "*slopes*" is meant, technically, that the base of the slope is to the height of the slope as $m : 1$); therefore, if the heights be a, b, the base of the slopes will be ma mb.



Let AB be any length of line, whose height at A is (a), and at B is (b). Then, if ADHB be taken as the half of the line, DE and HF will be the extent of the slopes on the one side, and DE and HF will be respectively equal to ma and mb; and, therefore, the area of AEFB will be $\frac{c + ma + mb}{2} \times \Pi$, where Π equals the length, but AEFB is only half the area; therefore the whole area of the block equals $(c + ma + mb) . \Pi$.

Example.—Required the superficial area of a portion of railways, 20 chains long and 12 feet high at one end, and 10 feet at the other, the slopes being 2 to 1 (the width of line being 33 feet); now $c = 33$, $a = 12$, $b = 10$, and $m = 2$, and $\Pi = 20$ chs.; therefore,

$$\frac{33 + 23 + 20}{66} \times 20 = 23{\cdot}333 \text{ sq. chains} = \overset{\text{A.}}{2}\ \overset{\text{R.}}{1}\ \overset{\text{P.}}{13.}$$

EXAMPLE.—What will be the superficial area of a cutting of a railroad, whose depths at every chain are 0, 2, 3, 5, 7, 9, 8, 6, 4, 2, 1, 0 feet; the other data as in last example?

The formula for the area is $(c+ma+mb)\text{n}$; in the two end areas, in the present example, where $a=0$, and $b=0$, ma and mb would be 0; and the areas will be $(c+mb)\text{n}$; in the one case, and $(c+ma)\text{n}$, in the other.

Now the whole area $=\frac{\text{n}}{66}\Big\}(33+4)+(33+4+6)+\&c.)$; or, by taking separately the whole of the *central area*, which will be (11 chains $\times\frac{33}{66}$), and using the formula (given in the first part, for calculating the areas of offsets, page 75), for the areas of the *slopes*, we have for their area $2\text{n}\,(\frac{4+6+10+14+18, \&c.}{66})$sq. chs.; therefore, the whole area $=\frac{1}{66}\times\Big\{33\times11+2\text{n}(4+6+10+\&c.)\Big\}$

$=8{\cdot}34$ square chains $=$ 0 A. 3 R. 13 P.

EXAMPLE.—What will be the area of a mile of railway, whose heights of embankment for the first quarter of a mile, at every 5 chains, are 12,* 25, 50, and 20 feet; for the next 25 chains, the line is level; and then, to the end, runs through a cutting, which rises gradually to the height of 40 feet, at half way, and descends to the end, having the other data as the preceding?

	A.	R.	P.
Ans.	15	3	15

* In this case, area of each slope equals $d\,(\frac{bg}{2}+\text{ch}+\text{dl}+\&c.)$ see page 76, where, in the given formula, the perpendicular at the starting point $=0$. In the section of a cutting or embankment, the area of the slopes would then be $d\,(\frac{bg}{2}+\text{ch}+\text{dl}+\text{em}+\frac{fn}{2})$

CHAP. XI.

CUTTINGS AND EMBANKMENTS.

There are two or three methods adopted in practice, in the calculation of these quantities, viz., the Prismoidal Formula, Bidder's Tables, M'Niel's, &c.

Prismoidal Formula.

Let c be any cutting, having, at one end, the height $= a$, at the other $= b$; length of cutting $= \text{n}$, slope 2 to 1, or generally m to 1.

Now the solid, thus cut off, assumes the form of an imperfect prism, which may properly be divided into three divisions; the *central* one, being a solid, generated by a plane of the given heights and length; moving along a plane, at right angles to it, to a distance, equal to the width of the railway; and the two slopes, equal to each other, being strictly frustra of pyramids; the height of the frustra being equal to the length of the cutting, and the sides of either end, being the sides of a right-angled triangle, whose base $=$ the height of the cutting at that end; and perpendicular, the proportion of the given slope to it.

Having the same data as above, *(See Diagram, Page 277.)*

the central contents $= \text{n}\,\frac{c}{2}\,(a+b)$, where $c =$ width of railway

$$\text{slopes} = \frac{\text{areas of the two ends} + \text{the mean area}}{3} \times \text{the length}$$

$$= \frac{\text{n}}{3}(ma^2 + mb^2 + mab)$$

$$\therefore \text{ The whole contents} = \frac{\text{n}}{2}\,c\,(a+b) + \frac{m}{3}(a^2+b^2+ab)$$

$$= \frac{\text{n}}{6}(3\,ac + 3\,bc + 2\,a^2m + 2\,b^2m + 2\,abm)$$

$$= \frac{\text{n}}{6}(ac + a^2m + bc + b^2m + 2\,ac + 2\,bc + m\,(a+b)^2)$$

$$= \frac{\text{n}}{6}(\overline{c+ma}.\ a + \overline{c+bm}.\ b + 4\,(\frac{a+b}{2}).(c + \frac{m}{2}.\ \overline{a+b})$$

which is the *prismoidal formula;* where $\overline{c+ma}.\ a$ and $\overline{c+mb}.\ b$ are areas of the two ends, and $4\,(\frac{a+b}{2})\,(c+\frac{m}{2}.\ \overline{a+b})$ is 4 times the area of a section midway; the whole area being equal to the sum of these three into $\frac{1}{6}$ the length, whether of cutting or embankment.

2. *Moseley's Formula.**

The central area $= \Pi \frac{c}{2}(a+b) = \frac{\Pi m}{6}(\frac{3c}{m}\cdot\overline{a+b})$

$$\text{slopes} = \frac{\Pi}{3}m\,(a^2+b^2+ab) = \frac{\Pi m}{6}(2a^2+2b^2+2ab)$$

$$\therefore \text{ whole contents} = \frac{\Pi m}{6}(\frac{3c}{m}(a+b)+2a^2+2b^2+2ab)$$

Now let $a = y_1$ and $b = y_2$, we have 1st contents $= \frac{\Pi m}{6}(\frac{3c}{m}(y_1+y_2)$ $+2y_1^2+2y_2^2+2y_1y_2)$.

Now, as the adjoining cutting must commence with the same height as this terminates, supposing the other height of the second cutting $= y_3$, and next height y_4, we have—

$$\text{area of 1st cutting} = \frac{\Pi m}{6}(\frac{3c}{m}(y_1+y_2)+2y_1^2+2y_2^2+2y_1y_2)$$

$$\text{2nd cutting} = \frac{\Pi m}{6}(\frac{3c}{m}(y_2+y_3)+2y_2^2+2y_3^2+2y_2y_3)$$

and the series $= \frac{\Pi m}{6}(\frac{3c}{m}(y_1+2y_2+2y_3 \cdots\cdots 2y_{n-1}+y_n)+2(y_1^2+2y_2^2+$ $2y_3^2 \cdots\cdot 2y_{n-1}^2+y_n^2)+2(y_1y_2+y_2y_3+y_3y_4+\cdots\cdots y_{n-1}\cdot y_n)$ or B,

$$\text{and } \therefore = \frac{\Pi m}{6}(\frac{3c}{m}(2\Sigma y_n \cdots (y_1+y_n)+2(2\Sigma y_n^2-(y_1^2+y_n^2)+2(\text{B}).$$

Now, when y_1 and y_n are $= 0$, which is the case in a complete cutting or embankment; we have—

$$\text{Solidity} = \frac{1}{6}\Pi m(\frac{3c}{m}\Sigma y_n+4\Sigma y_n^2+2\,(\text{B})),$$

when Σy_n = the sum of the simple quantities,
and Σy_n^2 = the sum of their squares,
and B = the sum of the continued products of the simple quantities.

---

* This formula, for which I am indebted to Professor Moseley, of King's College, I have had several opportunities of testing, and I have found it exceedingly useful. The calculations from the Prismoidal Formula, which is, geometrically deduced, are strictly accurate; but they are too tedious for general purposes. Bidder's tables, on the other hand, though convenient for common practice, are still, from being only calculated to full feet and limited to 50 feet, unfitted for contract estimates. Whereas Moseley's formula, being applicable to any height and any subdivision, and embracing any extent of cutting, whose heights are taken at any equal distances whatever, becomes especially useful for the final calculations; which are usually made at every two chains of distance, and to heights of one hundredth of a foot.

3. *Bidder's Formula,*

for computing the solid contents, is—

$$\left.\begin{array}{l}\text{for the slopes} = \frac{22}{27}\left((a+b)^2-ab\right)\\ \text{for the centre} = \frac{11}{9}\,(a+b)\end{array}\right\}\text{ in yards.}$$

This is for a chain in length, and in the centre for a foot wide; the slopes being one to one.

Now, it has been previously shown that—

$\frac{nc}{2}(a+b) =$ contents of centre.

Let $c = 1$ foot,

and $n =$ one chain or 66 feet, $\therefore \frac{66}{2}(a+b)$ in cubic feet $=$ centre.

$$\frac{66}{2} \text{ of } \frac{1}{27} \text{ or } \frac{11}{9}(a+b) \text{ in cubic yards} = \text{centre.}$$

And the slopes $= \frac{nm}{3}(a^2+b^2+ab)$.

Let $m = 1$, and $n = 66$ feet.

$\therefore$ slopes $= \frac{66}{3}$ feet, or $\frac{22}{27}(a^2+b^2+ab)$ in yards,

and $\therefore = \frac{22}{27}(\overline{a+b}^{\,2}-ab)$ cubic yards.

The central contents, thus obtained, would have to be multiplied by the length of the sections in chains, and the width of the line in feet; and the contents of the slopes, by the rates of m to 1; and by the length of the section in chains.

M'Neil's Tables are also for slopes of 1 to 1, and for base of 1 foot; but not for lengths of 1 chain.

The quantities obtained, therefore, in M'Neil's tables, would have to be multiplied, for the centre, by the width of railway in feet; and, for the slopes, by the length n (in feet).

These two tables, however, are only calculated for long sections, and for full feet, (for which they are invaluable,) principally, for computing the whole contents of a line previous to going to Parliament, to form the first estimate of the expense.

Example.—Required the solid contents of a cutting or embankment, whose several heights at each chain's length are, 0, 5, 10, 15, 20, 25, 30, 30, 24, 18, 15, 10, 0—slopes 2 to 1, and the width of railway, 30 feet.

BY BIDDER'S TABLES.

Lengths	Height at either end.	Central Contents	Contents of Slope
1·00	$\frac{0}{5}$	6·1	20
1·00	$\frac{5}{10}$	18·3	143
1·00	$\frac{10}{15}$	30·6	387
1·00	$\frac{15}{20}$	42·8	754
1·00	$\frac{20}{25}$	55·0	1243
1·00	$\frac{25}{30}$	67·2	1854
1·00	$\frac{30}{30}$	73·3	2200
1·00	$\frac{30}{24}$	66·0	1789
1·00	$\frac{24}{18}$	51·3	1085
1·00	$\frac{18}{15}$	40·3	667
1·00	$\frac{15}{10}$	30·6	387
1·00	$\frac{10}{0}$	12·2	82
		493·7	10611
		30	2
		14811·1	21222
			14811
			36033

493·7 being the number of cubic feet in the central column, for the width of one foot, which must, therefore, be multiplied by 30 for the whole width, and 10611 being the contents of the slopes for the ratio of one to one, which must, in this case, at the ratio of two to one, be doubled; the sum of the two, 36033, will be the number of cubic yards required.

BY MOSELEY'S FORMULA.

The sum of the simple quantities $= 202 = \Sigma y_n$
the sum of their squares $= 4400 = \Sigma y_n^2$
the sum of their products
(of each with its preceding) $= 4222 = B$

Now the formula $= \frac{\Pi m}{6}\left\{\frac{3c}{m} 2 \Sigma y_n) + 4 \Sigma y_n^2 + 2B\right\}$

$2 \Sigma y_n = 404$, $\frac{3c}{m} = 45$; $4 \Sigma y_n^2 = 176000$, $2B = 8444$

$\therefore \frac{2}{6} \cdot 66\,(404 \times 45 + 17600 + 8444) =$ cubical contents.

$404 \times 45 = 18180$
17600
8444

$44224 \times 22 = 972928$ cubic feet $= 36034$ cubic yards,

differing by one cubic yard only, from the previous result by Bidder's tables. The calculation of this by the *Prismoidal Formula* would be too tedious an operation. Of the two methods given, Bidder's tables have the advantage where the numbers are high, while the accuracy of calculation required for contract work, when the heights at every chain may be decimals of a foot, can only be done by Moseley's, Bidder's being calculated to feet only.

EXAMPLE.—Calculate, by Moseley's formula and Bidder's tables, the cubical contents of an embankment, whose heights at every chain are 0, 2, 3, 5, 7, 9, 8, 6, 4, 2, 1, 0; slopes 2 to 1; width of centre, 33 feet.

Ans. 5178 cubic yards.

EXAMPLE BY THE PRISMOIDAL FORMULA.—What are the contents in cubic yards of a cutting, where the heights at every 2 chains are, 0, 3, 5, 7·5, 8, 9·5, 8, 6, 4, 0, the width of the line being 33 feet, and the slopes 2 to 1.

$$\text{Now } \overline{c + ma}.\ a = \text{area at one end.}$$
$$\overline{c + mb}.\ b = \text{area at other.}$$
$$\frac{a+b}{2}\left(c + \frac{m}{2}\overline{a+b}\right) = \text{area midway.}$$

$\overline{33 + 6} \times 3$, $\overline{33 + 10} \times 5$, $\overline{33 + 15} \times 7{\cdot}5$, $\overline{33 + 16} \times 8$ &c., are the several end areas, and

$\frac{3}{2}.(33 + 3)$, $\frac{3+5}{2}.(33+3+5)$, $\frac{5+7{\cdot}5}{2}.(33 + 5 + 7{\cdot}5)$, &c., are the several areas midway.

Now the sum of the end areas $= 2404$ square feet, and that of the middle area $= 2380$.

Four times the middle area, *plus* twice the end areas (as the areas at every height serve for end areas to two sections), multiplied into $\frac{1}{6}$ the common length, will be the contents in cubic feet.

$$\frac{2\,(2404) + 4\,(2380) \times 132}{3} = 315216 \text{ cubic feet} = 11675 \text{ cubic}$$

yards, the solid contents required.

Instead of the preceding, however, there are two methods in common use among contractors, which, though apparently correct, are really far from being so.

The first is by taking the MEAN OF THE TWO END AREAS, which make the results *too much.*

The second, by taking THE AREA OF THE MEAN HEIGHT, which, on the other hand, makes it *too little.*

Now, in the proof of Bidder's formula, it has been shown that the contents of slopes $= \frac{n}{3} \left(m \left(\overline{a+b}^2 - ab\right)\right.$, and of the centre $= \frac{nc}{2} \cdot \overline{a+b}$

therefore the whole contents are $= n \left(\frac{m}{3} \left(\overline{a+b}^2 - ab\right) + \frac{a+b}{2} \cdot c\right)$

and by assuming x and y, as the unknown values of the respective excess and deficiency in the two erroneous methods above, and equating them with the true formula, we can obtain their respective values.

Of these two incorrect formulæ

the first $n \dfrac{\left(\overline{c+ma} \cdot a + \overline{c+mb} \cdot b\right)}{2} = \dfrac{3\,ac + 3\,ma^2 + 3\,bc + 3\,mb^2}{6}$

the second $= n \cdot \dfrac{a+b}{2} \left(c + \frac{m}{2} \cdot \overline{a+b}\right) = \dfrac{6\,ac + 6\,bc + 3ma^2 + 6\,mab + 3\,mb^2}{6}$

Finding the differences between these and the correct formula, which are the values of x and y respectively, we obtain

an excess for the first of $\frac{m}{6} \cdot \overline{a-b}^2$

a deficiency for the second of $\frac{m}{12} \cdot \overline{a-b}^2$

where a and b, as usual, represent the heights at the ends, and m the ratio of the slopes.

These corrections have to be multiplied by n, the length of the section.

THE END.

LONDON: D. CAHN, PRINTER.

CORRECTED AND OMITTED ANSWERS.

Page.	Ex.	
24	2	49 acres, 0 roods, and 10 perches.
25	4	231384 square yards.
29	3 & 4	For AF read AD.
		For DE read AF.
		For BC, 6·50, read FC, 6·50.
		For DB read AC.
32		For 6 read 0.
34	2	57 acres, 1 rood, 10 perches.
	3	9 acres, 2 roods, 0 perches.
35	3	10 acres, 1 rood, 26 perches.
46	3	Comes under Case 2.
48	2	Base $=$ 73·99 chains. $\left\{ \begin{array}{l} \angle \text{ at } B = 33^\circ 4' \\ \angle \text{ at } C = 29^\circ 6' \end{array} \right.$
	3	*In the question*, should be 70 chains and 50 chains.
120	4	14 acres, 1 rood, 6 perches.
122	2	16 acres, 2 roods, $12\frac{9}{16}$ perches.
	3	$79 \cdot 2 \cdot 7\frac{31}{100}$ acres Scotch $=$ 100 English. By 11200 sq. ft.
123	1	41 acres, 3 roods, 9 perches.
	2	61 acres, 2 roods, $36\frac{12}{25}$ perches.
	3	1944 English acres. $7\frac{3}{8}$ miles.
	4	65 acres, 0 roods, 35 perches.
124	1	280 sq. gads $+ 72\frac{1}{2}$ sq. feet, or 0 acres, 2 roods, 23 perches statute.
	2	56162711.
146	1	DE, 13 chains 17 links; $\angle$ DEF, $103^\circ 48' 20''$.
148	1	8 miles 3 furlongs and a half.
149	1	17 chains 62 links.
150	1	249 feet.
151	1	188 feet high; 90 feet wide.
152	1	Difference of height, 202 feet.
153	1	*In the question*, for $14^\circ 15'$ read $14^\circ 45'$. 249 feet.
181	2	$42^\circ 3' 25''$.
216	1	New Road should bear S. $24^\circ 38'$ W., 60 chains 82 links.
218	2	5 chains 42 links, and 6 chains 79 links.
227	1	12 acres, 3 roods, 9 perches.
	2	22 acres, 3 roods, 13 perches.
	2	7·04 chains.
230	2	$\left\{ \begin{array}{l} \text{A, 117 acres, and 12·72 chains frontage.} \\ \text{B, 67 acres, and 7·28 chains frontage.} \end{array} \right.$
257	1	$33\frac{1}{7}$, $45\frac{4}{7}$, $64\frac{4}{7}$ feet.
	2	$581\frac{3}{4}$ feet.

ERRATA.

In page 40 *for* $\dfrac{1}{\text{cos. A}}$ *read* $\dfrac{1}{-\text{cos. A}}$

for sec. A *read* —sec. A.

165 line 24, *for* cosec. DG *read* cosec. *d*. DG.

205 line 3, *for* them *read* these

for is *read* as.

APPENDIX.

TABLES

OF

LOGARITHMS,

OF

SINES, COSINES, AND TANGENTS.

AND

A TRAVERSE TABLE,

SIX PLACES OF DECIMALS.

TABLE OF LOGARITHMS.

This table extends to six places of logarithms, for all numbers from 1 to 10,000, which is amply sufficient for all ordinary practical purposes.

In looking for the logarithm of any number, it must be remembered, that,—as in natural numbers, the *position* of the decimal point alone determines the value of the numbers,—so, in logarithms, that value is represented by the number of digits in the index, *prefixed* to the logarithm, which is the logarithmic decimal point or characteristic.

Logarithms of natural numbers, therefore, are the same, whether those numbers be decimal or not; so that to find the logarithm of any given number, you must first find the logarithm of the figures independently of their integral value, and then prefix the proper Index, to represent the position of the decimal point.

To find the Logarithms.

Look for the first three significant figures in the columns of numbers, on the left of the page; then follow, *laterally*, through the columns of logarithms, till you come to the one, which is headed by the next figure in the given number. The logarithm thus found, which is the logarithm, *collateral*, with the first three significant figures, and, *vertically*, under the next figure, will be the required logarithm of the given number.

Example to find the logarithm of 4285. Look down the column of numbers for 428, then laterally to the column headed 5; then collaterally with 428, and, vertically, under 5, you will find the required logarithm 631951, which is the logarithm, whether the number be 4285 or 42·85 or ·0004285.

For the index, which represents the position of the decimal point, see chapter on logarithms, in the Introduction.

The method of finding the logarithms of numbers, *greater* than 10,000, or having more than four significant figures, will be found also fully explained there.

To find the number, corresponding to a given Logarithm.

Look in the columns of logarithms, for the nearest logarithm to the given one (omitting the index); then, looking, collaterally, to the column of numbers for the first three figures, and vertically, to the number on the top of the column for the fourth, you will obtain the corresponding number to four places of figures, which will be whole numbers, or decimals, according to the index of the logarithm.

For determining the position of the decimal point, as represented by this index, or for carrying out the number to more places of figures, see the chapter on the subject in the Introduction.

EXAMPLE—What is the natural number corresponding to the given logarithm 1·252130 ?

Looking in the columns of logarithms, you will find 252125 is the nearest logarithm ; collaterally, in the column of numbers you will find 178, the first three figures ; and vertically, at the head of the column 7, the fourth figure, which will give the required number 1787 ; having its integral value regulated by the given index 1, which will make 1787, all decimals, viz., ·1787.

LOGARITHMS OF NUMBERS FROM 1 TO 10,000.

No.	Log	No.	Log.	No.	Log.	No	Log.	No.	Log.
1	000000	21	322219	41	612784	61	785330	81	908485
2	301030	22	342423	42	623249	62	792392	82	913814
3	477121	23	361728	43	633468	63	799341	83	919078
4	602060	24	380211	44	643453	64	806180	84	924279
5	698970	25	397940	45	653213	65	812913	85	929419
6	778151	26	414973	46	662758	66	819544	86	934498
7	8450 8	27	431364	47	672098	67	826075	87	939519
8	903090	28	447158	48	681241	68	832509	88	944483
9	954243	29	462398	49	690196	69	838849	89	949390
10	000000	30	477121	50	698970	70	845098	90	954243
11	041393	31	491362	51	707570	71	851258	91	959041
12	079181	32	505150	52	716003	72	857333	92	963788
13	113943	33	518514	53	724276	73	863323	93	968483
14	146128	34	531479	54	732394	74	869232	94	973128
15	176091	35	544068	55	740363	75	875061	95	977724
16	204120	36	556303	56	748188	76	880814	96	982271
17	230449	37	568202	57	755875	77	886491	97	986772
18	255273	38	579784	58	763428	78	892095	98	991226
19	278754	39	591065	59	770852	79	897627	99	995635
20	301030	40	602060	60	778151	80	903090	100	000000

No	0	1	2	3	4	5	6	7	8	9
100	000000	000434	000868	001301	001734	002166	002598	003029	003461	003891
101	004321	004751	005181	005609	006038	006466	006894	007321	007748	008174
102	008600	009026	009451	009876	010300	010724	011147	011570	011993	012415
103	012837	013259	013680	014100	014521	014940	015360	015779	016197	016616
104	017033	017451	017868	018284	018700	019116	019532	019947	020361	020775
105	021189	021603	022016	022428	022841	023252	023664	024075	024486	024896
106	025306	025715	026125	026533	026942	027350	027757	028164	028571	028978
107	029384	029789	030195	030600	031004	031408	031812	032216	032619	033021
108	033424	033826	034227	034628	035029	035430	035830	036230	036629	037028
109	037426	037825	038223	038620	039017	039414	039811	040207	040602	040998
110	041393	041787	042182	042576	042969	043362	043755	044148	044540	044932
111	045323	045714	046105	046495	046885	047275	047664	048053	048442	048830
112	049218	049606	049993	050380	050766	051153	051538	051924	052309	052694
113	053078	053463	053846	054230	054613	054996	055378	055760	056142	056524
114	056905	057286	057666	058046	058426	058805	059185	059563	059942	060320
115	060698	061075	061452	061829	062206	062582	062958	063333	063709	064083
116	064458	064832	065206	065580	065953	066326	066699	067071	067443	067815
117	068186	068557	068928	069298	069668	070038	070407	070776	071145	071514
118	071882	072250	072617	072985	073352	073718	074085	074451	074816	075182
119	075547	075912	076276	076640	077004	077368	077731	078094	078457	078819
120	079181	079543	079904	080266	080626	080987	081347	081707	082067	082426
121	082785	083144	083503	083861	084219	084576	084934	085291	085647	086004
122	086360	086716	087071	087426	087781	088136	088490	088845	089198	089552
123	089905	090258	090611	090963	091315	091667	092018	092370	092721	093071
124	093422	093772	094122	094471	094820	095169	095518	095866	096215	096562
125	096910	097257	097604	097951	098298	098644	098990	099335	099681	100026
126	100371	100715	101059	101403	101747	102091	102434	102777	103119	103462
127	103804	104146	104487	104828	105169	105510	105851	106191	106531	106871
128	107210	107549	107888	108227	108565	108903	109241	109579	109916	110253
129	110590	110926	111263	111599	111934	112270	112605	112940	113275	113609
130	113943	114277	114611	114944	115278	115611	115943	116276	116608	116940
131	117271	117603	117934	118265	118595	118926	119256	119586	119915	120245
132	120574	120903	121231	121560	121888	122216	122544	122871	123198	123525
133	123852	124178	124504	124830	125156	125481	125806	126131	126456	126781
134	127105	127429	127753	128076	128399	128722	129045	129368	129690	130012
135	130334	130655	130977	131298	131619	131939	132260	132580	132900	133219
136	133539	133858	134177	134496	134814	135133	135451	135769	136086	136403
137	136721	137037	137354	137671	137987	138303	138618	138934	139249	139564
138	139879	140194	140508	140822	141136	141450	141763	142076	142389	142702
139	143015	143327	143639	143951	144263	144574	144885	145196	145507	145818
140	146128	146438	146748	147058	147367	147676	147985	148294	148603	148911
141	149219	149527	149835	150142	150449	150756	151063	151370	151676	151982
142	152288	152594	152900	153205	153510	153815	154120	154424	154728	155032
143	155336	155640	155943	156246	156549	156852	157154	157457	157759	158061
144	158362	158664	158965	159266	159567	159868	160168	160469	160769	161068
145	161368	161667	161967	162266	162564	162863	163161	163460	163758	164055
146	164353	164650	164947	165244	165541	165838	166134	166430	166726	167022
147	167317	167613	167908	168203	168497	168792	169086	169380	169674	169968
148	170262	170555	170848	171141	171434	171726	172019	172311	172603	172895
149	173186	173478	173769	174060	174351	174641	174932	175222	175512	175802

No	0	1	2	3	4	5	6	7	8	9
150	176091	176381	176670	176959	177248	177536	177825	178113	178401	178689
151	178977	179264	179552	179839	180126	180413	180699	180986	181272	181558
152	181844	182129	182415	182700	182985	183270	183555	183839	184123	184407
153	184691	184975	185259	185542	185825	186108	186391	186674	186956	187239
154	187521	187803	188084	188366	188647	188928	189209	189490	189771	190051
155	190332	190612	190892	191171	191451	191730	192010	192289	192567	192846
156	193125	193403	193681	193959	194237	194514	194792	195069	195346	195623
157	195899	196176	196453	196729	197005	197281	197556	197832	198107	198382
158	198657	198932	199206	199481	199755	200029	200303	200577	200850	201124
159	201397	201670	201943	202216	202488	202761	203033	203305	203577	203848
160	204120	204391	204663	204934	205204	205475	205746	206016	206286	206556
161	206826	207096	207365	207634	207904	208173	208441	208710	208979	209247
162	209515	209783	210051	210319	210586	210853	211121	211388	211654	211921
163	212188	212454	212720	212986	213252	213518	213783	214049	214314	214579
164	214844	215109	215373	215638	215902	216166	216430	216694	216957	217221
165	217484	217747	218010	218273	218536	218798	219060	219323	219585	219846
166	220108	220370	220631	220892	221153	221414	221675	221936	222196	222456
167	222716	222976	223236	223496	223755	224015	224274	224533	224792	225051
168	225309	225568	225826	226084	226342	226600	226858	227115	227372	227630
169	227887	228144	228400	228657	228913	229170	229426	229682	229938	230193
170	230449	230704	230960	231215	231470	231724	231979	232234	232488	232742
171	232996	233250	233504	233757	234011	234264	234517	234770	235023	235276
172	235528	235781	236033	236285	236537	236789	237041	237292	237544	237795
173	238046	238297	238548	238799	239049	239299	239550	239800	240050	240300
174	240549	240799	241048	241297	241546	241795	242044	242293	242541	242790
175	243038	243286	243534	243782	244030	244277	244525	244772	245019	245266
176	245513	245759	246006	246252	246499	246745	246991	247237	247482	247728
177	247973	248219	248464	248709	248954	249198	249443	249687	249932	250176
178	250420	250664	250908	251151	251395	251638	251881	252125	252368	252610
179	252853	253096	253338	253580	253822	254064	254306	254548	254790	255031
180	255273	255514	255755	255996	256237	256477	256718	256958	257198	257439
181	257679	257918	258158	258398	258637	258877	259116	259355	259594	259833
182	260071	260310	260548	260787	261025	261263	261501	261739	261976	262214
183	262451	262688	262925	263162	263399	263636	263873	264109	264346	264582
184	264818	265054	265290	265525	265761	265996	266232	266467	266702	266937
185	267172	267406	267641	267875	268110	268344	268580	268812	269046	269279
186	269513	269746	269980	270213	270446	270679	270912	271144	271377	271609
187	271842	272074	272306	272538	272770	273001	273233	273464	273696	273927
188	274158	274389	274620	274850	275081	275311	275542	275772	276002	276232
189	276462	276692	276921	277151	277380	277609	277838	278067	278296	278525
190	278754	278982	279211	279439	279667	279895	280123	280351	280578	280806
191	281033	281261	281488	281715	281942	282169	282396	282622	282849	283075
192	283301	283527	283753	283979	284205	284431	284656	284882	285107	285332
193	285557	285782	286007	286232	286456	286681	286905	287130	287354	287578
194	287802	288026	288249	288473	288696	288920	289143	289366	289589	289812
195	290035	290257	290480	290702	290925	291147	291369	291591	291813	292034
196	292256	292478	292699	292920	293141	293363	293584	293804	294025	294246
197	294466	294687	294907	295127	295347	295567	295787	296007	296226	296446
198	296665	296884	297104	297323	297542	297761	297979	298198	298416	298635
199	298853	299071	299289	299507	299725	299943	300161	300378	300595	300813

No	0	1	2	3	4	5	6	7	8	9
200	301030	301247	301464	301681	301898	302114	302331	302547	302764	302980
201	303196	303412	303628	303844	304059	304275	304491	304706	304921	305136
202	305351	305566	305781	305996	306211	306425	306639	306854	307068	307282
203	307496	307710	307924	308137	308351	308564	308778	308991	309204	309417
204	309630	309843	310056	310268	310481	310693	310906	311118	311330	311542
205	311754	311966	312177	312389	312600	312812	313023	313235	313445	313656
206	313867	314078	314289	314499	314710	314920	315130	315340	315551	315760
207	315970	316180	316390	316599	316809	317018	317227	317436	317646	317854
208	318063	318272	318481	318689	318898	319106	319314	319522	319730	319938
209	320146	320354	320562	320769	320977	321184	321391	321598	321805	322012
210	322219	322426	322633	322839	323046	323252	323458	323665	323871	324077
211	324282	324488	324694	324899	325105	325310	325516	325721	325926	326131
212	326336	326541	326745	326950	327155	327359	327563	327767	327972	328176
213	328380	328583	328787	328991	329194	329398	329601	329805	330008	330211
214	330414	330617	330819	331022	331225	331427	331630	331832	332034	332236
215	332438	332640	332842	333044	333246	333447	333649	333850	334051	334253
216	334454	334655	334856	335057	335257	335458	335658	335859	3360 9	336260
217	336460	336660	336860	337060	337260	337459	337659	337858	338058	338257
218	338456	338656	338855	339054	339253	339451	339650	339849	340047	340246
219	340444	340642	340841	341039	341237	341435	341632	341830	342028	342225
220	342423	342620	342817	343014	343212	343409	343606	343802	343999	344196
221	344392	344589	344785	344981	345178	345371	345570	345766	345962	346157
222	346353	346549	346744	346939	347135	347330	347525	347720	347915	348110
223	348305	348500	348694	348889	349083	349278	349472	349666	349860	350054
224	350248	350442	350636	350829	351023	351216	351410	351603	351796	351989
225	352183	352375	352568	352761	352954	353147	353339	353532	353724	353916
226	354108	354301	354493	354685	354876	355068	355260	355452	355643	355834
227	356026	356217	356408	356599	356790	356981	357172	357363	357554	357714
228	357935	358125	358316	358505	358696	358886	359076	359266	35 456	359646
229	359835	360025	360215	360404	360593	360783	360972	361161	361350	361539
230	361728	361917	362105	362294	362482	362671	362859	363048	363236	363424
231	363612	363800	363988	364176	364363	364551	364739	364926	365113	365301
232	365488	365675	365862	366049	366236	366423	366610	366796	366983	367169
233	367356	367542	367729	367915	368101	368287	368473	368659	368845	369030
234	369216	369401	369587	369772	369958	370143	370328	370513	370698	370883
235	371068	371253	371437	371622	371806	371991	372175	372360	372544	372728
236	372912	373096	373280	373464	373647	373831	374015	374198	374382	374565
237	374748	374932	375115	375298	375481	375664	375846	376029	376212	376394
238	376577	376759	376942	377124	377306	377488	377670	377852	378034	378216
239	378398	378580	378761	378943	379124	379306	379487	379668	379849	380030
240	380211	380392	380573	380754	380934	381115	381296	381476	381656	381837
241	382017	382197	382377	382557	382737	38 917	383097	383277	383456	383636
242	383815	383995	384174	384353	384533	384712	384891	385070	385249	385428
243	385606	385785	385964	386142	386321	386499	386677	386856	387034	387212
244	387390	387568	387746	387923	388101	388279	388456	388634	388811	388989
245	389166	389343	389520	389698	389875	390051	390228	390405	390582	390759
246	390935	391112	391288	391464	391641	391817	391993	392169	392345	392521
247	392697	392873	393048	393224	393400	393575	393751	393926	394101	394277
248	394452	394627	394802	394977	395152	395326	395501	395676	395850	396025
249	396199	396374	396548	396722	396896	297071	397245	397419	397592	397766

No	0	1	2	3	4	5	6	7	8	9
250	397940	398114	398287	398461	398634	398808	398981	399154	399328	399501
251	399674	399847	400020	400192	400365	400538	400711	400883	401056	401228
252	401401	401573	401745	401917	402089	402261	402433	402605	402777	402949
253	403121	403292	403464	403635	403807	403978	404149	404320	404492	404663
254	404834	405005	405176	405346	405517	405688	405858	406029	406199	406370
255	406540	406710	406881	407051	407221	407391	407561	407731	407901	408070
256	408240	408410	408579	408749	408918	409087	409257	409426	409595	409764
257	409933	410102	410271	410440	410609	410777	410946	411114	411283	411451
258	411620	411788	411956	412124	412293	412461	412629	412796	412964	413132
259	413300	413467	413635	413803	413970	414137	414305	414472	414639	414806
260	414973	415140	415307	415474	415641	415808	415974	416141	416308	416474
261	416641	416807	416973	417139	417306	417472	417638	417804	417970	418135
262	418301	418467	418633	418798	418964	419129	419295	419460	419625	419791
263	419956	420121	420286	420451	420616	420781	420945	421110	421275	421439
264	421604	421768	421933	422097	422261	422426	422590	422754	422918	423082
265	423246	423410	423574	423737	423901	424065	424228	424392	424555	424718
266	424882	425045	425208	425371	425534	425697	425860	426023	426186	426349
267	426511	426674	426836	426999	427161	427324	427486	427648	427811	427973
268	428135	428297	428459	428621	428783	428944	429106	429268	429429	429591
269	429752	429914	430075	430236	430398	430559	430720	430881	431042	431203
270	431364	431525	431685	431846	432007	432167	432328	432488	432649	432809
271	432969	433130	433290	433450	433610	433770	433930	434090	434249	434409
272	434569	434729	434888	435048	435207	435367	435526	435685	435844	436004
273	436163	436322	436481	436640	436800	436957	437116	437275	437433	437592
274	437751	437909	438067	438226	438384	438542	438701	438859	439017	439175
275	439333	439491	439648	439806	439964	440122	440279	440437	440594	440752
276	440909	441066	441224	441381	441538	441695	441852	442009	442166	442323
277	442480	442637	442793	442950	443106	443263	443419	443576	443732	443889
278	444045	444201	444357	444513	444669	444825	444981	445137	445293	445449
279	445604	445760	445915	446071	446226	446382	446537	446692	446848	447003
280	447158	447313	447468	447623	447778	447933	448088	448242	448397	448552
281	448706	448861	449015	449170	449324	449478	449633	449787	449941	450095
282	450249	450403	450557	450711	450865	451018	451172	451326	451479	451633
283	451786	451940	452093	452247	452400	452553	452706	452859	453012	453165
284	453318	453471	453624	453777	453930	454082	454235	454387	454540	454692
285	454845	454997	455150	455302	455454	455606	455758	455910	456062	456214
286	456366	456518	456670	456821	456973	457125	457276	457428	457579	457731
287	457882	458033	458184	458336	458487	458638	458789	458940	459091	459242
288	459392	459543	459694	459845	459995	460146	460296	460447	460597	460748
289	460898	461048	461198	461348	461499	461649	461799	461948	462098	462248
290	462398	462548	462697	462847	462997	463146	463296	463445	463594	463744
291	463893	464042	464191	464340	464490	464639	464788	464936	465085	465234
292	465383	465532	465680	465829	465977	466126	466274	466423	466571	466719
293	466868	467016	467164	467312	467460	467608	467756	467904	468052	468200
294	468347	468495	468643	468790	468938	469085	469233	469380	469527	469675
295	469822	469969	470116	470263	470410	470557	470704	470851	470998	471145
296	471292	471438	471585	471732	471878	472025	472171	472318	472464	472610
297	472756	472903	473049	473195	473341	473487	473633	473779	473925	474071
298	474216	474362	474508	474653	474799	474944	475090	475235	475381	475526
299	475671	475816	475962	476107	476252	476397	476542	476687	476832	476976

No	0	1	2	3	4	5	6	7	8	9
300	477121	477266	477411	477555	477700	477844	477989	478133	478278	478422
301	478566	478711	478855	478999	479143	479287	479431	479575	479719	479863
302	480007	480151	480294	480438	480582	480725	480869	481012	481156	481299
303	481443	481586	481729	481872	482016	482159	482302	482445	482588	482731
304	482874	483016	483159	483302	483445	483587	483730	483872	484015	484157
305	484300	484442	484585	484727	484869	485011	485153	485295	485437	485579
306	485721	485863	486005	486147	486289	486430	486572	486714	486855	486997
307	487138	487280	487421	487563	487704	487845	487986	488127	488269	488410
308	488551	488692	488833	488974	489114	489255	489396	489537	489677	489818
309	489958	490099	490239	490380	490520	490661	490801	490941	491081	491222
310	491362	491502	491642	491782	491922	492062	492201	492341	492481	492621
311	492760	492900	493040	493179	493319	493458	493597	493737	493876	494015
312	494155	494294	494433	494572	494711	494850	494989	495128	495267	495406
313	495544	495683	495822	495960	496099	496238	496376	496515	496653	496791
314	496930	497068	497206	497344	497483	497621	497759	497897	498035	498173
315	498311	498448	498586	498724	498862	498999	499137	499275	499412	499550
316	499687	499824	499962	500099	500236	500374	500511	500648	500785	500922
317	501059	501196	501333	501470	501607	501744	501880	502017	502154	502291
318	502427	502564	502700	502837	502973	503109	503247	503382	503518	503655
319	503791	503927	504063	504199	504335	504471	504607	504743	504878	505014
320	505150	505286	505421	505557	505693	505828	505964	506099	506234	506370
321	506505	506640	506776	506911	507046	507181	507316	507451	507586	507721
322	507856	507991	508126	508260	508395	508530	508664	508799	508934	509068
323	509203	509337	509471	509606	509740	509874	510009	510143	510277	510411
324	[illegible]	[illegible]	510813	510947	511081	511215	511349	511482	511616	511750
325	[illegible]	512017	512151	512284	512418	512551	512684	512818	512951	513084
326	513218	513351	513484	513617	513750	513883	514016	514149	514282	514415
327	514548	514681	514813	514946	515079	515211	515344	515476	515609	515741
328	515874	516006	516139	516271	516403	516535	516668	516800	516932	517064
329	517196	517328	517460	517592	517724	517855	517987	518119	518251	518382
330	518514	518646	518777	518909	519040	519171	519303	519434	519566	519697
331	519828	519959	520090	520221	520353	520484	520615	520745	520876	521007
332	521138	521269	521400	521530	521661	521792	521922	522053	522183	522314
333	522444	522575	522705	522835	522966	523096	523226	523356	523486	523616
334	523746	523876	524006	524136	524266	524396	524526	524656	524785	524915
335	525045	525174	525304	525434	525563	525693	525822	525951	526081	526210
336	526339	526469	526598	526727	526856	526985	527114	527243	527372	527501
337	527630	527759	527888	528016	528145	528274	528402	528531	528660	528788
338	528917	529045	529174	529302	529430	529559	529687	529815	529943	530072
339	530200	530328	530456	530584	530712	530840	530968	531096	531223	531352
340	531479	531607	531734	531862	531990	532117	532245	532372	532500	532627
341	532754	532882	533009	533136	533264	533391	533513	533645	533772	533899
342	534026	534153	534280	534407	534534	534661	534787	534914	535041	535167
343	535294	535421	535547	535674	535800	535927	536053	536180	536306	536432
344	536558	536685	536811	536937	537063	53718[illegible]	537315	537441	537567	537693
345	537819	537945	538071	538197	538322	538448	538574	538699	538825	538951
346	539076	539202	539327	539452	539578	539703	539829	539954	540079	540204
347	540329	540455	540580	540705	540830	540955	541080	541205	541330	541454
348	541579	541704	541829	541953	542078	542203	542327	542452	542576	542701
349	542825	542950	543074	543199	543323	543447	543571	543696	543820	043944

No	0	1	2	3	4	5	6	7	8	9
350	544068	544192	544316	544440	544564	544688	544812	544936	545060	545183
351	545307	545431	545555	545678	545802	545925	546049	546172	546296	546419
352	546543	546666	546789	546913	547036	547159	547282	547405	547529	547652
353	547775	547898	548021	548144	548267	548389	548512	548635	548758	548881
354	549003	549126	549249	549371	549494	549616	549739	549861	549984	550106
355	550228	550351	550473	550595	550717	550840	550962	551084	551206	551328
356	551450	551572	551694	551816	551938	552060	552181	552303	552425	552547
357	552668	552790	552911	553033	553155	553276	553398	553519	553640	553762
358	553883	554004	554126	554247	554368	554489	554610	554731	554852	554973
359	555094	555215	555336	555457	555578	555699	555820	555940	556061	556182
360	556303	556423	556544	556664	556785	556905	557026	557146	557267	557387
361	557507	557627	557748	557868	557988	558108	558228	558349	558469	558589
362	558709	558829	558948	559068	559188	559308	559428	559548	559667	559787
363	559907	560026	560146	560265	560385	560504	560624	560743	560863	560982
364	561101	561221	561340	561459	561578	561698	561817	561936	562055	562174
365	562293	562421	562531	562650	562769	562887	563006	563125	563244	563362
366	563481	563600	563718	563837	563955	564074	564192	564311	564429	564548
367	564666	564784	564903	565021	565139	565257	565376	565494	565612	565730
368	565848	565966	566084	566202	566320	566437	566555	566673	566791	566909
369	567026	567144	567262	567379	567497	567614	567732	567849	567967	568084
370	568202	568319	568436	568554	568671	568788	568905	569023	569140	569257
371	569374	569491	569608	569725	569842	569959	570076	570193	570309	570426
372	570543	570660	570776	570893	571010	571126	571243	571359	571476	571592
373	571709	571825	571942	572058	572174	572291	572407	572523	572639	572755
374	572872	572988	573104	573220	573336	573452	573568	573684	[illegible]	573915
375	574031	574147	574263	574379	574494	574610	574726	574841	[illegible]	575072
376	575188	575303	575419	575534	575650	575765	575880	575996	576111	576226
377	576341	576457	576572	576687	576802	576917	577032	577147	577262	577377
378	577492	577607	577722	577836	577951	578066	578181	578295	578410	578525
379	578639	578754	578868	578983	579097	579212	579326	579441	579555	579669
380	579784	579898	580012	580126	580241	580355	580469	580583	580697	580811
381	580925	581039	581153	581267	581381	581495	581608	581722	581836	581950
382	582063	582177	582291	582404	582518	582631	582745	582858	582972	583085
383	583199	583312	583426	583539	583652	583765	583879	583992	584105	584218
384	584331	584444	584557	584670	584783	584896	585009	585122	585235	585348
385	585461	585574	585686	585799	585912	586024	586137	586250	586362	586475
386	586587	586700	586812	586925	587037	587149	587262	587374	587486	587599
387	587711	587823	587935	588047	588160	588272	588384	588496	588608	588720
388	588832	588944	589056	589167	589279	589391	589503	589615	589726	589838
389	589950	590061	590173	590284	590396	590507	590619	590730	590842	590953
390	591065	591176	591287	591399	591510	591621	591732	591843	591955	592066
391	592177	592288	592399	592510	592621	592732	592843	592954	593064	593175
392	593286	593397	593508	593618	593729	593840	593950	594061	594171	594282
393	594393	594503	594614	594724	594834	594945	595055	595165	595276	595386
394	595496	595606	595717	595827	595937	596047	596157	596267	596377	596487
395	596597	596707	596817	596927	597037	597146	597256	597366	597476	597586
396	597695	597805	597914	598024	598134	598243	598353	598462	598572	598681
397	598791	598900	599009	599119	599228	599337	599446	599556	599665	599774
398	599883	599992	600101	600210	600319	600428	600537	600646	600755	600864
399	600973	601082	601191	601299	601408	601517	601625	601734	601843	601951

No.	0	1	2	3	4	5	6	7	8	9
400	602060	602169	602277	602386	602494	602603	602711	602819	602928	603036
401	603144	603253	603361	603469	603573	603686	603794	603902	604010	604118
402	604226	604334	604442	604550	604658	604766	604874	604982	605089	605197
403	605305	605413	605521	605628	605736	605844	605951	606059	606166	606274
404	606381	606489	606596	606704	606811	606919	607026	607133	607241	607348
405	607455	607562	607669	607777	607884	607991	608098	608205	608312	608419
406	608526	608633	608740	608847	608954	609061	609167	609274	609381	609488
407	609594	609701	609808	609914	610021	610128	610234	610341	610147	610554
408	610660	610767	610873	610979	611086	611192	611298	611405	611511	611617
409	611723	611829	611936	612042	612148	612254	612360	612466	612572	612678
410	612784	612890	612996	613102	613207	613313	613419	613525	613630	613736
411	613842	613947	614053	614159	614264	614370	614475	614581	614686	614792
412	614897	615003	615108	615213	615319	615424	615529	615634	615740	615845
413	615950	616055	616160	616265	616370	616476	616581	616686	616790	616895
414	617000	617105	617210	617315	617420	617525	617629	617734	617839	617943
415	618048	618153	618257	618362	618466	618571	618576	618780	618884	618989
416	619093	619198	619302	619406	619511	619615	619719	619824	619928	620032
417	620136	620240	620344	620448	620552	620656	620760	620864	620968	621072
418	621176	621280	621384	621488	621592	621695	621799	621903	622007	622110
419	622214	622318	622421	622525	622628	622732	622835	622939	623012	623146
420	623249	623353	623456	623559	623663	623766	623869	623973	624076	624179
421	624282	624385	624488	624591	624695	624798	624901	625004	625107	625210
422	625312	625415	625518	625621	625724	625827	625929	626032	626135	626238
423	[illegible]	626443	626546	626648	626751	626853	626956	627058	627161	627263
424	[illegible]	[illegible]	627571	627673	627775	627878	627980	928082	628185	628287
425	628389	628491	628593	628695	628797	628900	629002	629104	629206	629308
426	629410	629512	629613	629715	629817	629919	630021	630123	630224	630326
427	630428	630530	630631	630733	630835	630936	631038	631139	631241	631342
428	631444	631545	631647	631748	631819	631951	632052	632153	632255	632356
429	632457	632559	632660	632761	632862	632963	633064	633165	633266	633367
430	633468	633569	633670	633771	633872	633973	634074	634175	634276	634376
431	634477	634578	634679	634779	634880	634981	635081	635182	635283	635383
432	635484	635584	635685	635785	635886	635986	636087	636187	636287	636388
433	636488	636588	636688	636789	636889	636989	637089	637189	637290	637390
434	637490	637590	637690	637790	637890	637990	638090	638190	638290	638389
435	638489	638589	638689	638789	638888	638988	639088	639188	639287	639387
436	639486	639586	639686	639785	639885	639984	640084	640183	640283	640382
437	640481	640581	640680	640779	640879	640978	641077	641177	641276	641375
438	641474	641573	641672	641771	641871	641970	642069	642168	642267	642366
439	642465	642563	642662	642761	642860	642959	643058	643156	643255	643354
440	643453	643551	643650	643749	643847	643916	644044	644143	644242	644340
441	644439	644537	644636	644734	644832	644931	645029	645127	645226	645324
442	645422	645521	645619	645717	645815	645913	646011	646110	646208	646306
443	646404	646502	646500	646698	646796	646894	646992	647089	647187	647285
444	647383	647481	647579	647676	647774	647872	647969	648067	648165	648262
445	648360	648458	648555	648653	648750	648818	648945	649043	649140	649237
446	649335	649432	649530	649627	649724	649821	649919	650016	650113	650210
447	650308	650405	650502	650599	650696	650793	650890	650987	651084	651181
448	651278	651375	651472	651569	651666	651762	651859	651956	652053	652150
449	652246	652343	652440	652536	652633	652730	652826	652923	653019	653116

No.	0	1	2	3	4	5	6	7	8	9
450	653213	653309	653405	653502	653598	653695	653791	653888	653984	654080
451	654177	654273	654369	654465	654562	654658	654754	654850	654946	654042
452	655138	655235	655331	655427	655526	655619	655715	655810	655906	656002
453	656098	656194	656290	656386	656482	656577	656673	656769	656864	656960
454	657056	657152	657247	657343	657438	657534	657629	657725	657820	657916
455	658011	658107	658202	658298	658393	658488	658584	658679	658774	658870
456	658965	659060	659155	659250	659346	659441	659536	659631	659726	659821
457	659916	660011	660106	660201	660296	660391	660486	660581	660676	660771
458	660865	660960	661055	661150	661245	661339	661434	661529	661623	661718
459	661813	661907	622002	662096	662191	662286	662380	662475	662569	662663
460	662758	662852	662947	663041	663135	663230	663324	663418	663512	663607
461	663701	663795	663889	663983	664078	664172	664266	664360	664454	664548
462	664642	664736	664830	664924	665018	665112	665206	665299	665393	665487
463	665581	665675	665769	665862	665956	666050	666143	666237	666331	666424
464	666518	666612	666705	666799	666892	666986	667079	667173	667266	667360
465	667453	667546	667640	667733	667826	667920	668013	668106	668199	668293
466	668386	668479	668572	668665	668759	668852	668945	669038	669131	669224
467	669317	669410	669503	669596	669689	669782	669875	669967	670060	670153
468	670246	670339	670431	670524	670617	670710	670802	670895	670988	671080
469	671173	671265	671358	671451	671543	671636	671728	671821	671913	672005
470	672098	672190	672283	672375	672467	672560	672652	672744	672836	672929
471	673021	673113	673205	673297	673390	673482	673574	673666	673758	673850
472	673942	674034	674126	674218	674310	674402	674494	674586	674677	674769
473	674861	674953	675045	675137	675228	675320	675412	675503	[illegible]595	675687
474	675778	675870	675962	676053	676145	676236	676328	676419	[illegible]11	676602
475	676694	676785	676876	676968	677059	677151	677242	677333	677424	677516
476	677607	677698	677789	677881	677972	678063	678154	678245	678336	678427
477	678518	678609	678700	678791	677882	678073	679064	679155	679246	679337
478	679428	679519	679610	679700	679791	679882	679973	680063	680154	680245
479	680336	680426	680517	680607	680698	680789	680879	680970	681060	681151
480	681241	681332	681422	681513	681603	681693	681784	681874	681964	682055
481	682145	682235	682326	682416	682506	682596	682686	682777	682867	682957
482	683047	683137	683227	683317	683407	683497	683587	683677	683767	683857
483	683947	684037	684127	684217	684307	684396	684486	684576	684666	684756
484	684845	684935	685025	685114	685204	685294	685383	685473	685563	685652
485	685742	685831	685921	686010	686100	686189	686279	686368	686458	686547
486	686636	686726	686815	686904	686994	687083	687172	687261	687351	687440
487	687529	687618	687707	687796	687886	687975	688064	688153	688242	688331
488	688420	688509	688598	688687	688776	688865	688953	689042	689131	689220
489	689309	689398	689486	689575	689664	689753	689841	689930	690019	690107
490	690196	690285	690373	690462	690550	690639	690728	690816	699905	690993
491	691081	691170	691258	691347	691435	691524	691612	691700	691789	691877
492	691965	692053	692142	692230	692318	692406	692494	692583	692671	692759
493	692847	692935	693023	693111	693199	693287	693375	693463	693551	693639
494	693727	693815	693903	693991	694078	694166	694254	694342	694430	694517
495	694605	694693	694781	694868	694956	695044	695131	695219	695307	695294
496	695482	695569	695657	695744	695832	695919	696007	696094	696182	696269
497	696356	696444	696531	696618	696706	696793	696880	696968	697055	697142
498	697229	697317	697404	697491	697578	697665	697752	697839	697926	698014
499	698101	698188	698275	698362	698449	698535	698622	698709	698796	698883

No.	0	1	2	3	4	5	6	7	8	9
500	698970	699057	699144	699231	699317	699404	699491	699578	699664	699751
501	699838	699924	700011	700098	700184	700271	700358	700444	700531	700617
502	700704	700790	700877	700963	701050	701136	701222	701309	701395	701482
503	701568	701654	701741	701827	701913	701999	702086	702172	702258	702344
504	702431	702517	702603	702689	702775	702861	702947	703033	703119	703205
505	703291	703377	703463	703549	703635	703721	703807	703893	703979	704065
506	704151	704236	704322	704408	704494	704579	704665	704751	704837	704922
507	705008	705094	705179	705265	705350	705436	705522	705607	705693	705778
508	705864	705949	706035	706120	706206	706291	706376	706462	706547	706632
509	706718	706803	706888	706974	707059	707144	707229	707315	707400	707485
510	707570	707655	707740	707826	707911	707996	708081	708166	708251	708336
511	708421	708506	708591	708676	708761	708846	708931	709015	709100	709185
512	709270	709355	709440	709524	709609	709694	709779	709863	709948	710033
513	710117	710202	710287	710371	710456	710540	710625	710710	710794	710879
514	710963	711048	711132	711217	711301	711385	711470	711554	711639	711723
515	711807	711892	711976	712060	712144	712229	712313	712397	712481	712566
516	712650	712734	712818	712902	712986	713070	713154	713238	713326	713407
517	713491	713575	713659	713742	713826	713910	713994	714078	714162	714246
518	714330	714414	714497	714581	714665	714749	714833	714916	715000	715084
519	715167	715251	715335	715418	715502	715586	715669	715753	715836	715920
520	716003	716087	716170	716254	716337	716421	716504	716588	716671	716754
521	716838	716921	717004	717088	717171	717254	717338	717421	717504	717587
522	717671	717754	717837	717920	718003	718086	718169	718253	718336	718419
523	718502	718585	718668	718751	718834	718917	719000	719083	719165	719248
524	719331	719414	719497	719580	719663	719745	719828	719911	719994	720077
525	720159	720242	720325	720407	720490	720573	720655	720738	720821	720903
526	720986	721068	721151	721233	721316	721398	721481	721563	721646	721728
527	721811	721893	721975	722058	722140	722222	722305	722387	722469	722552
528	722634	722716	722798	722881	722963	723045	723127	723209	723291	723374
529	723456	723538	723620	723702	723784	723866	723948	724030	724112	724194
530	724276	724358	724440	724522	724604	724685	724767	724849	724931	725013
531	725095	725176	725258	725340	725422	725503	725585	725667	725748	725830
532	725912	725993	726075	726156	726238	726320	726401	726483	726564	726646
533	726727	726809	726890	726972	727053	727134	727216	727297	727379	727460
534	727541	727623	727704	727785	727866	727948	728029	728110	728191	728273
535	728354	728435	728516	728597	728678	728759	728841	728922	729003	729084
536	729165	729246	729327	729408	729489	729570	729651	729732	729813	729893
537	729974	730055	730136	730217	730298	730378	730459	730540	730621	730702
538	730782	730863	730944	731024	731105	731186	731266	731347	731428	731508
539	731589	731669	731750	731830	731911	731991	732072	732152	732233	732313
540	732394	732474	732555	732635	732715	732796	732876	732956	733037	733117
541	733197	733278	733358	733438	733518	733598	733679	733759	733839	733919
542	733999	734079	734160	734240	734320	734400	734480	734560	734640	734720
543	734800	734880	734960	735040	735120	735200	735279	735359	735439	735519
544	735599	735679	735759	735838	735918	735998	736078	736157	736237	736317
545	736397	736476	736556	736635	736715	736795	736874	736954	737034	737113
546	737193	737272	737352	737431	737511	737590	737670	737749	737829	737908
547	737987	738067	738146	738225	738305	738384	738463	738543	738622	738701
548	738781	738860	738939	739018	739097	739177	739256	739335	739414	739493
549	739572	739651	739731	739810	739889	739968	740047	740126	740205	740284

No	0	1	2	3	4	5	6	7	8	9
550	740363	740442	740521	740560	740678	740757	740836	740915	740994	741073
551	741152	741230	741309	741388	741467	741546	741624	741703	741782	741860
552	741939	742018	742096	742175	742254	742332	742411	742489	742568	742646
553	742725	742804	742882	742961	743039	743118	743196	743275	743353	743431
554	743510	743588	743667	743745	743823	743902	743980	744058	744136	744215
555	744293	744371	744449	744528	744606	744684	744762	744840	744919	744997
556	745075	745153	745231	745309	745387	745465	745543	745621	745699	745777
557	745855	745933	746011	746089	746167	746245	746323	746401	746479	746556
558	746634	746712	746790	746868	746945	747023	747101	747179	747256	747334
559	747412	747489	747567	747645	747722	747800	747878	747955	748033	748110
560	748188	748266	748343	748421	748498	748576	748653	748731	748880	748885
561	748963	749040	749118	749195	749272	749350	749427	749504	749582	749659
562	749736	749814	749891	749968	750045	750123	750200	750277	750354	750431
563	750508	750586	750663	750740	750817	750894	750971	751048	751125	751202
564	751279	751356	751433	751510	751587	751664	751741	751818	751895	751972
565	752048	752125	752202	752279	752356	752433	752509	752586	752663	752740
566	752816	752893	752970	753047	753123	753200	753277	753353	753430	753506
567	753583	753660	753736	753813	753889	753966	754042	754119	754195	754272
568	754348	754425	754501	754578	754654	754730	754807	754883	754960	755036
569	755112	755189	755265	755341	755417	755494	755570	755646	755722	755799
570	755875	755951	756027	756103	756180	756256	756332	756408	756484	756560
571	756636	756712	756788	756864	756940	757016	757092	757168	757244	757320
572	757396	757472	757548	757624	757700	757775	757851	757927	758003	758079
573	758155	758230	758306	758382	758458	758533	758609	758685	758761	758836
574	758912	758988	759063	759139	759214	759290	759366	759441	759517	759592
575	759668	759743	759819	759894	759970	760045	760121	760196	760272	760347
576	760422	760498	760573	760649	760724	760799	760875	760950	761025	761101
577	761176	761251	761326	761402	761477	761552	761627	761702	761778	761853
578	761928	762003	762078	762153	762228	762303	762378	762453	762529	762604
579	762679	762754	762829	762904	762978	763053	763128	763203	763278	763353
580	763428	763503	763578	763653	763727	763802	763877	763952	764027	764101
581	764176	764251	764326	764400	764475	764550	764624	764699	764774	764848
582	764923	761998	765072	765147	765221	765296	765370	765445	765520	765594
583	765669	765743	765818	765892	765966	766041	766115	766190	766264	766338
584	766413	766487	766562	766636	766710	766785	766859	766933	767007	767082
585	767156	767230	767304	767379	767453	767527	767601	767675	767749	767823
586	767898	767972	768046	768120	768194	768268	768342	768416	768490	768564
587	768638	768712	768786	768860	768934	769008	769082	769156	769230	769303
588	769377	769451	769325	769599	769673	769746	769820	769894	769968	770042
589	770115	770189	770263	770336	770410	770484	770557	770631	770705	770778
590	770852	770926	770999	771073	771146	771220	771293	771367	771440	771514
591	771587	771661	771734	771808	771881	771955	772028	772102	772175	772248
592	772322	772395	772468	772542	772615	772688	772762	772835	772908	772981
593	773055	773128	773201	773274	773348	773421	773494	773567	773640	773713
594	773786	773860	773933	774006	774079	774152	774225	774298	774371	774444
595	774517	774590	774663	774736	774809	774882	774955	775028	775100	775173
596	775246	775319	775392	775465	775538	775610	775683	775756	775829	775902
597	775974	776047	776120	776193	776265	776338	776411	776483	776556	776629
598	776701	776774	776846	776919	776992	777064	777137	777209	777282	777354
599	777427	777499	777572	777644	777717	777789	777862	777934	778006	778079

No.	0	1	2	3	4	5	6	7	8	9
600	778151	778224	778296	778368	778441	778513	778585	778658	778730	778802
601	778874	778947	779019	779091	779163	779236	779308	779380	779452	779524
602	779596	779669	779741	779813	779885	779957	780029	780101	780173	780245
603	780317	780389	780461	780533	780605	780677	780749	780821	780893	780965
604	781037	781109	781181	781253	781324	781396	781468	781540	781612	781684
605	781755	781827	781899	781971	782042	782114	782186	782258	782329	782401
606	782473	782544	782616	782688	782759	782831	782902	782974	783046	783117
607	783189	783260	783332	783403	783475	783546	783618	783689	783761	783832
608	783904	783975	784046	784118	784189	784261	784332	784403	784475	784546
609	784617	784689	784760	784831	784902	784974	785045	785116	785187	785259
610	785330	785401	785472	785543	785615	785686	785757	785828	785899	785970
611	786041	786112	786183	786254	786325	786396	786467	786538	786609	786680
612	786751	786822	786893	786964	787035	787106	787177	787248	787319	787390
613	787460	787531	787602	787673	787744	787815	787885	787956	788027	788098
614	788168	788239	788310	788381	788451	788522	788593	788663	788734	788804
615	788875	788946	789016	789087	789157	789228	789299	789369	789440	789510
616	789581	789651	789722	789792	789863	789933	790004	790074	790144	790215
617	790285	790356	790426	790496	790567	790637	790707	790778	790848	790918
618	790988	791059	791129	791199	791269	791340	791410	791480	791550	791620
619	791691	791761	791831	791901	791971	792041	792111	792181	792252	792322
620	792392	792462	792532	792602	792672	792742	792812	792882	792952	793022
621	793092	793162	793231	793301	793371	793441	793511	793581	793651	793721
622	793790	793860	793930	794000	794070	794139	794209	794279	794349	794418
623	794488	794558	794627	794697	794767	794836	794906	794976	795045	795115
624	795185	795254	795324	795393	795463	795532	795602	795672	795741	795811
625	795880	795949	796019	796088	796158	796227	796297	796366	796436	796505
626	796574	796644	796713	796782	796852	796921	796990	797060	797129	797198
627	797268	797337	797406	797475	797545	797614	797683	797752	797821	797890
628	797960	798029	798098	798167	798236	798305	798374	798443	798513	798582
629	798651	798720	798789	798858	798927	798996	799065	799134	799203	799272
630	799341	799409	799478	799547	799616	799685	799754	799823	799892	799961
631	800029	800098	800167	800236	800305	800373	800442	800511	800580	800648
632	800717	800786	800854	800923	800992	801061	801129	801198	801266	801335
633	801404	801472	801541	801609	801678	801747	801815	801884	801952	802021
634	802089	802158	802226	802295	802363	802432	802500	802568	802637	802705
635	802774	802842	802910	802979	803047	803116	803184	803252	803321	803389
636	803457	803525	803594	803662	803730	803798	803867	803935	804003	804071
637	804139	804208	804276	804344	804412	804480	804548	804616	804685	804753
638	804821	804889	804957	805025	805093	805161	805229	805297	805365	805433
639	805501	805569	805637	805705	805773	805841	805908	805976	806044	806112
640	806180	806248	806316	806384	806451	806519	806587	806655	806723	806790
641	806858	806926	806994	807061	807129	807197	807264	807332	807400	807467
642	807535	807603	807670	807738	807806	807873	807941	808008	808076	808143
643	808211	808279	808346	808414	808481	808549	808616	808684	808751	808818
644	808886	808953	809021	809088	809156	809223	809290	809358	809425	809492
645	809560	809627	809694	809762	809829	809896	809964	810031	810098	810165
646	810233	810300	810367	810434	810501	810569	810636	810703	810770	810837
647	810904	810971	811039	811106	811173	811240	811307	811374	811441	811508
648	811575	811642	811709	811776	811843	811910	811977	812044	812111	812178
649	812245	812312	812379	812445	812512	812579	812646	812713	812780	812847

No.	0	1	2	3	4	5	6	7	8	9
650	812913	812980	813047	813114	813181	813247	813314	813381	813448	813514
651	813581	813648	813714	813781	813848	813914	813981	814048	814114	814181
622	814248	814314	814381	814447	814514	814581	814647	814714	814780	814847
653	814913	814980	815046	815113	815179	815246	815312	815378	815445	815511
654	815578	815644	815711	815777	815843	815910	815976	816042	816109	816175
655	816241	816308	816374	816440	816506	816573	816639	816705	816771	816838
656	816904	816970	817036	817102	817169	817235	817301	817367	817433	817499
657	817565	817631	817698	817764	817830	817896	817962	818028	818094	818160
658	818226	818292	818358	818424	818490	818556	818622	818688	818754	818820
659	818885	818951	819017	819083	819149	819215	819281	819346	819412	819478
660	819544	819610	819676	819741	819807	819873	819939	820004	820070	820136
661	820201	820267	820313	820399	820464	820530	820595	820661	820727	820792
662	820858	820924	820989	821055	821120	821186	821251	821317	821382	821448
663	821514	821579	821645	821710	821775	821841	821906	821972	822037	822103
664	822168	822233	822299	822364	822430	822495	822560	822626	822691	822756
665	822822	822887	822952	823018	823083	823148	823213	823279	823344	823409
666	823474	823539	823605	823670	823735	823800	823865	823930	823996	824061
667	824126	824191	824256	824321	824386	824451	824516	824581	824646	824711
668	824776	824841	824906	824971	825036	825101	825166	825231	825296	825361
669	825426	825491	825556	825621	825686	825751	825815	825880	825945	826010
670	826075	826140	826204	826269	826334	826399	826464	826528	826593	826658
671	826723	826787	826852	826917	826981	827046	827111	827175	827240	827305
672	827369	827434	827499	827563	827628	827692	827757	827821	827886	827951
673	828015	828080	828144	828209	828273	828338	828402	828467	828531	828595
674	828660	828724	828789	828853	828918	828982	829046	829111	829175	829239
675	829304	829368	829432	829497	829561	829625	829690	829754	829818	829882
676	829947	830011	830075	830139	830204	830268	830332	830396	830460	830525
677	830589	830653	830717	830781	830845	830909	830973	831037	831102	831166
678	831230	831294	831358	831422	831486	831550	831614	831678	831742	831806
679	831870	831934	831998	832062	832126	832189	832253	832317	832381	832445
680	832509	832573	832637	832700	832764	832828	832892	832956	833020	833083
681	833147	833211	833275	833338	833402	833466	833530	833593	833657	833721
682	833784	833848	833912	833975	834039	834103	834166	834230	834294	834357
683	834421	834484	834548	834611	834675	834739	834802	834866	834929	834993
684	835056	835120	835183	835247	835310	835373	835437	835500	835564	835627
685	835691	835754	835817	835881	835944	836007	836071	836134	836197	836261
686	836324	836387	836451	836514	836577	836641	836704	836767	836830	836894
687	836957	837020	837083	837146	837210	837273	837336	837399	837462	837525
688	837588	837652	837715	837778	837841	837904	837967	838030	838093	838156
689	838219	838282	838345	838408	838471	838534	838597	838660	838723	838786
690	838849	838912	838975	839038	839101	839164	839227	839289	839352	839415
661	839478	839541	839604	839667	839729	839792	839855	839918	839981	840043
692	840106	840169	840232	840294	840357	840420	840482	840545	840608	840671
693	840733	840796	840859	840921	840984	841046	841109	841172	841234	841297
694	841359	841422	841485	841547	841610	841672	841735	841797	841860	841922
695	841985	842047	842110	842172	842235	842297	842360	842422	842484	842547
696	842609	842672	842734	842796	842859	842921	842983	843046	843108	843170
697	843233	843295	843357	843420	843482	843544	843606	843669	843731	843793
698	843855	843918	843980	844042	844104	844166	844229	844291	844353	844415
699	844477	844539	844601	844664	844726	844788	844850	844912	844974	845036

No.	0	1	2	3	4	5	6	7	8	9
700	845098	845160	845222	845284	845346	845408	845470	845532	845594	845656
701	845718	845780	845842	845904	845966	846028	846090	846151	846213	846275
702	846337	846399	846461	846523	846585	846646	846708	846770	846832	846894
703	846955	847017	847079	847141	847202	847264	847326	847388	847449	847511
704	847573	847634	847696	847758	847819	847881	847943	848004	848066	848128
705	848189	848251	848312	848374	848435	848497	848559	848620	848682	848743
706	848805	848866	848928	848989	849051	849112	849174	849235	849297	849358
707	849419	849481	849542	849604	849665	849726	849788	849849	849911	849972
708	850033	850095	850156	850217	850279	850340	850401	850462	850524	850585
709	850646	850707	850769	850830	850891	850952	851014	851075	851136	851197
710	851258	851320	851381	851442	851503	851564	851625	851686	851747	851809
711	851870	851931	851992	852053	852114	852175	852236	852297	852358	852419
712	852480	852541	852602	852663	852724	852785	852846	852907	852968	853029
713	853090	853150	853211	853272	853333	853394	853455	853516	853577	853637
714	853698	853759	853820	853881	853941	854002	854063	854124	854185	854245
715	854306	854367	854428	854488	854549	854610	854670	854731	854792	854852
716	854913	854974	855034	855095	855156	855216	855277	855337	855398	855459
717	855519	855580	855640	855701	855761	855822	855882	855943	856003	856064
718	856124	856185	856245	856306	856366	856427	856487	856548	856608	856668
719	856729	856789	856850	856910	856970	857031	857091	857152	857212	857272
720	857332	857393	857453	857513	857574	857634	857694	857755	857815	857875
721	857935	857995	858056	858116	858176	858236	858297	858357	858417	858477
722	858537	858597	858657	858718	858778	858838	858898	858958	859018	859078
723	859138	859198	859258	859318	859379	859439	859499	859559	859619	859679
724	859739	859799	859859	859918	859978	860038	860098	860158	860218	860278
725	860338	860398	860458	860518	860578	860637	860697	860757	860817	860877
726	860937	860996	861056	861116	861176	861236	861295	861355	861415	861475
727	861534	861594	861654	861714	861773	861833	861893	861952	862012	862072
728	862131	862191	862251	862310	862370	862430	862489	862549	862608	862668
729	862728	862787	862847	862906	862966	863025	863085	863144	863204	863263
730	863323	863382	863442	863501	863561	863620	863680	863739	863799	863858
731	863917	863977	864036	864096	864155	864214	864274	864333	864392	864452
732	864511	864570	864630	864689	864748	864808	864867	864926	864985	865045
733	865104	865163	865222	865282	865341	865400	865459	865519	865578	865637
734	865696	865755	865814	865874	865933	865992	866051	866110	866169	866228
735	866287	866346	866405	866465	866524	866583	866642	866701	866760	866819
736	866878	866937	866996	867055	867114	867173	867232	867291	867350	867409
737	867467	867526	867585	867644	867703	867762	867821	867880	867939	867998
738	868056	868115	868174	868233	868292	868350	868409	868468	868527	868586
739	868644	868703	868762	868821	868879	868938	868997	869056	869114	869173
740	869232	869290	869349	869408	869466	869525	869584	869642	869701	869760
741	869818	869877	869935	869994	870053	870111	870170	870228	870287	870345
742	870404	870462	870521	870579	870638	870696	870755	870813	870872	870930
743	870989	871047	871106	871164	871223	871281	871339	871398	871456	871515
744	871573	871631	871690	871748	871806	871865	871923	871981	872040	872098
745	872156	872215	872273	872331	872389	872448	872506	872564	872622	872681
746	872739	872797	872855	872913	872972	873030	873088	873146	873204	873262
747	873321	873379	873437	873495	873553	873611	873669	873727	873785	873844
748	873902	873960	874018	874076	874134	874192	874250	874308	874366	874424
749	874482	874540	874598	874656	874714	874772	874830	874888	874945	875003

No	0	1	2	3	4	5	6	7	8	9
750	875061	875119	875177	875235	875293	875351	875409	875466	875524	875582
751	875640	875698	875756	875813	875871	875929	875987	876045	876102	876160
752	876218	876276	876333	876391	876449	876507	876564	876622	876680	876737
753	876795	876853	876910	876968	877026	877083	877141	877199	877256	877314
754	877371	877429	877487	877544	877602	877659	877717	877774	877832	877889
755	877947	878004	878062	878119	878177	878234	878292	878349	878407	878464
756	878522	878579	878637	878694	878752	878809	878866	878924	878981	879039
757	879096	879153	879211	879268	879325	879383	879440	879497	879555	879612
758	879669	879726	879784	879841	879898	879956	880013	880070	880127	880185
759	880242	880299	880356	880413	880471	880528	880585	880642	880699	880756
760	880814	880871	880928	880985	881042	881099	881156	881213	881271	881328
761	881385	881442	881499	881556	881613	881670	881727	881784	881841	881898
762	881955	882012	882069	882126	882183	882240	882297	882354	882411	882468
763	882525	882581	882638	882695	882752	882809	882866	882923	882980	883037
764	883093	883150	883207	883264	883321	883377	883434	883491	883548	883605
765	883661	883718	883775	883832	883888	883945	884002	884059	884115	884172
766	884229	884285	884342	884399	884455	884512	884569	884625	884682	884739
767	884795	884852	884909	884965	885022	885078	885135	885192	885248	885305
768	885361	885418	885474	885531	885587	885644	885700	885757	885813	885870
769	885926	885983	886039	886096	886152	886209	886265	886321	886378	886434
770	886491	886547	886604	886660	886716	886773	886829	886885	886942	886998
771	887054	887111	887167	887223	887280	887336	887392	887449	887505	887561
772	887617	887674	887730	887786	887842	887898	887955	888011	888067	888123
773	888179	888236	888292	888348	888404	888460	888516	888573	888629	888685
774	888741	888797	888853	888909	888965	889021	889077	889134	889190	889246
775	889302	889358	889414	889470	889526	889582	889638	889694	889750	889806
776	889862	889918	889974	890030	890086	890141	890197	890253	890309	890365
777	890421	890477	890533	890589	890645	890700	890756	890812	890868	890924
778	890980	891035	891091	891147	891203	891259	891314	891370	891426	891482
779	891537	891593	891649	891705	891760	891816	891872	891928	891983	892039
780	892095	892150	892206	892262	892317	892373	892429	892484	892540	892595
781	892651	892707	892762	892818	892873	892929	892985	893040	893096	893151
782	893207	893262	893318	893373	893429	893484	893540	893595	893651	893706
783	893762	893817	893873	893928	893984	894039	894094	894150	894205	894261
784	894316	894371	894427	894482	894538	894593	894648	894704	894759	894814
785	894870	894925	894980	895036	895091	895146	895201	895257	895312	895367
786	895423	895478	895533	895588	895644	895699	895754	895809	895864	895920
787	895975	896030	896085	896140	896195	896251	896306	896361	896416	896471
788	896526	896581	896636	896692	896747	896802	896857	896912	896967	897022
789	897077	897132	897187	897242	897297	897352	897407	897462	897517	897572
790	897627	897682	897737	897792	897847	897902	897957	898012	898067	898122
791	898176	898231	898286	898341	898396	898451	898506	898561	898615	898670
792	898725	898780	898835	898890	898944	898999	899054	899109	899164	899218
793	899273	899328	899383	899437	899492	899547	899602	899656	899711	899766
794	899821	899875	899930	899985	900039	900094	900149	900203	900258	900312
795	900367	900422	900476	900531	900586	900640	900695	900749	900804	900859
796	900913	900968	901022	901077	901131	901186	901240	901295	901349	901404
797	901458	901513	901567	901622	901676	901731	901785	901840	901894	901948
798	902003	902057	902112	902166	902221	902275	902329	902384	902438	902492
799	902547	902601	902655	902710	902764	902818	902873	902927	902981	903036

No.	0	1	2	3	4	5	6	7	8	9
800	903090	903144	903199	903253	903307	903361	903416	903470	903524	903578
801	903633	903687	903741	903795	903819	903904	903958	904012	904066	904120
802	904174	904229	904283	904337	904391	904445	904499	904553	904607	904661
803	904716	904770	904824	904878	904932	904986	905040	905094	905148	905202
804	905256	905310	905364	905418	905472	905526	905580	905634	905688	905742
805	905796	905850	905904	905958	906012	906066	906119	906173	906227	906281
806	906335	906389	906443	906497	906551	906604	906658	906712	906766	906820
807	906874	906927	906981	907035	907089	907143	907196	907250	907304	907358
808	907411	907465	907519	907573	907626	907680	907734	907787	907841	907895
809	907949	908002	908056	908110	908163	908217	908270	908324	908378	908431
810	908485	908539	908592	908646	908699	908753	908807	908860	908914	908967
811	909021	909074	909128	909181	909235	909289	909342	909396	909449	909503
812	909556	909610	909663	909716	909770	909823	909877	909930	909984	910037
813	910091	910144	910197	910251	910304	910358	910411	910464	910518	910571
814	910624	910678	910731	910784	910838	910891	910944	910998	911051	911104
815	911158	911211	911264	911317	911371	911424	911477	911530	911584	911637
816	911690	911743	911797	911850	911903	911956	912009	912063	912116	912169
817	912222	912275	912323	912381	912435	912488	912541	912594	912647	912700
818	912753	912806	912859	912913	912966	913019	913072	913125	913178	313231
819	913284	913337	913390	913443	913496	913549	913602	913655	913708	913761
820	913814	913867	913920	913973	914026	914079	914132	914184	914237	914290
821	914343	914396	914449	914502	914555	914608	914660	914713	914766	914819
822	914872	914925	914977	915030	915083	915136	915189	915241	915294	915347
823	915400	915453	915505	915558	915611	915664	915716	915769	915822	915875
824	915927	915980	916033	916085	916138	916191	916243	916296	916349	916401
825	916454	916507	916559	916612	916664	916717	916770	916822	916875	916927
826	916980	917033	917085	917138	917190	917243	917295	917348	917400	917453
827	917506	917558	917611	917663	917716	917768	917820	917873	917925	917978
828	918030	918083	918135	918188	918240	918293	918345	918397	918450	918502
829	918555	918607	918659	918712	918764	918816	918869	918921	918973	919026
830	919078	919130	919183	919235	919287	919340	919392	919444	919496	919549
831	919601	919653	919706	919758	919810	919862	919914	919967	920019	920071
832	920123	920176	920228	920280	920332	920384	920436	920489	920541	920593
833	920645	920697	920749	920801	920853	920906	920958	921010	921062	921114
834	921166	921218	921270	921322	921374	921426	921478	921530	921582	921634
835	921686	921738	921790	921842	921894	921946	921998	922050	922102	922151
836	922206	922258	922310	922362	922414	922466	922518	922570	922622	922674
837	922725	922777	922829	922881	922933	922985	923037	923089	923140	923192
838	923244	923296	923348	923399	923451	923503	923555	923607	923658	923710
839	923762	923814	923865	923917	923969	924021	924072	924124	924176	924228
840	924279	924331	924383	924434	924486	924538	924589	924641	924693	924744
841	924796	924848	924899	924951	925003	925054	925106	925157	925209	925261
842	925312	925364	925415	925467	925518	925570	925621	925673	925725	925776
843	925828	925879	925931	925982	926034	926085	926137	926188	926240	926291
844	926342	926394	926445	926497	926548	926600	926651	926702	926754	926805
845	926857	926908	926959	927011	927062	927114	927165	927216	927268	927319
846	927370	927422	927473	927524	927576	927627	927678	927730	927781	927832
847	927883	927935	927986	928037	928088	928140	928191	928242	928293	928345
848	928396	928447	928498	928549	928601	928652	928703	928754	928805	928857
849	928908	928959	929010	929061	929112	929163	929215	929266	929317	929368

No	0	1	2	3	4	5	6	7	8	9
850	929419	929470	929521	929572	929623	929674	929725	929776	929827	929879
851	929930	929881	930032	930083	930134	930185	930236	930287	930338	930389
852	930440	930491	930542	930592	930643	930694	930745	930796	930847	930898
853	930949	931000	931051	931102	931153	931204	931254	931305	931356	931407
854	931458	931509	931560	931610	931661	931712	931763	931814	931865	931915
855	931966	932017	932068	932118	932169	932220	932271	932322	932372	932423
856	932474	932524	932575	932626	332677	932727	932778	932829	932879	932930
857	932981	933031	933082	933133	933183	933234	933285	933335	933386	933437
858	933487	933538	933589	933639	933690	933740	933791	933841	933892	933913
859	933993	934044	934094	934145	934195	934246	934296	931347	934394	934448
860	934498	934549	934599	934650	934700	934751	934801	934852	934902	934953
861	935003	935054	935104	935154	935205	935255	935306	935356	935406	935457
862	935507	935558	935608	935658	935709	935759	935809	935860	935910	935960
863	936011	936061	936111	936162	936212	936262	936313	936363	936413	936463
864	936514	936564	936614	936665	936715	936765	936815	936865	936916	936966
865	937016	937066	937117	937167	937217	937267	937317	937367	937418	937468
866	937518	937568	937618	937668	937718	937769	937819	937869	937919	937969
867	938019	938069	938119	938169	938219	938269	938320	938370	938420	938470
868	938520	938570	938620	938670	938720	938770	938820	938870	938919	938970
869	939020	939070	939120	939170	939220	939270	939320	939369	939419	939469
870	939519	939569	939619	939669	939719	939769	939819	939869	939918	939968
871	940018	940068	940118	940168	940218	940267	940317	940367	940417	940467
872	949516	940566	940616	940666	940716	940765	940815	940865	940915	940964
873	941014	941064	941114	941163	941213	941263	941313	941362	941412	941462
874	941511	941561	941611	941660	941710	941760	941809	941859	941909	941958
875	942008	942058	942107	942157	942207	942256	942306	942355	942405	942455
876	942504	942554	942603	942653	942702	942752	942801	942851	942901	942950
877	943000	943049	943099	943148	943198	943247	943297	943346	943396	943445
878	943495	943544	943593	943643	943692	943742	943791	943841	943890	943939
879	943989	944038	944088	944137	944186	944236	944285	944335	944384	944433
880	944483	944532	944581	944631	944680	944729	944779	944828	944877	944927
881	944976	945025	945074	945124	945173	945222	945272	945321	945370	945419
882	945469	945518	945567	945616	945665	945715	945764	945813	945862	945912
883	945961	946010	946059	946108	946157	946207	946256	946305	946354	946403
884	946452	946501	946551	946600	946649	946698	946747	946796	946845	946894
885	946943	946992	947041	947090	947140	947189	947238	947287	947336	947385
886	947434	947483	947532	947581	947630	947679	947728	947777	947826	947875
887	947924	947973	948022	948070	948119	948168	948217	948266	948315	948364
888	948413	948462	948511	948560	948609	948657	948706	948755	948804	948853
889	948902	948951	948999	949048	949097	949146	949195	949244	949292	949341
890	949390	949439	949488	949536	949585	949634	949683	949731	949780	949829
891	949878	949926	949975	950024	950073	950121	950170	950219	950267	950316
892	950365	950414	950462	950511	950560	950608	950657	950706	950754	950803
893	950851	950900	950949	950997	951046	951095	951143	951192	951240	951289
894	951338	951386	951435	951483	951532	951580	951629	951677	951726	951775
895	951823	951872	951920	951969	952017	952066	952114	952163	952211	952260
896	952308	952356	952405	952453	952502	952550	952599	952647	952696	952744
897	952792	952841	952889	952938	952986	953034	953083	953131	953180	953228
898	953276	953325	953373	953421	953470	953518	953566	953615	953663	953711
899	953760	953808	953856	953905	953953	954001	954049	954098	954146	954194

No.	0	1	2	3	4	5	6	7	8	9
900	954243	954291	954339	954387	954435	954484	954532	954380	954628	954677
901	954725	954773	954821	954869	954918	954966	955014	955062	955110	955158
902	955207	955255	955303	955351	955399	955447	955495	955543	955592	955640
903	955688	955736	955784	955832	955880	955928	955976	956024	956072	956120
904	956168	956216	956265	956313	956361	956409	956457	956505	956553	956601
905	956649	956697	956745	956793	956840	956888	956936	956984	957032	957080
906	957128	957176	957224	957272	957320	957368	957416	957464	957512	957559
907	957607	957655	957703	957751	957799	957847	957894	957942	957990	958038
908	958086	958134	958181	958229	958277	958325	958373	958421	958468	958516
909	958564	958612	958659	958707	958755	958803	958850	958898	958946	958994
910	959041	959089	959137	959185	959232	959280	959328	959375	959423	959471
911	959518	959566	959614	959661	959709	959757	959804	959852	959900	959947
912	959995	960042	960090	960138	960185	960233	960280	960328	960376	960423
913	960471	960518	960566	960613	960661	960709	960756	960804	960851	960899
914	960946	960994	961041	961089	961136	961184	961231	961279	961326	961374
915	961421	961469	961516	961563	961611	961658	961706	961753	961801	961848
916	961895	961943	961990	962038	962085	962132	962180	962227	962275	962322
917	962369	962417	962464	962511	962559	962606	962653	962701	962748	962795
918	962843	962890	962937	962985	963032	963079	963126	963174	963221	963268
919	963316	963363	963410	963457	963504	963552	963599	963646	963693	963741
920	963788	963835	963882	963929	963977	964024	964071	964118	964165	964212
921	964260	964307	964354	964401	964448	964495	961542	964590	964637	964684
922	964731	964778	964825	964872	964919	964966	965013	965061	965108	965155
923	965202	965249	965296	965343	965390	965437	965481	965531	965578	965625
924	965672	965719	965766	965813	965860	965907	965954	966001	966048	966095
925	966142	966189	966236	966283	966329	966376	966423	966470	966517	966564
926	966611	966658	966705	966752	966799	966845	966892	966939	966986	967033
927	967080	967127	967173	967220	967267	967314	967361	967408	967454	967501
928	967548	967595	967642	967688	967735	967782	967829	967875	967922	967969
929	968016	968062	968109	968156	968203	968249	968296	968343	968390	968436
930	968483	968530	968576	968623	968670	968716	968763	968810	968856	968903
931	968950	968996	969043	969090	969136	969183	969229	969276	969323	969369
932	969416	969463	969509	969556	969602	969649	969695	969742	969789	969835
933	969882	969928	969975	970021	970068	970114	970161	970207	970254	970300
934	970347	970393	970440	970486	970533	970579	970626	970672	970719	970765
935	970812	970858	970904	970951	970997	971044	971090	971137	971183	971229
936	971276	971322	971369	971415	971461	971508	971554	971601	971647	971693
937	971740	971786	971832	971879	971925	971971	972018	972064	972110	972157
938	972203	972249	972295	972342	972388	972434	972481	972527	972573	972619
939	972666	972712	972758	972804	972851	972897	972943	972989	973035	973082
940	973128	973174	973220	973266	973313	973359	973405	973451	973497	973543
941	973590	973636	973682	973728	973774	973820	973866	973913	973959	974005
942	974051	974097	974143	974189	974235	974281	974327	971374	974420	974466
943	974512	974558	974604	974650	974696	974742	974788	974834	974880	974926
944	974972	975018	975064	975110	975156	975202	975248	975294	975340	975386
945	975432	975478	975524	975570	975616	975662	975707	975753	975799	975845
946	975891	975937	975983	976029	976075	976121	976167	976212	976258	976304
947	976350	976396	976442	976488	976533	976579	976625	976671	976717	976763
948	976808	976854	976900	976946	976992	977037	977083	977129	977175	977220
949	977266	977312	977358	977403	977449	977495	977541	977586	977632	977678

No.	0	1	2	3	4	5	6	7	8	9
950	977724	977769	977815	977861	977906	977952	977998	978043	978089	978135
951	978181	978226	978272	978317	978363	978409	478454	978500	978546	978591
952	978637	978683	978728	978774	978819	978865	978911	978956	979022	979047
953	979093	979138	979184	979230	979275	979321	979366	979412	979457	979503
954	979548	979594	979639	979685	979730	979776	979821	979867	979912	979958
955	980003	980049	980094	980140	980185	980231	980276	980322	980367	980412
956	980458	980503	980549	980594	980640	980685	980730	980776	980821	980867
957	980912	980957	981003	981048	981093	981139	981184	981229	981275	981320
958	981366	981411	981456	981501	981547	981592	981637	981683	981728	981773
959	981819	981864	981909	981954	982000	982045	982090	982135	982181	982226
960	982271	982316	982362	982407	982452	982497	982543	982588	982633	982678
961	982723	982769	982814	982859	982904	982949	982994	983040	983085	983130
962	983175	983220	983265	983310	983356	983401	983446	983491	983536	983581
963	983626	983671	983716	983762	983807	983852	983897	983942	983987	984032
964	984077	984122	984167	984212	984257	984302	984347	984392	984437	984482
965	984527	984572	984617	984662	984707	984752	984797	984842	984887	984932
966	984977	985022	985067	985112	985157	985202	985247	985292	985337	985382
967	985426	985471	985516	985561	985606	985651	985696	985741	985786	985830
968	985875	985920	985965	986010	986055	986100	986144	986189	986234	986279
969	986324	986369	986413	986458	986503	986548	986593	986637	986682	986727
970	986772	986817	986861	986906	986951	986996	987040	987085	987130	987175
971	987219	987264	987309	987353	987398	987443	987488	987532	987577	987622
972	987666	987711	987756	987800	987845	987890	987934	987979	988024	988068
973	988113	988157	988202	988247	988291	988336	988381	988425	988470	988514
974	988559	988604	988648	988693	988737	988782	988826	988871	988916	988960
975	989005	989049	989094	989138	989183	989227	989272	989316	989361	989405
976	989450	989494	989539	989583	989628	989672	989717	989761	989806	989850
977	989895	989939	989983	990028	990072	890117	990161	990206	990250	990294
978	990339	990383	990428	990472	990516	990561	990605	990650	990694	990738
979	990783	990827	990871	990916	990960	991004	991049	991093	991137	991182
980	991226	991270	991315	991359	991403	991448	991492	991536	991580	991625
981	991669	991713	991758	991802	991846	991890	991935	991979	992023	992067
982	992111	992156	992200	992244	992288	992333	992377	992421	992465	992509
983	992554	992598	992642	992686	992730	992774	992819	992863	992907	992951
984	992995	993039	993083	993127	993172	993216	993260	993304	993348	993392
985	993436	993480	993524	993568	993613	993657	993701	993745	993789	993833
986	993877	993921	993965	994009	994053	994097	994141	994185	994229	994273
987	994317	994361	994405	994449	994493	994537	994581	994625	994669	994713
988	994757	994801	994845	994889	994933	994977	995021	995065	995108	995152
989	995196	995240	995284	995328	995372	995416	995460	995504	995547	995591
990	995635	995679	995723	995767	995811	995854	995898	995942	995986	996030
991	996074	996117	996161	996205	996249	996293	996337	996380	996424	996468
992	996512	996555	996599	996643	996687	996731	996774	996818	996862	996906
993	996949	996993	997037	997080	997124	997168	997212	997255	997299	997343
994	997386	997430	997474	997517	997561	997605	997648	997692	997736	997779
995	997823	997867	997910	997954	997998	998041	998085	998129	998172	998216
996	998259	998303	998347	998390	998434	998477	998521	998564	998608	998652
997	998695	998739	998782	998826	998869	998913	998956	999000	999043	999087
998	999131	999174	999218	999261	999305	999348	999392	999435	999474	999522
999	999565	999609	999652	999696	999739	999783	999826	999870	999913	999957

TABLE II.

LOGARITHMIC SINES, COSINES, TANGENTS, AND COTANGENTS,

Calculated to the natural radius of 10,000 *millions, or the logarithmic radius of* 10 *index.*

THE secant and cosecant will be at once obtained from the tables, by the following formula, which will be found in the chapter on Trigonometry, page 38.

$$\text{sec.} = \frac{\text{rad.}^2}{\text{cos.}} \quad \text{therefore, log. sec.} = 20 - \text{log. cos.}$$

$$\text{cosec.} = \frac{\text{rad.}^2}{\text{sin.}} \quad \text{log. cosec.} = 20 - \text{log. sine.}$$

PROBLEM I.

To find the sine, tangent, &c., of a given arc.

If the degrees in the arc be *less than* 45 *degrees*, look at the top of the page for the degrees, reading downwards in the *left* hand column for the minutes, then, collaterally with the minutes, in the column headed sines, tangents, &c., at the *top* of the page, will be found the logarithmic sine or tangent required.

If the degrees be *more* than 45° look at the bottom of the page for the degrees, reading upwards, in the right hand column, for the minutes; then, collaterally, in the column, which has the proper heading, at the *bottom* of the page, will be found the logarithmic sine, or tangent required.

EXAMPLE 1.—It is required to find the sine of the arc of 29 degrees 45 minutes.

This arc being less than 45 degrees, look for 29 degrees at the top of the page, then reading downwards in the left hand, find 45 minutes; collaterally with this, in the column of sines, will be found 9·695671, the logarithmic sine required.

Example 2.—It is required to find the tangent of the arc of 54 degrees 30 minutes.

This being above 45 degrees, look for 54 degrees, in the bottom of the page, and for 30 minutes, in the right hand column; then, in the column, headed tangents, at the *bottom* of the page, will be found, collaterally with the 30 minutes, the required tangent 10·146732.

When the given angle is in seconds, look out in the tables for the logarithmic sines, tangents, &c., of the next lower and next higher minutes; then say,—

As the difference between the next lower and next greater arc, in seconds : the difference between their corresponding logarithmic sines or tangents : : the excess, in seconds, of the given arc above the next lower in the tables : the logarithmic excess of the sine or tangent of the given angle, above that of the next lower angle in the tables.

Add this excess to the next lower, in the case of the sine, tangent, and secant, subtracting it for their complements, and you obtain the logarithmic sine, or tangent required.

Or, multiply the difference of the next lower and next higher logarithmic sines and tangents by the seconds in the given angle, and divide by 60, for the excess aforesaid.

Example —What is the sine of the angle of $15^\circ\,45'\,30''$?

$$\begin{array}{rl}
\text{log. sin. } 15{\cdot}46 = & 9{\cdot}434122 \\
\text{log. sin. } \underline{15{\cdot}45} = & \underline{9{\cdot}433675} \\
\text{diff. } 1' = & 447
\end{array}$$

$$\text{As } 1' \text{ or } 60'' : 30'' : 447 : x$$

$$447 \times \frac{30}{60} = 223 = \text{excess}$$

$$\begin{array}{rl}
\text{log. sin. } 15{\cdot}45 = & 9{\cdot}433675 \\
\text{excess} = & \underline{223} \\
\text{log. sin. } 15^\circ\,45'\,30'' = & 9{\cdot}433898
\end{array}$$

PROBLEM II.

To find the number of degrees and minutes of an arc, when the logarithmic sine or tangent is given.

If the given logarithmic sine or tangent be found, or nearly so, in the tables, observe, whether the column, in which it is found, is headed sine or tangent, at the bottom or the top of the page; if at the *top*, look there for the number of degrees, and downwards in the left

hand column, for the minutes; if, on the contrary, at the *bottom*, the degrees will be found there, and, reading upwards in the right hand column, will be found the number of minutes required, placed collaterally with the given logarithmic sine or tangent.

EXAMPLE—What are the degrees and minutes of an arc, whose sine is 9·937895?

This will be found in the second column of logarithms, being headed sines, at the *bottom* of the page, where will be found 60 degrees; then tracing, collaterally, to the right hand column of minutes, will be found 5, making the required angle 60 degrees 5 minutes.

When the given logarithmic sine is not to be found exactly in the tables, and *the calculations are carried out to seconds* say,—As the difference between the next lower and next higher logarithmic sine or tangent, : the difference between their corresponding angles in the tables, in seconds, : : the difference between the next lower logarithmic sine or tangent and the given one, : excess in seconds, of the required angle.

Add this, in the case of the sine, tangent, or secant, and *vice versa*, to the next lower angle in the table, and you obtain the required angle in seconds.

Or, multiply the logarithmic excess by 60 seconds, and divide by the logarithmic difference for the actual excess in seconds.

EXAMPLE 2.—How many degrees, minutes, and seconds, are there in the arc, whose tangent is 10·246305?

next higher tan.	$10{\cdot}246474 =$ log.	$60^\circ\ 27'$.	given tan. $10{\cdot}246305$
next lower tan.	$10{\cdot}246180 =$ log.	$60^\circ\ 26'$.	$= 10{\cdot}246180$
diff.	294	$60''$	log. excess $= 125$

As $294 : 60'' : 125 : x =$ actual excess

$$x = \frac{125 \times 60}{294} = 25 \text{ seconds which added to}$$

the lower, makes the required angle $= 60^\circ\ 26'\ 25.''$

	0°				1°				
′	Sine	Cosine	Tan.	Cotan	Sine	Cosine	Tan.	Cotan.	
0		10·000000			8·241855	9·999934	8·241921	11·758079	60
1	6·463726	10·000000	6·463726	13·536274	8·249033	9·999932	8·249102	11·750898	59
2	6·764756	10·000000	6·764756	13·235344	8·256094	9·999929	8·256165	11·743835	58
3	6·940847	10·000000	6·940847	13·059153	8·263042	9·999927	8·263115	11·736885	57
4	7·065786	10·000000	7·065786	12·934214	8·269881	9·999925	8·269956	11·730044	56
5	7·162696	10·000000	7·162696	12·837304	8·276614	9·999922	8·276691	11·723309	55
6	7·241877	9·999999	7·241878	12·758122	8·283243	9·999920	8·283323	11·716677	54
7	7·308824	9·999999	7·308825	12·691175	8·289773	9·999918	8·289856	11·710144	53
8	7·366816	9·999999	7·366817	12·633183	8·296207	9·999915	8·296292	11·703708	52
9	7·417968	9·999999	7.417970	12.582030	8·302546	9·999913	8·302634	11·697366	51
10	7·463726	9·999998	7·463727	12·536273	8·308794	9·999910	8·308884	11·691116	50
11	7·505118	9·999998	7·505120	12·494880	8·314954	9·999907	8·315046	11·684954	49
12	7·542906	9·999997	7·542909	12·457091	8·321027	9·999905	8·321122	11·678878	48
13	7·577668	9·999997	7·577672	12·422328	8·327016	9·999902	8·327114	11·672886	47
14	7·609853	9·999996	7·609857	12·390143	8·332924	9·999899	8·333025	11·666975	46
15	7·639816	9·999996	7·639820	12·360180	8·338753	9·999897	8·338856	11·661144	45
16	7·667845	9·999995	7·667849	12·332151	8·344504	9·999894	8·344610	11·655390	44
17	7·694173	9·999995	7·694179	12·305821	8·350181	9·999897	8·350289	11·649711	43
18	7·718997	9·999994	7·719003	12·280997	8·355783	9·999888	8·355895	11·644105	42
19	7·742478	9·999993	7·742484	12·257516	8·361315	9·999885	8·361430	11·638570	41
20	7·764754	9·999993	7·764761	12·235239	8·366777	9·999882	8·366895	11·633105	40
21	7·785943	9·999992	7·785951	12·214049	8·372171	9·999879	8.372292	11·627708	39
22	7·806146	9·999991	7·806155	12·193845	8·377499	9·999876	8·377622	11·622378	38
23	7·825451	9·999990	7·825460	12·174540	8·382762	9·999873	8·382889	11·617111	37
24	7·843934	9·999989	7·843944	12·156056	8·387962	9·999870	8·388092	11·611908	36
25	7·861662	9·999989	7·861674	12·138326	8·393101	9·999867	8·393234	11·606766	35
26	7·878695	9·999988	7·878708	12·121292	8·398179	9·999864	8·398315	11·601685	34
27	7·895085	9·999987	7·895099	12·104901	8·403199	9·999861	8·403338	11·596662	33
28	7·910879	9·999986	7·910894	12·089106	8·408161	9·999858	8·408304	11·591696	32
29	7·926119	9·999985	7·926134	12·073866	8·413068	9·999854	8·413213	11·586787	31
30	7·940842	9·999983	7·940858	12·059142	8·417919	9·999851	8·418068	11·581932	30
31	7·955082	9·999982	7·955100	12·044900	8·422717	9·999848	8·422869	11·577131	29
32	7·968870	9·999981	7·968889	12·031111	8·427462	9·999845	8·427618	11·572382	28
33	7·982233	9·999980	7·982253	12·017747	8·432156	9·999841	8·432315	11·567685	27
34	7·995198	9·999979	7·995219	12·004781	8·436800	9·999838	8·436962	11·563038	26
35	8·007787	9·999977	8·007809	11·992191	8·441394	9·999834	8·441560	11·558440	25
36	8·020021	9·999976	8·020044	11·979956	8·445941	9·999831	8·446110	11·553890	24
37	8·031919	9·999975	8·031945	11·968055	8·450440	9·999827	8·450613	11·549387	23
38	8·043501	9·999973	8·043527	11·956473	8·454893	9·999824	8·455070	11·544930	22
39	8·054781	9·999972	8·054809	11·945191	8·459301	9·999820	8·459481	11·540519	21
40	8·065776	9·999971	8·065806	11·934194	8·463665	9·999816	8·463849	11·536151	20
41	8·076500	9·999969	8·076531	11·923469	8·467985	9·999813	8·468172	11·531828	19
42	8·086965	9·999968	8·086997	11·913003	8·472263	9·999809	8·472454	11·527446	18
43	8·097183	9·999966	8·097217	11·902783	8·476498	9·999805	8·476693	11·523307	17
44	8·107167	9·999964	8·107203	11·892797	8·480693	9·999801	8·480892	11·519108	16
45	8·116926	9·999963	8·116963	11·883037	8·484848	9·999797	8·485050	11·514950	15
46	8·126471	9·999961	8·126510	11·873490	8·488963	9·999794	8·489170	11·510830	14
47	8·135810	9·999959	8·135851	11·864149	8·493040	9·999790	8·493250	11·506750	13
48	8·144953	9·999958	8·144996	11·855004	8·497078	9·999786	8·497293	11·502707	12
49	8·153907	9·999956	8·153952	11·846048	8·501080	9·999782	8·501298	11·498702	11
50	8·162681	9·999954	8·162727	11·837273	8·505045	9·999778	8·505267	11·494733	10
51	8·171280	9·999952	8·171328	11·828672	8·508974	9·999774	8·509200	11·490800	9
52	8·179713	9·999950	8·179763	11·820237	8·512867	9·999769	8·513098	11·486902	8
53	8·187985	9·999948	8·188036	11·811964	8·516726	9·999765	8·516961	11·483039	7
54	8·196103	9·999946	8·196156	11·803844	8·520551	9·999761	8·520790	11·479210	6
55	8·204070	9·999944	8·204126	11·795874	8·524343	9·999757	8·524586	11·475414	5
56	8·211895	9·999942	8·211953	11·788047	8·528102	9·999753	8·528349	11·471651	4
57	8·219582	9·999940	8·219641	11·780359	8·531828	9·999748	8·532080	11·467920	3
58	8·227134	9·999938	8·227195	11·772805	8·535523	9·999744	8·535779	11·464221	2
59	8·234557	9·999936	8·234621	11·765379	8·539186	9·999740	8·539447	11·460553	1
60	8·241855	9·999934	8·241921	11·758079	8·542819	9·999735	8·543084	11·456916	0
	Cosine	Sine	Cotan.	Tan.	Cosine	Sine	Cotan.	Tan.	′
	89°				88°				

	2°				3°				
′	Sine	Cosine	Tan.	Cotan.	Sine	Cosine	Tan.	Cotan	
0	8·542819	9·999735	8·543084	11·456916	8·718800	9·999404	8·719396	11·280604	60
1	8·546422	9·999731	8·550691	11·453309	8·721204	9·999398	8·721806	11·278194	59
2	8·549994	9·999726	8·550268	11·459732	8·723595	9·999391	8·724204	11·275796	58
3	8·553539	9·999722	8·553817	11·446183	8·725972	9·999384	8·726588	11·273412	57
4	8·557054	9·999717	8·557336	11·442664	8·728337	9·999378	8·728959	11·271041	56
5	8·560540	9·999713	8·560828	11·439479	8·730688	9·999371	8·731317	11·268683	55
6	8·563999	9·999708	8·564291	11·435709	8·733027	9·999364	8·733663	11·266337	54
7	8·567431	9·999704	8·567727	11·432273	8·735354	9·999357	8·735996	11·264004	53
8	8·570836	9·999699	8·571137	11·428863	8·737667	9·999350	8·738317	11·261683	52
9	8·574214	9·999694	8·574520	11·425480	8·739969	9·999343	8·740626	11·259374	51
10	8·577566	9·999689	8·577877	11·422123	8·742259	9·999336	8·742922	11·257078	50
11	8·580892	9·999685	8·581208	11·418792	8·744536	9·999329	8·745207	11·254793	49
12	8·584193	9·999680	8·584514	11·415486	8·746802	9·999322	8·747479	11·252521	48
13	8·587469	9·999675	8·587795	11·412205	8·749055	9·999315	8·749740	11·250260	47
14	8·590721	9·999670	8·591051	11·408949	8·751297	9·999308	8·751989	11·248011	46
15	8·593948	9·999665	8·594233	11·405717	8·753528	9·999301	8·754227	11·245773	45
16	8·597152	9·999660	8·597492	11·402508	8·755747	9·999294	8·756453	11·243547	44
17	8·600332	9·999655	8·600677	11·399323	8·757955	9·999287	8·758668	11·241332	43
18	8·603489	9·999650	8·603839	11·396161	8·760151	9·999279	8·760872	11·239128	42
19	8·606623	9·999645	8·606978	11·393022	8·762337	9·999272	8·763065	11·236935	41
20	8·609734	9·999640	8·610094	11·389906	8·764511	9·999265	8·765246	11·234754	40
21	8·612823	9·999635	8·613189	11·386811	8·766675	9·999257	8·767417	11·232583	39
22	8·615891	9·999629	8·616262	11·383738	8·768828	9·999250	8·769578	11·230422	38
23	8·618937	9·999624	8·619313	11·380687	8·770970	9·999242	8·771727	11·228273	37
24	8·621962	9·999619	8·622343	11·377657	8·773101	9·999235	8·773866	11·226131	36
25	8·624965	9·999614	8·625352	11·374648	8·775223	9·999227	8·775995	11·224005	35
26	8·627948	9·999608	8·628340	11·371660	8·777333	9·999220	8·778114	11·221886	34
27	8·630911	9·999603	8·631308	11·368692	8·779431	9·999212	8·780222	11·219778	33
28	8·633854	9·999597	8·634256	11·365744	8·781524	9·999205	8·782320	11·217680	32
29	8·636776	9·999592	8·637184	11·362816	8·783605	9·999197	8·784408	11·215592	31
30	8·639680	9·999586	8·640093	11·359907	8·785675	9·999189	8·786486	11·213574	30
31	8·642563	9·999581	8·642982	11·357018	8·787736	9·999181	8·788554	11·211446	29
32	8·645428	9·999575	8·645853	11·354147	8·789787	9·999174	8·790613	11·209387	28
33	8·648274	9·999570	8·648704	11·351296	8·791828	9·999166	8·792662	11·207338	27
34	8·651102	9·999564	8·651537	11·318463	8·793859	9·999158	8·794701	11·205299	26
35	8·653911	9·999558	8·654352	11·345648	8·795881	9·999150	8·796731	11·203269	25
36	8·656702	9·999553	8·657149	11·342851	8·797894	9·999142	8·798752	11·201248	24
37	8·659475	9·999547	8·659928	11·340072	8·799897	9·999134	8·800763	11·199237	23
38	8·662230	9·999541	8·662689	11·337311	8·801892	9·999126	8·802765	11·197235	22
39	8·664968	9·999535	8·665433	11·334567	8·803876	9·999118	8·804758	11·195242	21
40	8·667689	9·999529	8·668160	11·331840	8·805852	9·999110	8·806742	11·193258	20
41	8·670393	9·999524	8·670870	11·329130	8·807819	9·999102	8·808717	11·191283	19
42	8·673080	9·999518	8·673563	11·326437	8·809777	9·999094	8·810683	11·189317	18
43	8·675751	9·999512	8·676239	11·323761	8·811726	9·999086	8·812641	11·187359	17
44	8·678405	9·999506	8·678900	11·321100	8·813667	9·999077	8·814589	11·185411	16
45	8·681043	9·999500	8·681544	11·318456	8·815599	9·999069	8·816529	11·183471	15
46	8·683665	9·999493	8·684172	11·315828	8·817522	9·999061	8·818461	11·181539	14
47	8·686272	9·999487	8·686784	11·313216	8·819436	9·999053	8·820384	11·179616	13
48	8·688863	9·999481	8·689381	11·310619	8·821343	9·999044	8·822298	11·177702	12
49	8·691438	9·999475	8·691963	11·308037	8·823240	9·999036	8·824205	11·175795	11
50	8·693998	9·999469	8·694529	11·305471	8·825130	9·999027	8·826103	11·173897	10
51	8·696543	9·999463	8·697081	11·302919	8·827011	9·999019	8·827992	11·172008	9
52	8·699073	9·999456	8·699617	11·300383	8·828884	9·999010	8·829874	11·170126	8
53	8·701589	9·999450	8·702139	11·297861	8·830749	9·999002	8·831748	11·168252	7
54	8·704090	9·999443	8·704646	11·295354	8·832607	9·998993	8·833613	11·166387	6
55	8·706577	9·999437	8·707140	11·292860	8·834456	9·998984	8·835471	11·164529	5
56	8·709049	9·999431	8·709618	11·290382	8·836297	9·998976	8·837321	11·162679	4
57	8·711507	9·999424	8·712083	11·287917	8·838130	9·998967	8·839163	11·160837	3
58	8·713952	9·999418	8·714534	11·285466	8·839956	9·998958	8·840998	11·159002	2
59	8·716383	9·999411	8·716972	11·283028	8·841774	9·998950	8·842825	11·157175	1
60	8·718800	9·999404	8·719396	11·280604	8·843585	9·998941	8·844644	11·155356	0
	Cosine	Sine	Cotan.	Tan.	Cosine	Sine	Cotan.	Tan.	′
	87°				86°				

	4°				5°				
′	Sine	Cosine	Tan.	Cotan.	Sine	Cosine	Tan.	Cotan.	
0	8·843585	9·998941	8·844644	11·155356	8·940296	9·998344	8·941952	11·058048	60
1	8·845387	9·998932	8·846455	11·153545	8·941738	9·998333	8·943404	11·056596	59
2	8·847183	9·998923	8·848260	11·151740	8·943174	9·998322	8·944852	11·055148	58
3	8·848971	9·998914	8·850057	11·149943	8·944606	9·998311	8·946295	11·053705	57
4	8·850751	9·998905	8·851846	11·148154	8·946034	9·998300	8·947734	11·052266	56
5	8·852525	9·998896	8·853628	11·146372	8·947456	9·998289	8·949168	11·050832	55
6	8·854291	9·998887	8·855403	11·144597	8·948874	9·998277	8·950597	11·049403	54
7	8·856049	9·998878	8·857171	11·142829	8·950287	9·998266	8·952021	11·047979	53
8	8·857801	9·998869	8·858932	11·141068	8·951696	9·998255	8·953441	11·046559	52
9	8·859546	9·998860	8·860686	11·139314	8·953100	9·998243	8·954856	11·045144	51
10	8·861283	9·998851	8·862433	11·137567	8·954499	9·998232	8·956267	11·043733	50
11	8·863014	9·998841	8·864173	11·135827	8·955894	9·998220	8·957651	11·042326	49
12	8·864738	9·998832	8·865906	11·134094	8·957281	9·998209	8·959075	11·040925	48
13	8·866455	9·998823	8·867632	11·132368	8·958670	9·998197	8·960473	11·039527	47
14	8·868165	9·998813	8·869351	11·130649	8·960052	9·998186	8·961866	11·038134	46
15	8·869868	9·998804	8·871064	11·128936	8·961429	9·998174	8·963255	11·036745	45
16	8·871565	9·998795	8·872770	11·127230	8·962801	9·998163	8·964639	11·035361	44
17	8·873255	9·998785	8·874469	11·125531	8·964170	9·998152	8·966019	11·033981	43
18	8·874938	9·998776	8·876162	11·123838	8·965534	9·998139	8·967394	11·032606	42
19	8·876615	9·998766	8·877849	11·122151	8·966893	9·998128	8·968766	11·031234	41
20	8·878285	9·998757	8·879529	11·120471	8·968249	9·998116	8·970133	11·029867	40
21	8·879949	9·998747	8·881202	11·118798	8·969600	9·998104	8·971496	11·028504	39
22	8·881607	9·998738	8·882869	11·117131	8·970947	9·998092	8·972855	11·027145	38
23	8·883258	9·998728	8·884530	11·115470	8·972289	9·998080	8·974209	11·025791	37
24	8·884903	9·998718	8·886185	11·113815	8·973628	9·998068	8·975560	11·024440	36
25	8·886542	9·998708	8·887833	11·112167	8·974962	9·998056	8·976906	11·023094	35
26	8·888174	9·998699	8·889476	11·110524	8·976293	9·998044	8·978248	11·021752	34
27	8·889801	9·998689	8·891112	11·108888	8·977619	9·998032	8·979586	11·020414	33
28	8·891421	9·998679	8·892742	11·107258	8·978941	9·998020	8·980921	11·019079	32
29	8·893035	9·998669	8·894366	11·105634	8·980259	9·998008	8·982251	11·017749	31
30	8·894643	9·998659	8·895984	11·104016	8·981573	9·998996	8·983577	11·016423	30
31	8·896246	9·998649	8·897596	11·102404	8·982883	9·997984	8·984899	11·015101	29
32	8·897842	9·998639	8·899203	11·100797	8·984189	9·997972	8·986217	11·013783	28
33	8·899432	9·998629	8·900803	11·099197	8·985491	9·997959	8·987532	11·012468	27
34	8·901917	9·998619	8·902398	11·097602	8·986789	9·997947	8·988842	11·011158	26
35	8·902596	9·998609	8·903987	11·096013	8·988083	9·997935	8·990149	11·009851	25
36	8·904169	9·998599	8·905579	11·094430	8·989374	9·997922	8·991551	11·008549	24
37	8·905736	9·998589	8·907147	11·092853	8·990660	9·997910	8·992750	11·007250	23
38	8·907297	9·998578	8·908719	11·091281	8·991913	9·997897	8·994045	11·005955	22
39	8·908853	9·998568	8·910285	11·089715	8·993222	9·997885	8·995337	11·004663	21
40	8·910404	9·998558	8·911846	11·088154	8·994497	9·997872	8·996624	11·003376	20
41	8·911949	9·998548	8·913401	11·086599	8·995768	9·997860	8·997908	11·002092	19
42	8·913488	9·998537	8·914951	11·085049	8·997036	9·997847	8·999188	11·000812	18
43	8·915022	9·998527	8·916495	11·083505	8·998299	9·997835	9·000465	10·999535	17
44	8·916550	9·998516	8·918034	11·081966	8·999560	9·997822	9·001738	10·998262	16
45	8·918073	9·998506	8·919568	11·080432	9·000816	9·997809	9·003007	10·996993	15
46	8·919591	9·998495	8·921096	11·078904	9·002069	9·997797	9·004272	10·995728	14
47	8·921103	9·998485	8·922619	11·077381	9·003318	9·997784	9·005534	10·994466	13
48	8·922610	9·998474	8·924136	11·075864	9·004563	9·997771	9·006792	10·993208	12
49	8·924112	9·998464	8·925649	11·074351	9·005805	9·997758	9·008047	10·991953	11
50	8·925609	9·998453	8·927156	11·072844	9·007044	9·997745	9·009298	10·990702	10
51	8·927100	9·998442	8·928658	11·071342	9·008278	9·997732	9·010546	10·989454	9
52	8·928587	9·998431	8·930155	11·069845	9·009510	9·997719	9·011790	10·988210	8
53	8·930068	9·998421	8·931647	11·068353	9·010737	9·997706	9·013031	10·986969	7
54	8·931544	9·998410	8·933134	11·066866	9·011962	9·997693	9·014268	10·985732	6
55	8·933015	9·998399	8·934616	11·065384	9·013182	9·997680	9·015502	10·984498	5
56	8·934481	9·998388	8·936093	11·063907	9·014400	9·997667	9·016732	10·983268	4
57	8·935942	9·998377	8·937565	11·062435	9·015613	9·997654	9·017959	10·982041	3
58	8·937398	9·998366	8·939032	11·060968	9·016824	9·997641	9·019183	10·980817	2
59	8·938850	9·998355	8·940494	11·059506	9·018031	9·997628	9·020403	10·979597	1
60	8·940296	9·998344	8·941952	11·058048	9·019235	9·997614	9·021620	10·978380	0
	Cosine	Sine	Cotan.	Tan.	Cosine	Sine	Cotan.	Tan.	′
	85°				84°				

	6°				7°				
′	Sine	Cosine	Tan.	Cotan.	Sine	Cosine	Tan.	Cotan.	
0	9·019235	9·997614	9·021620	10·978380	9·085894	9·996751	9·089144	10·910856	60
1	9·020435	9·997601	9·022834	10·977166	9·086922	9·996735	9·090187	10·909813	59
2	9·021632	9·997588	9·024044	10·975956	9·087947	9·996720	9·091228	10·908772	58
3	9·022825	9·997574	9·025251	10·974749	9·088970	9·996704	9·092266	10·907734	57
4	9·024016	9·997561	9·026455	10·973545	9·089990	9·996688	9·093302	10·906698	56
5	9·025203	9·997547	9·027655	10·972345	9·091008	9·996673	9·094336	10·905664	55
6	9·026386	9·997534	9·028852	10·971148	9·092024	9·996657	9·095367	10·904633	54
7	9·027567	9·997520	9·030046	10·969954	9·093037	9·996641	9·096395	10·903605	53
8	9·028744	9·997507	9·031237	10·968763	9·094047	9·996625	9·097422	10·902578	52
9	9·029918	9·997493	9·032425	10·967575	9·095056	9·996610	9·098446	10·901554	51
10	9·031089	9·997480	9·033609	10·966391	9·096062	9·996594	9·999468	10·900532	50
11	9·032257	9·997466	9·034791	10·965209	9·097065	9·996578	9·090487	10·899513	49
12	9·033421	9·997452	9·035969	10·964031	9·098066	9·996562	9·091504	10·898496	48
13	9·034582	9·997439	9·037144	10·962856	9·099065	9·996546	9·102519	10·897481	47
14	9·035741	9·997425	9·038316	10·961684	9·100062	9·996530	9·103532	10·896468	46
15	9·036896	9·997411	9·039485	10·960515	9·101056	9·996514	9·104542	10·895458	45
16	9·038048	9·997397	9·040651	10·959349	9·102048	9·996498	9·105550	10·894450	44
17	9·039197	9·997383	9·041813	10·958187	9·103037	9·996482	9·106556	10·893444	43
18	9·040342	9·997369	9·042973	10·957087	9·104025	9·996465	9·107559	10·892441	42
19	9·041485	9·997355	9·044130	10·955870	9·105010	9·996449	9·108560	10·891440	41
20	9·042625	9·997341	9·045284	10·954716	9·105992	9·996433	9·109559	10·890441	40
21	9·043762	9·997327	9·046434	10·953566	9·106973	9·996417	9·110556	10·889444	39
22	9·044895	9·997313	9·047582	10·952418	9·107951	9·996400	9·111551	10·888449	38
23	9·046026	9·997299	9·048727	10·951273	9·108927	9·996384	9·112543	10·887457	37
24	9·047154	9·997285	9·049869	10·950131	9·109901	9·996368	9·113533	10·886467	36
25	9·048277	9·997271	9·051008	10·948992	9·110873	9·996351	9·114521	10·885479	35
26	9·049400	9·997257	9·052144	10·947856	9·111842	9·996335	9·115507	10·884493	34
27	9·050519	9·997242	9·053277	10·946723	9·112809	9·996318	9·116491	10·883509	33
28	9·051635	9·997228	9·054407	10·945593	9·113774	9·996302	9·117472	10·882528	32
29	9·052749	9·997214	9·055535	10·944465	9·114737	9·996285	9·118452	10·881548	31
30	9·053859	9·997199	9·056659	10·943341	9·115698	9·996269	9·119429	10·880571	30
31	9·054966	9·997185	9·057781	10·942219	9·116656	9·996252	9·120404	10·879596	29
32	9·056071	9·997170	9·058900	10·941100	9·117613	9·996235	9·121377	10·878623	28
33	9·057172	9·997156	9·060016	10·939984	9·118567	9·996219	9·122348	10·877652	27
34	9·058271	9·997141	9·061130	10·938870	9·119519	9·996202	9·123317	10·876683	26
35	9·059367	9·997127	9·062244	10·937760	9·120469	9·996185	9·124284	10·875716	25
36	9·060460	9·997112	9·063348	10·936652	9·121417	9·996168	9·125249	10·874751	24
37	9·061551	9·997098	9·064453	10·935547	9·122362	9·996151	9·126211	10·873789	23
38	9·062639	9·997083	9·065556	10·934444	9·123306	9·996134	9·127172	10·872828	22
39	9·063724	9·997068	9·066655	10·933345	9·124248	9·996117	9·128130	10·871870	21
40	9·064806	9·997053	9·067752	10·932248	9·125187	9·996100	9·129087	10·870913	20
41	9·065885	9·997039	9·068846	10·931154	9·126125	9·996083	9·130041	10·869959	19
42	9·066962	9·997024	9·069938	10·930062	9·127060	9·996066	9·130994	10·869006	18
43	9·068036	9·997009	9·071027	10·928073	9·127993	9·996049	9·131944	10·868056	17
44	9·069107	9·996994	9·072113	10·927887	9·128925	9·996032	9·132893	10·867107	16
45	9·070176	9·996979	9·073197	10·926803	9·129854	9·996015	9·133839	10·866161	15
46	9·071242	9·996964	9·074278	10·925722	9·130781	9·995998	9·134784	10·865216	14
47	9·072306	9·996949	9·075356	10·924644	9·131706	9·995980	9·135726	10·864274	13
48	9·073366	9·996934	9·076432	10·923568	9·132630	9·995963	9·136667	10·863333	12
49	9·074424	9·996919	9·077505	10·922495	9·133551	9·995946	9·137605	10·862395	11
50	9·075480	9·996904	9·078576	10·921424	9·134470	9·995928	9·138542	10·861458	10
51	9·076533	9·996889	9·079644	10·920356	9·135387	9·995911	9·139476	10·860524	9
52	9·077583	9·996874	9·080710	10·919290	9·136303	9·995894	9·140409	10·859591	8
53	9·078631	9·996858	9·081773	10·918227	9·137216	9·995876	9·141340	10·858660	7
54	9·079676	9·996843	9·082833	10·917167	9·138128	9·995859	9·142269	10·857731	6
55	9·080719	9·996828	9·083891	10·916109	9·139037	9·995841	9·143196	10·856804	5
56	9·081759	9·996812	9·084947	10·915053	9·139944	9·995823	9·144121	10·855879	4
57	9·082797	9·996797	9·086000	10·914000	9·140850	9·995806	9·145044	10·854956	3
58	9·083832	9·996782	9·087050	10·912950	9·141754	9·995788	9·145966	10·854034	2
59	9·084864	9·996766	9·088098	10·911902	9·142655	9·995771	9·146885	10·853115	1
60	9·085894	9·996751	9·089144	10·910856	9·143555	9·995753	9·147803	10·852197	0
	Cosine	Sine	Cotan.	Tan.	Cosine	Sine	Cotan.	Tan.	′
	83°				82°				

	8°				9°				
′	Sine	Cosine	Tan.	Cotan.	Sine	Cosine	Tan.	Cotan.	
0	9·143555	9·995753	9·147803	10·852197	9·194332	9·994620	9·199713	10·800287	60
1	9·144453	9·995735	9·148718	10·851282	9·195129	9·994600	9·200529	10·799471	59
2	9·145349	9·995717	9·149632	10·850368	9·195925	9·994580	9·201345	10·798655	58
3	9·146243	9·995699	9·150544	10·849456	9·196719	9·994560	9·202159	10·797841	57
4	9·147136	9·995681	9·151454	10·848546	9·197511	9·994540	9·202971	10·797029	56
5	9·148026	9·995664	9·152363	10·847637	9·198302	9·994519	9·203782	10·796218	55
6	9·148915	9·995646	9·153269	10·846731	9·199091	9·994499	9·204592	10·795408	54
7	9·149802	9·995628	9·154174	10·845826	9·199879	9·994479	9·205400	10·794600	53
8	9·150686	9·995610	9·155077	10·844923	9·200666	9·994459	9·206207	10·793793	52
9	9·151569	9·995591	9·155978	10·844022	9·201451	9·994438	9·207013	10·792987	51
10	9·152451	9·995573	9·156877	10·843123	9·202234	9·994418	9·207817	10·792183	50
11	9·153330	9·995555	9·157775	10·842225	9·203017	9·994398	9·208619	10·791381	49
12	9·154208	9·995537	9·158671	10·841329	9·203797	9·994377	9·209420	10·790580	48
13	9·155083	9·995519	9·159565	10·840435	9·204577	9·994357	9·210220	10·789780	47
14	9·155957	9·995501	9·160457	10·839543	9·205354	9·994336	9·211018	10·788982	46
15	9·156830	9·995482	9·161347	10·838653	9·206131	9·994316	9·211815	10·788185	45
16	9·157700	9·995464	9·162236	10·837764	9·206906	9·994295	9·212611	10·787389	44
17	9·158569	9·995446	9·163123	10·836877	9·207679	9·994274	9·213405	10·786595	43
18	9·159435	9·995427	9·164008	10·835992	9·208452	9·994254	9·214198	10·785802	42
19	9·160301	9·995409	9·164892	10·835108	9·209222	9·994233	9·214989	10·785011	41
20	9·161164	9·995390	9·165774	10·834226	9·209992	9·994212	9·215780	10·784220	40
21	9·162025	9·995372	9·166654	10·833346	9·210760	9·994191	9·216568	10·783432	39
22	9·162885	9·995353	9·167532	10·832468	9·211526	9·994171	9·217356	10·782644	38
23	9·163743	9·995334	9·168409	10·831591	9·212291	9·994150	9·218142	10·781858	37
24	9·164600	9·995316	9·169284	10·830716	9·213055	9·994129	9·218926	10·781074	36
25	9·165454	9·995297	9·170157	10·829843	9·213818	9·994108	9·219710	10·780290	35
26	9·166307	9·995278	9·171029	10·828971	9·214579	9·994087	9·220492	10·779508	34
27	9·167159	9·995260	9·171899	10·828101	9·215338	9·994066	9·221272	10·778728	33
28	9·168008	9·995241	9·172767	10·827233	9·216097	9·994045	9·222052	10·777948	32
29	9·168856	9·995222	9·173634	10·826366	9·216854	9·994024	9·222830	10·777170	31
30	9·169702	9·995203	9·174499	10·825501	9·217609	9·994003	9·223607	10·776393	30
31	9·170547	9·995184	9·175362	10·824638	9·218363	9·993982	9·224382	10·775618	29
32	9·171389	9·995165	9·176224	10·823776	9·219116	9·993960	9·225156	10·774844	28
33	9·172230	9·995146	9·177084	10·822916	9·219868	9·993939	9·225929	10·774071	27
34	9·173070	9·995127	9·177942	10·822058	9·220618	9·993918	9·226700	10·773300	26
35	9·173908	9·995108	9·178799	10·821201	9·221367	9·993897	9·227471	10·772529	25
36	9·174744	9·995089	9·179655	10·820345	9.222115	9·993875	9·228239	10·771761	24
37	9·175578	9·995070	9·180508	10·819492	9·222861	9·993854	9·229007	10·770993	23
38	9·176411	9·995051	9·181360	10·818640	9·223606	9·993832	9·229773	10·770227	22
39	9·177242	9·995032	9·182211	10·817789	9·224349	9·993811	9·230539	10·769461	21
40	9·178072	9·995013	9·183059	10·816941	9·225092	9·993789	9·231302	10·768698	20
41	9·178900	9·994993	9·183907	10·816093	9·225833	9·993768	9·232065	10·767935	19
42	9·179726	9·994974	9·184752	10·815248	9·226573	9·993746	9·232826	10·767174	18
43	9·180551	9·994955	9·185597	10·814403	9·227311	9·993725	9·233586	10·766414	17
44	9·181374	9·994935	9·186439	10·813561	9·228048	9·993703	9·234345	10·765655	16
45	9·182196	9·994916	9·187280	10·812720	9·228784	9·993681	9·235103	10·764897	15
46	9·183016	9·994896	9·188120	10·811880	9·229518	9·993660	9·235859	10·764141	14
47	9·183834	9·994877	9·188958	10·811042	9·230252	9·993638	9·236614	10·763386	13
48	9·184651	9·994857	9·189794	10·810206	9·230984	9·993616	9·237368	10·762632	12
49	9·185466	9·994838	9·190629	10·809371	9·231715	9·993594	9·238120	10·761880	11
50	9·186280	9·994818	9·191462	10·808538	9·232444	9·993572	9·238872	10·761128	10
51	9·187092	9·994798	9·192294	10·807706	9·233172	9·993550	9·239622	10·760378	9
52	9·187903	9·994779	9·193124	10·806876	9·233899	9·993528	9·240371	10·759629	8
53	9·188712	9·994759	9·193953	10·806047	9·234625	9·993506	9·241118	10·758882	7
54	9·189519	9·994739	9·194780	10·805220	9·235349	9·993484	9·241865	10·758135	6
55	9·190325	9·994720	9·195606	10·804394	9·236073	9·993462	9·242610	10·757390	5
56	9·191130	9·994700	9·196430	10·803570	9·236795	9·993440	9·243354	10·756646	4
57	9·191933	9·994680	9·197253	10·802747	9·237515	9·993418	9·244097	10·755903	3
58	9·192734	9·994660	9·198074	10·801926	9·238235	9·993396	9·244839	10·755161	2
59	9·193534	9·994640	9·198894	10·801106	9·238953	9·993374	9·245579	10·754421	1
60	9·194332	9·994620	9·199713	10·800287	9·239670	9·993351	9·246319	10·753681	0
	Cosine	Sine	Cotan.	Tan.	Cosine	Sine	Cotan.	Tan.	′
	81°				80°				

	10°				11°				
′	Sine	Cosine	Tan.	Cotan.	Sine	Cosine	Tan.	Cotan.	
0	9·239670	9·993351	9·246319	10·753681	9·280599	9·991947	9·288652	10·711348	60
1	9·240386	9·993329	9·247057	10·752943	9·281248	9·991922	9·289326	10·710674	59
2	9·241101	9·993307	9·247794	10·752206	9·281897	9·991897	9·289999	10·710001	58
3	9·241814	9·993284	9·248530	10·751470	9·282544	9·991873	9·290671	10·709329	57
4	9·242526	9·993262	9·249264	10·750736	9·283190	9·991848	9·291342	10·708658	56
5	9·243237	9·993240	9·249998	10·750002	9·283836	9·991823	9·292013	10·707987	55
6	9·243947	9·993217	9·250730	10·749270	9·284480	9·991799	9·292682	10·707318	54
7	9·244656	9·993195	9·251461	10·748539	9·285124	9·991774	9·293350	10·706650	53
8	9·245363	9·993172	9·252191	10·747809	9·285766	9·991749	9·294017	10·705983	52
9	9·246069	9·993149	9·252920	10·747080	9·286408	9·991724	9·294684	10·705316	51
10	9·246775	9·993127	9·253648	10·746352	9·287048	9·991699	9·295349	10·704651	50
11	9·247478	9·993104	9·254374	10·745626	9·287688	9·991674	9·296013	10·703987	49
12	9·248181	9·993081	9·255100	10·744900	9·288326	9·991649	9·296677	10·703323	48
13	9·248883	9·993059	9·255824	10·744176	9·288964	9·991624	9·297339	10·702661	47
14	9·249583	9·993036	9·256547	10·743453	9·289600	9·991599	9·298001	10·701999	46
15	9·250282	9·993013	9·257269	10·742731	9·290236	9·991574	9·298662	10·701338	45
16	9·250980	9·992990	9·257990	10·742010	9·290870	9·991549	9·299322	10·700678	44
17	9·251677	9·992967	9·258710	10·741290	9·291504	9·991524	9·299980	10·700020	43
18	9·252373	9·992944	9·259429	10·740571	9·292137	9·991498	9·300638	10·699362	42
19	9·253067	9·992921	9·260146	10·739854	9·292768	9·991473	9·301295	10·698705	41
20	9·253761	9·992898	9·260863	10·739137	9·293399	9·991448	9·301951	10·698049	40
21	9·254453	9·992875	9·261578	10·738422	9·294029	9·991422	9·302607	10·697393	39
22	9·255144	9·992852	9·262292	10·737708	9·294658	9·991397	9·303261	10·696739	38
23	9·255834	9·992829	9·263005	10·736995	9·295286	9·991372	9·303914	10·696086	37
24	9·256523	9·992806	9·263717	10·736283	9·295913	9·991346	9·304567	10·695433	36
25	9·257211	9·992783	9·264428	10·735572	9·296539	9·991321	9·305218	10·694782	35
26	9·257898	9·992759	9·265138	10·734862	9·297164	9·991295	9·305869	10·694131	34
27	9·258583	9·992736	9·265847	10·734153	9·297788	9·991270	9·306519	10·693481	33
28	9·259268	9·992713	9·266555	10·733445	9·298412	9·991244	9·307168	10·692832	32
29	9·259951	9·992690	9·267261	10·732739	9·299034	9·991218	9·307816	10·692184	31
30	9·260633	9·992666	9·267967	10·732033	9·299655	9·991193	9·308463	10·691537	30
31	9·261314	9·992643	9·268671	10·731329	9·300276	9·991167	9·309109	10·690891	29
32	9·261994	9·992619	9·269375	10·730625	9·300895	9·991141	9·309754	10·690246	28
33	9·262673	9·992596	9·270077	10·729923	9·301514	9·991115	9·300399	10·689601	27
34	9·263351	9·992572	9·270779	10·729221	9·302132	9·991090	9·311042	10·688958	26
35	9·264027	9·992549	9·271479	10·728521	9·302748	9·991064	9·311685	10·688315	25
36	9·264703	9·992525	9·272178	10·727822	9·303364	9·991038	9·312327	10·687673	24
37	9·265377	9·992501	9·272876	10·727124	9·303979	9·991012	9·312968	10·687032	23
38	9·266051	9·992478	9·273573	10·726427	9·304593	9·990986	9·313608	10·686392	22
39	9·266723	9·992454	9·274269	10·725731	9·305207	9·990960	9·314247	10·685753	21
40	9·267395	9·992430	9·274964	10·725036	9·305819	9·990934	9·314885	10·685115	20
41	9·268065	9·992406	9·275658	10·724342	9·306430	9·990908	9·315523	10·684477	19
42	9·268734	9·992382	9·276351	10·723649	9·307041	9·990882	9·316159	10·683841	18
43	9·269402	9·992359	9·277043	10·722957	9·307650	9·990855	9·316795	10·683205	17
44	9·270069	9·992335	9·277734	10·722266	9·308259	9·990829	9·317430	10·682570	16
45	9·270735	9·992311	9·278424	10·721576	9·308867	9·990803	9·318064	10·681936	15
46	9·271400	9·992287	9·279113	10·720887	9·309474	9·990777	9·318697	10·681303	14
47	9·272064	9·992263	9·279801	10·720199	9·310080	9·990750	9·319330	10·680670	13
48	9·272726	9·992239	9·280488	10·719512	9·310685	9·990724	9·319961	10·680039	12
49	9·273388	9·992214	9·282174	10·718826	9·311289	9·990697	9·320592	10·679408	11
50	9·274049	9·992190	9·281858	10·718142	9·311893	9·990671	9·321222	10·678778	10
51	9·274708	9·992106	9·282542	10·717458	9·312495	9·990645	9·321851	10·678149	9
52	9·275367	9·992142	9·283225	10·716775	9·313097	9·990618	9·322479	10·677521	8
53	9·276025	9·992118	9·283907	10·716093	9·313698	9·990591	9·323106	10·676894	7
54	9·276681	9·902093	9·284588	10·715412	9·314297	9·990565	9·323733	10·676267	6
55	9·277337	9·992069	9·285268	10·714732	9·314897	9·990538	9·324358	10·675642	5
56	9·277991	9·992044	9·285947	10·714053	9·315495	9·990511	9·324983	10·675017	4
57	9·278645	9·992020	9·286624	10·713376	9·316092	9·990485	9·325607	10·674393	3
58	9·279297	9·991996	9·287301	10·712699	9·316689	9·990458	9·326231	10·673769	2
59	9·279948	9·991971	9·287977	10·712023	9·317284	9·990431	9·326853	10·673147	1
60	9·280599	9·991947	9·288652	10·711348	9·317879	9·990404	9·327475	10·672525	0
	Cosine	Sine	Cotan.	Tan.	Cosine	Sine	Cotan.	Tan.	′
	79°				78°				

	12°				13°				
′	Sine	Cosine	Tan.	Cotan.	Sine	Cosine	Tan.	Cotan.	
0	9·317879	9·990404	9·327475	10·672525	9·352088	9·988724	9·363364	10·636636	60
1	9·318473	9·990378	9·328095	10·671905	9·352635	9·988695	9·363940	10·636060	59
2	9·319066	9·990351	9·328715	10·671285	9·353181	9·988666	9·364515	10·635485	58
3	9·319658	9·990324	9·329334	10·670666	9·353726	9·988636	9·365090	10·634910	57
4	9·320249	9·990297	9·329953	10·670047	9·354271	9·988607	9·365664	10·634336	56
5	9·320840	9·990270	9·330570	10·669430	9·354815	9·988578	9·366237	10·633763	55
6	9·321430	9·990243	9·331187	10·668813	9·355358	9·988548	9·366810	10·633190	54
7	9·322019	9·990215	9·331803	10·668197	9·355901	9·988519	9·367382	10·632618	53
8	9·322607	9·990185	9·332418	10·667582	9·356443	9·988489	9·367953	10·632047	52
9	9·323194	9·990161	9·333033	10·666967	9·356984	9·988460	9·368524	10·631476	51
10	9·323780	9·990134	9·333646	10·666354	9·357524	9·988430	9·369094	10·630906	50
11	9·324366	9·990107	9·334259	10·665741	9·358064	9·988401	9·369663	10·630337	49
12	9·324950	9·990079	9·334871	10·665129	9·358603	9·988371	9·370232	10·629768	48
13	9·325534	9·990052	9·335482	10·664518	9·359141	9·988342	9·370799	10·629201	47
14	9·326117	9·990025	9·336093	10·663907	9·359678	9·988312	9·371367	10·628633	46
15	9·326700	9·989997	9·336702	10·663298	9·360215	9·988282	9·371933	10·628067	45
16	9·327281	9·989970	9·337311	10·662689	9·360752	9·988252	9·372499	10·627501	44
17	9·327862	9·989942	9·337919	10·662081	9·361287	9·988223	9·373064	10·626936	43
18	9·328442	9·989915	9·338527	10·661473	9·361822	9·988193	9·373629	10·626371	42
19	9·329021	9·989887	9·339133	10·660867	9·362356	9·988163	9·374193	10·625807	41
20	9·329599	9·989860	9·339739	10·660261	9·362889	9·988133	9·374756	10·625244	40
21	9·330176	9·989832	9·340344	10·659656	9·363422	9·988103	9·375319	10·624681	39
22	9·330753	9·989804	9·340948	10·659052	9·363954	9·988073	9·375881	10·624119	38
23	9·331329	9·989777	9·341552	10·658448	9·364485	9·988043	9·376442	10·623558	37
24	9·331903	9·989749	9·342155	10·657845	9·365016	9·988013	9·377003	10·622997	36
25	9·332478	9·989721	9·342757	10·657243	9·365546	9·987983	9·377563	10·622437	35
26	9·333051	9·989693	9·343358	10·656642	9·366075	9·987953	9·378122	10·621878	34
27	9·333624	9·989665	9·343958	10·656042	9·366604	9·987922	9·378681	10·621319	33
28	9·334195	9·989637	9·344558	10·655442	9·367131	9·987892	9·379239	10·620761	32
29	9·334767	9·989610	9·345157	10·654843	9·367659	9·987862	9·379797	10·620203	31
30	9·335337	9·989582	9·345755	10·654245	9·368185	9·987832	9·380354	10·619646	30
31	9·335906	9·989553	9·346353	10·653647	9·368711	9·987801	9·380910	10·619090	29
32	9·336475	9·989525	9·346949	10·953051	9·369236	9·987771	9·381466	10·618534	28
33	9·337043	9·989497	9·347545	10·652455	9·369761	9·987740	9·382020	10·617980	27
34	9·337610	9·989469	9·348141	10·651859	9·370285	9·987710	9·382575	10·617425	26
35	9·338176	9·989441	9·348735	10·651265	9·370808	9·987679	9·383129	10·616871	25
36	9·338742	9·989413	9·349329	10·650671	9·371330	9·987649	9·383682	10·616318	24
37	9·339307	9·989385	9·349922	10·650078	9·371852	9·987618	9·384234	10·615766	23
38	9·339871	9·989356	9·350514	10·649486	9·372373	9·987588	9·384786	10·615214	22
39	9·340434	9·989328	9·351106	10·648894	9·372894	9·987557	9·385337	10·614663	21
40	9·340996	9·989300	9·351697	10·648303	9·373414	9·987526	9·385888	10·614112	20
41	9·341558	9·989271	9·352287	10·647713	9·373933	9·987496	9·386438	10·613562	19
42	9·342119	9·989243	9·352876	10·647124	9·374452	9·987465	9·386987	10·613013	18
43	9·342679	9·989214	9·353465	10·646535	9·374970	9·987434	9·387536	10·612464	17
44	9·343239	9·989186	9·354053	10·645947	9·375487	9·987403	9·388084	10·611916	16
45	9·343797	9·989157	9·354640	10·645360	9·376003	9·987372	9·388631	10·611369	15
46	9·344355	9·989128	9·355227	10·644773	9·376519	9·987341	9·389178	10·610822	14
47	9·344912	9·980100	9·356813	10·644187	9·377035	9·987310	9·389724	10·610276	13
48	9·345469	9·989071	9·356398	10·643602	9·377549	9·987279	9·390270	10·609730	12
49	9·346024	9·989042	9·356982	10·643018	9·378063	9·987248	9·390815	10·609185	11
50	9·346579	9·989014	9·357566	10·642434	9·378577	9·987217	9·391360	10·608640	10
51	9·347134	9·988985	9·358149	10·641851	9·379089	9·987186	9·391903	10·608097	9
52	9·347687	9·988956	9·358731	10·641269	9·379601	9·987155	9·392447	10·607553	8
53	9·348240	9·988927	9·359313	10·640687	9·380113	9·987124	9·392989	10·607011	7
54	9·348792	9·988898	9·359893	10·640107	9·380624	9·987092	9·393531	10·606469	6
55	9·349343	9·988869	9·360474	10·639526	9·381134	9·987061	9·394073	10·605927	5
56	9·349893	9·988840	9·361053	10·638947	9·381642	9·987030	9·394614	10·605386	4
57	9·350443	9·983811	9·361632	10·638368	9·382152	9·986998	9·395154	10·604846	3
58	9·350992	9·988782	9·362210	10·637790	9·382661	9·986967	9·395694	10·604306	2
59	9·351540	9·988753	9·362787	10·637213	9·383168	9·986936	9·396233	10·603767	1
60	9·352088	9·988724	9·363364	10·636636	9·383675	9·986904	9·396771	10·603229	0
	Cosine	Sine	Cotan.	Tan.	Cosine	Sine	Cotan.	Tan.	′
	77°				76°				

	14°				15°				
′	Sine	Cosine	Tan.	Cotan.	Sine	Cosine	Tan.	Cotan.	
0	9·383675	9·986904	9·396771	10·603229	9·412996	9·984944	9·428052	10·571948	60
1	9·384182	9·986873	9·397309	10·602691	9·413467	9·984910	9·428558	10·571442	59
2	9·384687	9·986841	9·397846	10·602154	9·413938	9·984876	9·429062	10·570938	58
3	9·385192	9·986809	9·398383	10·601617	9·414408	9·984842	9·429566	10·570434	57
4	9·385697	9·986778	9·398919	10·601081	9·414878	9·984808	9·430070	10·569930	56
5	9·386201	9·986746	9·399455	10·600545	9·415347	9·984774	9·430573	10·569427	55
6	9·386704	9·986714	9·399990	10·600010	9·415815	9·984740	9·431075	10·568925	54
7	9·387207	9·986683	9·400524	10·599476	9·416283	9·984706	9·431577	10·568423	53
8	9·387709	9·986651	9·401058	10·598942	9·416751	9·984672	9·432079	10·567921	52
9	9·388210	9·986619	9·401591	10·598409	9·417217	9·984638	9·432580	10·567420	51
10	9·388711	9·986587	9·402124	10·597876	9·417684	9·984603	9·433080	10·566920	50
11	9·389211	9·986555	9·402656	10·597344	9·418150	9·984569	9·433580	10·566420	49
12	9·389711	9·986523	9·403187	10·596813	9·418615	9·984535	9·434080	10·565920	48
13	9·390210	9·986491	9·403718	10·596282	9·419079	9·984500	9·434579	10·565421	47
14	9·390708	9·986459	9·404249	10·595751	9·419544	9·984466	9·435078	10·564922	46
15	9·391206	9·986427	9·404778	10·595222	9·420007	9·984432	9·435576	10·564424	45
16	9·391703	9·986395	9·405308	10·594692	9·420470	9·984397	9·436073	10·563927	44
17	9·392199	9·986363	9·405836	10·594164	9·420933	9·984363	9·436570	10·563430	43
18	9·392695	9·986331	9·406364	10·593636	9·421395	9·984328	9·437067	10·562933	42
19	9·393191	9·986299	9·406892	10·593108	9·421857	9·984294	9·437563	10·562437	41
20	9·393685	9·986266	9·407419	10·592581	9·422318	9·984259	9·438059	10·561941	40
21	9·394179	9·986234	9·407945	10·592055	9·422778	9·984224	9·438554	10·561446	39
22	9·394673	9·986202	9·408471	10·591529	9·423238	9·984190	9·439048	10·560952	38
23	9·395166	9·986169	9·408996	10·591004	9·423697	9·984155	9·439543	10·560457	37
24	9·395658	9·986137	9·409521	10·590479	9·424156	9·984120	9·440036	10·559964	36
25	9·396150	9·986104	9·410045	10·589955	9·424615	9·984085	9·440529	10·559471	35
26	9·396641	9·986072	9·410569	10·589431	9·425073	9·984050	9·441022	10·558978	34
27	9·397132	9·986039	9·411092	10·588908	9·425530	9·984015	9·441514	10·558486	33
28	9·397621	9·986007	9·411615	10·588385	9·425987	9·983981	9·442006	10·557994	32
29	9·398111	9·985974	9·412137	10·587863	9·426443	9·983946	9·442497	10·557503	31
30	9·398600	9·985942	9·412658	10·587342	9·426899	9·983911	9·442988	10·557012	30
31	9·399088	9·985909	9·413179	10·586821	9·427354	9·983875	9·443479	10·556521	29
32	9·399575	9·985876	9·413699	10·586301	9·427809	9·983840	9·443968	10·556032	28
33	9·400062	9·985843	9·414219	10·585781	9·428263	9·983805	9·444458	10·555542	27
34	9·400549	9·985811	9·414738	10·585262	9·428717	9·983770	9·444947	10·555053	26
35	9·401035	9·985778	9·415257	10·584743	9·429170	9·983735	9·445435	10·554565	25
36	9·401520	9·985745	9·415775	10·584225	9·429623	9·983700	9·445923	10·554077	24
37	9·402005	9·985712	9·416293	10·583707	9·430075	9·983664	9·446411	10·553589	23
38	9·402489	9·985679	9·416810	10·583190	9·430527	9·983629	9·446898	10·553102	22
39	9·402972	9·985646	9·417326	10·582674	9·430978	9·983594	9·447384	10·552616	21
40	9·403455	9·985613	9·417842	10·582158	9·431429	9·983558	9·447870	10·552130	20
41	9·403938	9·985580	9·418358	10·581642	9·431879	9·983523	9·448356	10·551644	19
42	9·404420	9·985547	9·418873	10·581127	9·432329	9·983487	9·448841	10·551159	18
43	9·404901	9·985514	9·419387	10·580613	9·432778	9·983452	9·449326	10·550674	17
44	9·405382	9·985480	9·419901	10·580099	9·433226	9·983416	9·449810	10·550190	16
45	9·405862	9·985447	9·420415	10·579585	9·433675	9·983381	9·450294	10·549706	15
46	9·406341	9·985414	9·420927	10·579073	9·434122	9·983345	9·450777	10·549223	14
47	9·406820	9·985381	9·421440	10·578560	9·434569	9·983309	9·451260	10·548740	13
48	9·407299	9·985347	9·421952	10·578048	9·435016	9·983273	9·451743	10·548257	12
49	9·407777	9·985314	9·422463	10·577537	9·435462	9·983238	9·452225	10·547775	11
50	9·408254	9·985280	9·422974	10·577026	9·435908	9·983202	9·452706	10·547294	10
51	9·408731	9·985247	9·423484	10·576516	9·436353	9·983166	9·453187	10·546813	9
52	9·409207	9·985213	9·423993	10·576007	9·436798	9·983130	9·453668	10·546332	8
53	9·409682	9·985180	9·424503	10·575497	9·437242	9·983094	9·454148	10·545852	7
54	9·410157	9·985146	9·425011	10·574989	9·437686	9·983058	9·454628	10·545372	6
55	9·410632	9·985113	9·425519	10·574481	9·438129	9·983022	9·455107	10·544893	5
56	9·411106	9·985079	9·426027	10·573973	9·438572	9·982986	9·455586	10·544414	4
57	9·411579	9·985045	9·426534	10·573466	9·439014	9·982950	9·456064	10·543936	3
58	9·412052	9·985011	9·427041	10·572959	9·439456	9·982914	9·456542	10·543458	2
59	9·412524	9·984978	9·427547	10·572453	9·439897	9·982878	9·457019	10·542981	1
60	9·412996	9·984944	9·428052	10·571948	9·440338	9·982842	9·457496	10·542504	0
	Cosine	Sine	Cotan.	Tan.	Cosine	Sine	Cotan.	Tan.	′
	75°				74°				

	16°				17°				
′	Sine	Cosine	Tan.	Cotan.	Sine	Cosine	Tan.	Cotan.	
0	9·440338	9·982842	9·457496	10·542504	9·465935	9·980596	9·485339	10·514661	60
1	9·440778	9·982806	9·457973	10·542027	9·466348	9·980558	9·485791	10·514209	59
2	9·441218	9·982769	9·458449	10·541551	9·466761	9·980519	8·486242	10·513758	58
3	9·441658	9·982733	9·458925	10·541075	9·467173	9·980480	9·486693	10·513307	57
4	9·442096	9·982696	9·459400	10·540600	9·467585	9·980442	9·487143	10·512857	56
5	9·442535	9·982660	9·459875	10·540125	9·467996	9·980403	9·487593	10·512407	55
6	9·442973	9·982624	9·460349	10·539651	9·468407	9·980364	9·488043	10·511957	54
7	9·443410	9·982587	9·460823	10·539177	9·468817	9·980325	9·488492	10·511508	53
8	9·443847	9·982551	9·461297	10·538703	9·469227	9·980286	9·488941	10·511059	52
9	9·444284	9·982514	9·461770	10·538230	9·469637	9·980247	9·489390	10·510610	51
10	9·444720	9·982477	9·462242	10·537758	9·470046	9·980208	9·489838	10·510162	50
11	9·445155	9·982441	9·462715	10·537285	9·470455	9·980169	9·490286	10·509714	49
12	9·445590	9·982404	9·463186	10·536814	9·470863	9·980130	9·490733	10·509267	48
13	9·446025	9·982367	9·463658	10·536342	9·471271	9·980091	9·491180	10·508820	47
14	9·446459	9·982331	9·464128	10·535872	9·471679	9·980052	9·491627	10·508373	46
15	9·446893	9·982294	9·464599	10·535401	9·472086	9·980012	9·492073	10·507927	45
16	9·447326	9·982257	9·465069	10·534931	9·472492	9·979973	9·492519	10·507481	44
17	9·447759	9·982220	9·465539	10·534461	9·472898	9·979934	9·492965	10·507035	43
18	9·448191	9·982183	9·466008	10·533992	9·473304	9·979895	9·493410	10·506590	42
19	9·448623	9·982146	9·466477	10·533523	9·473710	9·979855	9·493854	10·506146	41
20	9·449054	9·982109	9·466945	10·533055	9·474115	9·979816	9·494299	10·505701	40
21	9.449485	9·982072	9·467413	10·532587	9·474519	9·979776	9·494743	10·505257	39
22	9 449915	9·982035	9·467880	10·532120	9·474923	9·979737	9·495186	10·504814	38
23	9·450345	9·981998	9·468347	10·531653	9·475327	9·979697	9·495630	10·504370	37
24	9·450775	9·981961	9·468814	10·531186	9·475730	9·979658	9·496073	10·503927	36
25	9·451204	9·981924	9·469280	10·530720	9·476133	9·979618	9·496515	10·503485	35
26	9·451632	9·981886	9·469746	10·530254	9·476536	9·979579	9·496957	10·503043	34
27	9·452060	9·981849	9·470211	10·529789	9·476938	9·979539	9·497399	10·502601	33
28	9·452488	9·981812	9·470676	10·529324	9·477340	9·979499	9·497841	10·502159	32
29	9·452915	9·981774	9·471141	10·528859	9·477741	9·979459	9·498282	10·501718	31
30	9·453342	9·981737	9·471605	10·528395	9·478142	9·979420	9·498722	10·501278	30
31	9·453768	9·981700	9·472069	10·527931	9·478542	9·979380	9·499163	10·500837	29
32	9·454194	9·981662	9·472532	10·527468	9·478942	9·979340	9·499603	10·500397	28
33	9·454619	9·981626	9·472995	10·527005	9·479342	9·979300	9·500042	10·499958	27
34	9·455044	9·981587	9·473457	10·526543	9·479741	9·979260	9·500481	10·499519	26
35	9·455469	9·981549	9·473919	10·526081	9·480140	9·979220	9·500920	10·499080	25
36	9·455893	9·981512	9·474381	10·525619	9·480539	9·979180	9·501359	10·498641	24
37	9·456316	9·981474	9·474842	10·525158	9·480937	9·979140	9·501797	10·498203	23
38	9·456739	9·981436	9·475303	10·524697	9·481334	9·979100	9·502235	10·497765	22
39	9·457162	9·981399	9·475763	10·524237	9·481731	9·979059	9·502672	10·497328	21
40	9·457584	9·981361	9·476223	10·523777	9·482128	9·979019	9·503109	10·496891	20
41	9·458006	9·981323	9·476683	10·523317	9·482525	9·978979	9·503546	10·496454	19
42	9·458427	9·981285	9·477142	10·522858	9·482921	9·978939	9·503982	10·496018	18
43	9·458848	9·981247	9·477601	10·522399	9·483316	9·978898	9·504418	10·495582	17
44	9·459268	9·981209	9·478059	10·521941	9·483712	9·978858	9·504854	10·495146	16
45	9·459688	9·981171	9·478517	10·521483	9·484107	9·978817	9·505289	10·494711	15
46	9·460108	9·981133	9·478975	10·521025	9·484501	9·978777	9·505724	10·494276	14
47	9·460527	9·981095	9·479432	10·520568	9·484895	9·978737	9·506159	10·493841	13
48	9·460946	9·981057	9·479889	10·520111	9·485289	9·978696	9·506593	10·493407	12
49	9·461364	9·981019	9·480345	10·519655	9·485682	9·978655	9·507027	10·492973	11
50	9·461782	9·980981	9·480801	10·519199	9·486075	9·978615	9·507460	10·492540	10
51	9·462199	9·980942	9·481257	10·518743	9·486467	9·978574	9·507893	10·492107	9
52	9·462616	9·980904	9·481712	10·518288	9·486860	9·978533	9·508326	10·491674	8
53	9·463032	9·980866	9·482167	10·517833	9·487251	9·978493	9·508759	10·491241	7
54	9·463148	9·980827	9·482621	10·517379	9·487643	9·978452	9·509191	10·490809	6
55	9·463864	9·980789	9·483075	10·516925	9·488034	9·978411	9·509622	10·490378	5
56	9·464279	9.980750	9·483529	10·516471	9·488424	9·978370	9·510054	10·489946	4
57	9·464694	9·980712	9·483982	10·516018	9·488814	9·978329	9·510485	10·489515	3
58	9·465108	9·980673	9·484435	10·515565	9·489204	9·978288	9·510916	10·489084	2
59	9·465522	9·980635	9·484887	10·515113	9·489593	9·978247	9·511346	10·488654	1
60	9·465935	9·980596	9·485339	10·514661	9·489982	9·978206	9·511776	10·488224	0
	Cosine	Sine	Cotan.	Tan.	Cosine	Sine	Cotan.	Tan.	′
	73°				72°				

	18°				19°				
′	Sine	Cosine	Tan.	Cotan.	Sine	Cosine	Tan.	Cotan.	
0	9·489982	9·978206	9·511776	10·488224	9·512642	9·975670	9·536972	10·463028	60
1	9·490371	9·978165	9·512206	10·487794	9·513009	9·975627	9·537382	10·462618	59
2	9·490759	9·978124	9·512635	10·487365	9·513375	9·975583	9·537792	10·462208	58
3	9·491147	9·978083	9·513064	10·486936	9·513741	9·975539	9·538202	10·461798	57
4	9·491535	9·978042	9·513493	10·486507	9·514107	9·975496	9·538611	10·461389	56
5	9·491922	9·978001	9·513921	10·486079	9·514472	9·975452	9·539020	10·460980	55
6	9·490308	9·977959	9·514349	10·485651	9·514837	9·975408	9·539429	10·460571	54
7	9·492695	9·977918	9·514777	10·485223	9·515202	9·975365	9·539837	10·460163	53
8	9·493081	9·977877	9·515204	10·484796	9·515566	9·975321	9·540245	10·459755	52
9	9·493466	9·977835	9·515631	10·484369	9·515930	9·975277	9·540653	10·459347	51
10	9·493851	9·977794	9·516057	10·483943	9·516294	9·975233	9·541061	10·458939	50
11	9·494236	9·977752	9·516484	10·483516	9·516657	9·975189	9·541468	10·458532	49
12	9·494621	9·977711	9·516910	10·483090	9·517020	9·975140	9·541875	10·458125	48
13	9·495005	9·977669	9·517335	10·482665	9·517382	9·975101	9·542281	10·457719	47
14	9·495388	9·977628	9·517761	10·482239	9·517745	9·975057	9·542688	10·457312	46
15	9·495772	9·977586	9·518186	10·481814	9·518107	9·975013	9·543094	10·456906	45
16	9·496154	9·977544	9·518610	10·481390	9·518468	9·974969	9·543499	10·456501	44
17	9·496537	9·977503	9·519034	10·480966	9·518829	9·974925	9·543905	10·456095	43
18	9·496919	9·977461	9·519458	10·480542	9·519190	9·974880	9·544310	10·455690	42
19	9·497301	9·977419	9·519882	10·480118	9·519551	9·974836	9·544715	10·455285	41
20	9·497682	9·977377	9·520305	10·479695	9·519911	9·974792	9·545119	10·454881	40
21	9·498064	9·977335	9·520728	10·479272	9·520271	9·974748	9·545524	10·454476	39
22	9·498444	9·977293	9·521151	10·478849	9·520631	9·974703	9·545928	10·454072	38
23	9 498825	9·977251	9·521573	10·478427	9·520990	9·974659	9·546331	10·453669	37
24	9 499204	9·977209	9·521995	10·478005	9·521349	9·974614	9·546735	10·453265	36
25	9·499584	9·977167	9·522417	10·477583	9·521707	9·974570	9·547138	10·452862	35
26	9·499963	9·977125	9·522838	10·477162	9·522066	9·974525	9·547540	10·452460	34
27	9·500342	9·977083	9·523259	10·476741	9·522424	9·974481	9·547943	10·452057	33
28	9·500721	9·977041	9·523680	10·476320	9·522781	9·974436	9·548345	10·451655	32
29	9·501099	9·976999	9·524100	10·475900	9·523138	9·974391	9·548747	10·451253	31
30	9·501476	9·976957	9·524520	10·475480	9·523495	9·974347	9·549149	10·450851	30
31	9·501854	9·976914	9·524940	10·475060	9·523852	9·974302	9·549550	10·450450	29
32	9·502231	9·976872	9·525360	10·474641	9·524208	9·974257	9·549951	10·450049	28
33	9·502607	9·976830	9·525778	10·474222	9·524564	9·974212	9·550352	10·449648	27
34	9·502984	9·976787	9·526197	10·473803	9·524920	9·974167	9·550752	10·449248	26
35	9·503360	9·976745	9·526615	10·473385	9·525275	9·974122	9·551153	10·448847	25
36	9·503735	9·976702	9·527033	10·472967	9·525630	9·974077	9·551552	10·448448	24
37	9·504110	9·976660	9·527451	10·472549	9·525984	9·974032	9·551952	10·448048	23
38	9·504485	9·976617	9·527868	10·472132	9·526339	9·973987	9·552351	10·447649	22
39	9·504860	9·976574	9·528285	10·471715	9·526693	9·973942	9·552750	10·447250	21
40	9·505234	9·976532	9·528702	10·471298	9·527046	9·973897	9·553149	10·446851	20
41	9·505608	9·976489	9·529119	10·470881	9·527400	9·973852	9·553548	10·446452	19
42	9·505981	9·976446	9·529535	10·470465	9·527753	9·973807	9·553946	10·446054	18
43	9·506354	9·976404	9·529951	10·470049	9·528105	9·973761	9·554344	10·445656	17
44	9·506727	9·976361	9·530366	10·469634	9·528458	9·973716	9·554741	10·445259	16
45	9·507099	9·976318	9·530781	10·469219	9·528810	9·973671	9·555139	10·444861	15
46	9·507471	9·976275	9·531196	10·468804	9·529161	9·973625	9·555536	10·444464	14
47	9·507843	9·976232	9·531611	10·468389	9·529513	9·973580	9·555933	10·444067	13
48	9·508214	9·976189	9·532025	10·467975	9·529864	9·973535	9·556329	10·443671	12
49	9·508585	9·976146	9·532439	10·467561	9·530215	9·973489	9·556725	10·443275	11
50	9·508956	9·976103	9·532853	10·467147	9·530565	9·973444	9·557121	10·442879	10
51	9·509326	9·976060	9·533266	10·466734	9·530915	9·973398	9·557517	10·442483	9
52	9·509696	9·976017	9·533679	10·466321	9·531265	9·973352	9·557913	10·442087	8
53	9·510065	9·975974	9·534092	10·465905	9·531614	9·973307	9·558308	10·441692	7
54	9·510434	9·975930	9·534504	10·465496	9·531963	9·973261	9·558703	10·441297	6
55	9·510803	9·975887	9·534916	10·465084	9·532312	9·973215	9·559097	10·440903	5
56	9·511172	9·975844	9·535328	10·464672	9·532661	9·973169	9·559491	10·440509	4
57	9·511540	9·975800	9·535739	10·464261	9·533009	9·973124	9·559885	10·440115	3
58	9·511907	9·975757	9·536150	10·463850	9·533357	9·973078	9·560279	10·439721	2
59	9·512275	9·975714	9·536561	10·463439	9·533704	9·973032	9·560673	10·439327	1
60	9·512642	9·975670	9·536972	10·463028	9·534052	9·972986	9·561066	10·438934	0
	Cosine	Sine	Cotan.	Tan.	Cosine	Sine	Cotan.	Tan.	′
	71°				70°				

	20°				21°				
′	Sine	Cosine	Tan.	Cotan.	Sine	Cosine	Tan.	Cotan.	
0	9·534052	9·972986	9·561066	10·438934	9·554329	9·970152	9·584177	10·415823	60
1	9·534399	9·972940	9·561459	10·438541	9·554658	9·970103	9·584555	10·415445	59
2	9·534745	9·972894	9·561851	10·438149	9·554987	9·970055	9·584932	10·415068	58
3	9·535092	9·972848	9·562244	10·437756	9·555315	9·970006	9·585309	10·414691	57
4	9·535438	9·972802	9·562636	10·437364	9 555643	9·969957	9·585686	10·414314	56
5	9·535783	9·972755	9·563028	10·436972	9·555971	9·969909	9·586062	10·413938	55
6	9·536129	9·972709	9·563419	10·436581	9·556299	9·969860	9·586439	10·413561	54
7	9·536474	9·972663	9·563811	10·436189	9·556626	9·969811	9·586815	10·413185	53
8	9·536818	9.972667	9·564202	10·435798	9·556953	9·969762	9·587190	10·412819	52
9	9·537163	9·972570	9·564593	10·435407	9·557280	9·969714	9·587566	10·412434	51
10	9·537507	9·972524	9·564983	10·435017	9·557606	9·969665	9·587941	10·412059	50
11	9·537851	9·972478	9·565373	10·434627	9·557932	9·969616	9·588316	10·411684	49
12	9·538194	9·972431	9·565763	10·434237	9·558258	9·969567	9·588691	10·411309	48
13	9·538538	9.972385	9·566153	10·433847	9·558583	9·969518	9·589066	10·410934	47
14	9·538880	9·972338	9·566542	10·433458	9·558909	9·969469	9·589440	10·410560	46
15	9·539223	9·972291	9·566932	10·433063	9·559234	9·969420	9·589814	10·410186	45
16	9·539565	9·972245	9·567320	10·432680	9·559558	9·969370	9·590188	10·409812	44
17	9·539907	9·972198	9·567709	10·432291	9·559883	9·969321	9·590562	10·409438	43
18	9·540249	9·972151	9·568098	10·431902	9·560207	9·969272	9·590935	10·409065	42
19	9·540590	9·972105	9·568486	10·431514	9·560531	9·969223	9·591308	10·408692	41
20	9·540931	9·972058	9·568873	10·431127	9·560855	9·969173	9·591681	10·408319	40
21	9·541272	9·972011	9·569261	10·430739	9·561178	9·969124	9·592054	10·407946	39
22	9·541613	9·971964	9 569648	10·430352	9·561501	9·969075	9·592426	10·407574	38
23	9·541953	9·971917	9·570035	10·429965	9·561824	9·969025	9·592799	10·407201	37
24	9·542293	9·971870	9·570422	10·429578	9·562146	9·968976	9·593171	10·406829	36
25	9·542632	9·971823	9·570809	10·429191	9·562468	9·968926	9·593542	10·406458	35
26	9·542971	9·971776	9·571195	10·428805	9·562790	9·968877	9·593914	10·406086	34
27	9·543310	9·971729	9·571581	10·428419	9·563112	9·968827	9·594285	10·405715	33
28	9·543649	9·971682	9·571967	10·428033	9·563433	9·968777	9·594656	10·405344	32
29	9·543987	9·971635	9·572352	10·427648	9·563755	9·968728	9·595027	10·404973	31
30	9·544325	9·971588	9·572738	10·427262	9·564075	9·968678	9·595398	10·404602	30
31	9·544663	9·971540	9·573123	10·426877	9·564396	9·968628	9·595768	10·404232	29
32	9·545000	9·971493	9·573507	10·426493	9·564716	9·968578	9·596138	10·403862	28
33	9·545338	9·971446	9·573892	10·426108	9·565036	9·968528	9·596508	10·403492	27
34	9·545674	9·971398	9 574276	10·425724	9·565356	9·968479	9·596878	10·403122	26
35	9·546011	9·971351	9 574660	10·425340	9·565676	9·968429	9·597247	10·402753	25
36	9·546347	9 971303	9·575044	10·424956	9·565995	9·968379	9·597616	10·402384	24
37	9·546683	9·971256	9 575427	10·424573	9·566314	9·968329	9·597985	10·402015	23
38	9 547019	9·971208	9 575810	10·424190	9·566632	9·968278	9·598354	10·401646	22
39	9·547354	9·971161	9·576193	10·423807	9·566951	9·968228	9·598722	10·401278	21
40	9·547689	9·971113	9·576576	10·423424	9·567269	9·968178	9·599091	10·400909	20
41	9·548024	9·971066	9·576959	10·423041	9·567587	9·968128	9·599459	10·400541	19
42	9·548359	9·971018	9·577341	10·422659	9·567904	9·968078	9·599827	10·400173	18
43	9·548693	9·970970	9·577723	10·422277	9·568222	9·968027	9·600194	10·399806	17
44	9·549027	9·970922	9·578104	10·421896	9·568539	9·967977	9·600562	10·399438	16
45	9·549360	9·970874	9 578486	10·421514	9·568856	9·967927	9·600929	10·399071	15
46	9·549693	9·970827	9·578867	10·421133	9·569172	9·967876	9·601296	10·398704	14
47	9·550026	9·970779	9·579248	10·420752	9·569488	9·967826	9·601663	10·398337	13
48	9·550359	9·970731	9·579629	10·420371	9·569804	9·967775	9·602029	10·397971	12
49	9·550692	9·970683	9·580009	10·419991	9·570120	9·967725	9·602395	10·397605	11
50	9·551024	9·970635	9 580389	10·419611	9·570435	9·967674	9·602761	10·397239	10
51	9·551356	9·970586	9 580769	10·419231	9·570751	9·967624	9·603127	10·396873	9
52	9·551687	9·970538	9 581149	10·418851	9·571066	9·967573	9·603493	10·396507	8
53	9·552018	9·970490	9 581528	10·418472	9·571380	9·967522	9·603858	10·396142	7
54	9·552349	9·970442	9 581907	10·418093	9·571695	9·967471	9·604223	10·395777	6
55	9·552680	9·970394	9·582286	10·417714	9·572009	9·967421	9·604588	10·395412	5
56	9·553010	9·970345	9 582665	10·417335	9·572323	9·967370	9·604953	10·395047	4
57	9·553341	9·970297	9 583044	10·416956	9·572636	9·967319	9·605317	10·394683	3
58	9·553670	9·970249	9·583422	10·416578	9·572950	9·967268	9·605682	10·394318	2
59	9 554000	9·970200	9 583800	10·416200	9·573263	9·967217	9·606046	10·393954	1
60	9·554329	9·970152	9·584177	10·415823	9·573575	9·967166	9·606410	10·393590	0
	Cosine	Sine	Cotan.	Tan.	Cosine	Sine	Cotan.	Tan.	′
	69°				68°				

	22°				23°				
′	Sine	Cosine	Tan.	Cotan.	Sine	Cosine	Tan.	Cotan	
0	9·573575	9·967166	9·606410	10·393590	9·591878	9·964026	9·627852	10·372148	60
1	9·573888	9·967115	9·606773	10·393227	9·592176	9·963972	9·628203	10·371797	59
2	9·574200	9·967064	9·607137	10·392863	9·592473	9·963919	9·628554	10·371446	58
3	9·574512	9·967013	9·607500	10·392500	9·592770	9·963865	9·628905	10·371095	57
4	9·574824	9·966961	9·607863	10·392137	9·593067	9·963811	9·629255	10·370745	56
5	9·575136	9·966910	9·608225	10·391775	9·593363	9·963757	9·629606	10·370394	55
6	9·575447	9·966859	9·608588	10·391412	9·593659	9·963704	9·629956	10·370044	54
7	9·575758	9·966808	9·608950	10·391050	9·593955	9·963650	9·630306	10·369694	53
8	9·576069	9·966756	9·609312	10·390688	9·594251	9·963596	9·630656	10·369344	52
9	9·576379	9·966705	9·609674	10·390326	9·594547	9·963542	9·631005	10·368995	51
10	9·576689	9·966653	9·610036	10·389964	9·594842	9·963488	9·631355	10·368645	50
11	9·576999	9·966602	9·610397	10·389603	9·595137	9·963434	9·631704	10·368296	49
12	9·577309	9·966550	9·610759	10·389241	9·595432	9·963379	9·632053	10·367947	48
13	9·577618	9·966499	9·611120	10·388880	9·595727	9·963325	9·632402	10·367598	47
14	9·577927	9·966447	9·611480	10·388520	9·596021	9·963271	9·632750	10·367250	46
15	9·578236	9·966395	9·611841	10·388159	9·596315	9·963217	9·633099	10·366901	45
16	9·578545	9·966344	9·612201	10·387799	9·596609	9·963163	9·633447	10·366553	44
17	9·578853	9·966292	9·612561	10·387439	9·596903	9·963108	9·633795	10·366205	43
18	9·579162	9·966240	9·612921	10·387079	9·597196	9·963054	9·634143	10·365857	42
19	9·579470	9·966188	9·613281	10·386719	9·597490	9·962999	9·634490	10·365510	41
20	9·579777	9·966136	9·613641	10·386359	9·597783	9·962945	9·634838	10·365162	40
21	9·580085	9·966085	9·614000	10·386000	9·598075	9·962890	9·635185	10·364815	39
22	9·580392	9·966033	9·614359	10·385641	9·598368	9·962836	9·635532	10·364468	38
23	9·580699	9·965981	9·614718	10·385282	9·598660	9·962781	9·635879	10·364121	37
24	9·581005	9·965929	9·615077	10·384923	9·598952	9·962727	9·636226	10·363774	36
25	9·581312	9·965876	9·615435	10·384565	9·599244	9·962672	9·636572	10·363428	35
26	9·581618	9·965824	9·615793	10·384207	9·599536	9·962617	9·636919	10·363081	34
27	9·581924	9·965772	9·616151	10·383849	9·599827	9·962562	9·637265	10·362735	33
28	9·582229	9·965720	9·616509	10·383491	9·600118	9·962508	9·637611	10·362389	32
29	9·582535	9·965668	9·616867	10·383133	9·600409	9·962453	9·637956	10·362044	31
30	9·582840	9·965615	9·617224	10·382776	9·600700	9·962398	9·638302	10·361698	30
31	9·583145	9·965563	9·617582	10·382418	9·600990	9·962343	9·638647	10·361353	29
32	9·583449	9·965511	9·617939	10·382061	9·601280	9·962288	9·638992	10·361008	28
33	9·583754	9·965458	9·618295	10·381705	9·601570	9·962233	9·639337	10·360663	27
34	9·584058	9·965406	9·618652	10·381348	9·601860	9·962178	9·639682	10·360318	26
35	9·584361	9·965353	9·619008	10·380992	9·602150	9·962123	9·640027	10·359973	25
36	9·584665	9·965301	9·619364	10·380636	9·602439	9·962067	9·640371	10·359629	24
37	9·584968	9·965248	9·619720	10·380280	9·602728	9·962012	9·640716	10·359284	23
38	9·585272	9·965195	9·620076	10·379924	9·603017	9·961957	9·641060	10·358940	22
39	9·585574	9·965143	9·620432	10·379568	9·603305	9·961902	9·641404	10·358596	21
40	9·585877	9·965090	9·620787	10·379213	9·603594	9·961846	9·641747	10·358253	20
41	9·586179	9·965037	9·621142	10·378858	9·603882	9·961791	9·642091	10·357909	19
42	9·586482	9·964984	9·621497	10·378503	9·604170	9·961735	9·642434	10·357566	18
43	9·586783	9·964931	9·621852	10·378148	9·604457	9·961680	9·642777	10·357223	17
44	9·587085	9·964879	9·622207	10·377793	9·604745	9·961624	9·643120	10·356880	16
45	9·587386	9·964826	9·622561	10·377439	9·605032	9·961569	9·643463	10·356537	15
46	9·587688	9·964773	9·622915	10·377085	9·605319	9·961513	9·643806	10·356194	14
47	9·587989	9·964720	9·623269	10·376731	9·605606	9·961458	9·644148	10·355852	13
48	9·588289	9·964666	9·623623	10·376377	9·605892	9·961402	9·644490	10·355510	12
49	9·588590	9·964613	9·623976	10·376024	9·606179	9·961346	9·644832	10·355168	11
50	9·588890	9·964560	9·624330	10·375670	9·606465	9·961290	9·645174	10·354826	10
51	9·589190	9·964507	9·624683	10·375317	9·606751	9·961235	9·645516	10·354484	9
52	9·589489	9·964454	9·625036	10·374964	9·607036	9·961179	9·645857	10·354143	8
53	9·589789	9·964400	9·625388	10·374612	9·607322	9·961123	9·646199	10·353801	7
54	9·590088	9·964347	9·625741	10·374259	9·607607	9·961067	9·646540	10·353460	6
55	9·590387	9·964294	9·626093	10·373907	9·607892	9·961011	9·646881	10·353119	5
56	9·590686	9·964240	9·626445	10·373555	9·608177	9·960955	9·647222	10·352778	4
57	9·590984	9·964187	9·626797	10·373203	9·608461	9·960899	9·647562	10·352438	3
58	9·591282	9·964133	9·627149	10·372851	9·608745	9·960843	9·647903	10·352097	2
59	9·591580	9·964080	9·627501	10·372499	9·609029	9·960786	9·648243	10·351757	1
60	9·591878	9·964026	9·627852	10·372148	9·609313	9·960730	9·648583	10·351417	0
	Cosine	Sine	Cotan.	Tan.	Cosine	Sine	Cotan.	Tan.	′
	67°				66°				

	24°				25°				
′	Sine	Cosine	Tan.	Cotan.	Sine	Cosine	Tan.	Cotan.	
0	9·609313	9 960730	9·648583	10·351417	9·625948	9·957276	9·668673	10·331327	60
1	9 609597	9 960674	9·648923	10 351077	9·626219	9·957217	9·669002	10·330998	59
2	9·609880	9 960618	9·649990	10·350737	9·626490	9·957158	9·669332	10·330663	58
3	9 610164	9 960561	9 649602	10 350398	9·626760	9·957099	9·669661	10·330339	57
4	9 610447	9·260505	9 649942	10·350058	9·627030	9·957040	9·669991	10·330009	56
5	9·610729	9 960448	9 650281	10 349719	9·627300	9·956981	9·670320	10·329680	55
6	9 611012	9 960392	9 650620	10 349380	9·627570	9·956921	9·670649	10·329351	54
7	9·611294	9·960335	9 650959	10 349041	9·627840	9·956862	9·670977	10·328023	53
8	9 611576	9·960279	9 651297	10·348703	9·628109	9·956803	9·671306	10·328694	52
9	9 611858	9·960222	9·651636	10·348364	9·628378	9·956744	9·671635	10·328365	51
10	9·612140	9 960165	9·651974	10 348026	9·628647	9·956684	9·671963	10·328037	50
11	9 612421	9·960109	9 652312	10·347688	9·628916	9·956625	9·672291	10·327709	49
12	9·612702	9·960052	9·652650	10·347350	9·629185	9·956566	9·672619	10·327381	48
13	9·612983	9·959995	9·652988	10·347012	9·629453	9·956506	9·672947	10·327053	47
14	9 613264	9·959938	9·653326	10·346674	9·629721	9·956447	9·673274	10·326726	46
15	9 613545	9·959882	9·653663	10·346337	9·629989	9·956387	9·673602	10·326398	45
16	9·613825	9·959825	9·654000	10·346000	9·630257	9·956327	9·673929	10·326071	44
17	9·614105	9·959768	9 654337	10·345663	9·630524	9·956268	9·674257	10·325743	43
18	9·614385	9·959711	9 654674	10·345326	9·630792	9·956208	9·674584	10·325416	42
19	9·614665	9·959654	9·655011	10·344989	9·631059	9·956148	9·674911	10·325089	41
20	9·614944	9·959596	9·655348	10 344652	9·631326	9·956089	9·675237	10·324763	40
21	9 615223	9·959539	9 655684	10·344316	9·631593	9·956029	9·675564	10·324436	39
22	9 615502	9·959489	9·656020	10·343980	9·631859	9·955969	9·675890	10·324110	38
23	9·615781	9·959425	9·656356	10·348644	9·632125	9·955909	9·676217	10·323783	37
24	9 616060	9 959368	9·656692	10·343308	9·632392	9·955849	9·676543	10·323457	36
25	9·616338	9·959310	9·657028	10·342972	9·632658	9·955789	9·676869	10·323131	35
26	9·616616	9 959253	9·657364	10·342636	9·632923	9·955729	9·677194	10·322806	34
27	9 616894	9·959195	9·657699	10·342301	9·633189	9·955669	9·677520	10·322480	33
28	9 617172	9·959138	9·658034	10·341966	9·633454	9·955609	9·677846	10·322154	32
29	9·617450	9·959080	9·658369	10 341631	9·633719	9·955548	9·678171	10·321829	31
30	9·617727	9·959023	9·658704	10 341296	9·633984	9·955488	9·678496	10·321504	30
31	9 618004	9·958965	9·659039	10·340961	9·634249	9·955428	9·678821	10·321179	29
32	9 618281	9·958908	9·659373	10·340327	9·634514	9·955368	9·679146	10·320854	28
33	9 618558	9·958850	9·659708	10 340292	9·634778	9·955307	9·679471	10·320529	27
34	9·618834	9·958792	9·660042	10·339958	9·635042	9·955247	9·679795	10·320205	26
35	9·619110	9·958734	9·660376	10·339624	9·635306	9·955186	9·680120	10·319880	25
36	9·619386	9·958677	9·660710	10 339296	9·635570	9·955126	9·680444	10·319556	24
37	9 619662	9·958619	9·661043	10·338957	9·635834	9·955065	9·680768	10·319232	23
38	9 619938	9·958561	9·661377	10·338623	9·636097	9·955005	9·681092	10·318908	22
39	9·620213	9·958503	9 661710	10·338290	9·636360	9·954944	9·681416	10·318584	21
40	9·620488	9·958445	9·662043	10·337957	9·636623	9·954883	9·681740	10·318260	20
41	9·620763	9·958387	9·662376	10 337624	9·636886	9·954823	9·682063	10·317937	19
42	9·621038	9·958329	9·662709	10·337291	9·637148	9·954762	9·682387	10·317613	18
43	9·621313	9·958271	9·663042	10·336958	9·637411	9·954701	9·682710	10·317290	17
44	9·621587	9·958213	9·663375	10·336625	9·637673	9·954640	9·683033	10·316967	16
45	9·621861	9·958154	9·663707	10 336293	9·637935	9·954579	9·683356	10·316644	15
46	9·622135	9·958096	9·664039	10 335961	9·638197	9·954518	9·683679	10·316321	14
47	9 622409	9·958038	9 664371	10·335629	9·638458	9·954457	9·684001	10·315999	13
48	9·622682	9·957979	9·664703	10 335297	9·638720	9·954396	9·684324	10·315676	12
49	9·622956	9·957921	9·665035	10·334965	9·638981	9·954334	9·684646	10·315354	11
50	9·623229	9·957863	9·665366	10·334634	9·639242	9·954275	9·684968	10·315032	10
51	9·633502	9·957804	9·665698	10·334302	9·639503	9·954213	9·685290	10·314710	9
52	9·623774	9·957746	9·666029	10·333971	9·639764	9·954152	9·685612	10·314388	8
53	9·624047	9·957687	9·666360	10·333640	9·640024	9·954090	9·685934	10·314066	7
54	9·624319	9·957628	9·666691	10·333309	9·640284	9·954029	9·686255	10·313745	6
55	9·624591	9·957570	9·667021	10·332979	9·640544	9·953968	9·686577	10·313423	5
56	9·624863	9·957511	9·667352	10·332648	9·640804	9·953906	9·686898	10·313102	4
57	9·625135	9·957452	9·667682	10·332318	9·641064	9·953845	9·687219	10·312781	3
58	9·625406	9 957393	9·668013	10·331987	9·641324	9·953783	9·687540	10·312460	2
59	9·625677	9·957335	9·668343	10·331657	9·641583	9·953722	9·687861	10·312139	1
60	9·625948	9·957276	9·668673	10·331327	9·641842	9·953660	9·688182	10·311818	0
	Cosine	Sine	Cotan.	Tan.	Cosine	Sine	Cotan.	Tan.	′
	65°				64°				

	26°				27°				
′	Sine	Cosine	Tan.	Cotan.	Sine	Cosine	Tan.	Cotan.	
0	9·641842	9·953660	9·688182	10·311818	9·657047	9·949881	9·707166	10·292834	60
1	9·642101	9·953599	9·688502	10·311498	9·657295	9·949816	9·707478	10·292522	59
2	9·642360	9·953537	9·688823	10·311177	9·657542	9·949752	9·707790	10·292210	58
3	9·642618	9·953475	9·689143	10·310857	9·657790	9·949688	9·708102	10·291898	57
4	9·642877	9·953413	9·689463	10·310537	9·658037	9·949623	9·708414	10·291586	56
5	9·643135	9·953352	9·689783	10·310217	9·658284	9·949558	9·708726	10·291274	55
6	9·643393	9·953290	9·690103	10·309897	9·658531	9·949494	9·709037	10·290963	54
7	9·643650	9·953228	9·690423	10·309577	9·658778	9·949429	9·709349	10·290651	53
8	9·643908	9·953166	9·690742	10·309258	9·659025	9·949364	9·709660	10·290340	52
9	9·644165	9·953104	9·691062	10·308938	9·659271	9·949300	9·709971	10·290029	51
10	9·644423	9·953042	9·691381	10·308619	9·659517	9·949235	9·710282	10·289718	50
11	9·644680	9·952980	9·691700	10·308300	9·659763	9·949170	9·710593	10·289407	49
12	9·644936	9·952918	9·692019	10·307981	9·660009	9·949105	9·710904	10·289096	48
13	9·645193	9·952855	9·692338	10·307662	9·660255	9·949040	9·711215	10·288785	47
14	9·645450	9·952793	9·692656	10·307344	9·660501	9·948975	9·711525	10·288475	46
15	9·645706	9·952731	9·692975	10·307025	9·660746	9·948910	9·711836	10·288164	45
16	9·645962	9·952669	9·693293	10·306707	9·660991	9·948845	9·712146	10·287854	44
17	9·646218	9·952606	9·693612	10·306388	9·661236	9·948780	9·712456	10·287544	43
18	9·646474	9·952544	9·693930	10·306070	9·661481	9·948715	9·712766	10·287234	42
19	9·646729	9·952481	9·694248	10·305752	9·661726	9·948650	9·713076	10·286924	41
20	9·646984	9·952419	9·694566	10·305434	9·661970	9·948584	9·713386	10·286614	40
21	9.647240	9·952356	9·694883	10·305117	9·662214	9·948519	9·713696	10·286304	39
22	9.647494	9·952294	9·695201	10·304799	9·662459	9·948454	9·714005	10·285995	38
23	9·647749	9·952231	9·695518	10·304482	9·662703	9·948388	9·714314	10·285686	37
24	9·648004	9·952168	9·695836	10·304164	9·662946	9·948323	9·714624	10·285376	36
25	9·648258	9·952106	9·696153	10·303847	9·663190	9·948257	9·714933	10·285067	35
26	9·648512	9·952043	9·696470	10·303530	9·663433	9·948192	9·715242	10·284758	34
27	9·648766	9·951980	9·696787	10·303213	9·663677	9·948126	9·715551	10·284449	33
28	9·649020	9·951917	9·697103	10·302897	9·663920	9·948060	9·715860	10·284140	32
29	9·649274	9·951854	9·697420	10·302580	9·664163	9·947995	9·716168	10·283832	31
30	9·649527	9·951791	9·697736	10·302264	9·664406	9·947929	9·716477	10·283523	30
31	9·649781	9·951728	9·698053	10·301947	9·664648	9·947863	9·716785	10·283215	29
32	9·650034	9·951665	9·698369	10·301631	9·664891	9·947797	9·717093	10·282907	28
33	9·650287	9·951602	9·698685	10·301315	9·665133	9·947731	9·717401	10·282599	27
34	9·650539	9·951539	9·699001	10·300999	9·665375	9·947665	9·717709	10·282291	26
35	9·650792	9·951476	9·699316	10·300684	9·665617	9·947600	9·718017	10·281983	25
36	9·651044	9·951412	9·699632	0·300368	9·665859	9·947533	9·718325	10·281675	24
37	9·651297	9·951349	9·699947	10·300053	9·666100	9·947467	9·718633	10·281367	23
38	9·951549	9·951286	9·700263	10·299737	9·666342	9·947401	9·718940	10·281060	22
39	9·651800	9·951222	9·700578	10·299422	9·666583	9·947335	9·719248	10·280752	21
40	9·652052	9·951159	9·700893	10·299107	9·666824	9·947269	9·719555	10·280445	20
41	9·652304	9·951096	9·701208	10·298792	9·667065	9·947203	9·719862	10·280138	19
42	9·652555	9·951032	9·701523	10·298477	9·667305	9·947136	9·720169	10·279831	18
43	9·652806	9·950968	9·701837	10·298163	9·667546	9·947070	9·720476	10·279524	17
44	9·653057	9·950905	9·702152	10·297848	9·667786	9·947004	9·720783	10·279217	16
45	9·653308	9·950841	9·702466	10·297534	9·668027	9·946937	9·721089	10·278911	15
46	9·653558	9·950778	9·702781	10·297219	9·668267	9·946871	9·721396	10·278604	14
47	9·653808	9·950714	9·703095	10 296905	9·668506	9·946804	9·721702	10·278298	13
48	9·654059	9·950650	9·703409	10·296591	9·668746	9·946738	9·722009	10·277991	12
49	9·654309	9·950586	9·703722	10·296278	9·668986	9·946671	9·722315	10·277685	11
50	9·654558	9·950522	9·704036	10·295964	9·669225	9·946604	9·722621	10·277379	10
51	9·654808	9·950458	9·704350	10·295650	9·669464	9·946538	9·722927	10·277073	9
52	9·655058	9·950394	9·704663	10·295337	9·669703	9·946471	9·723232	10·276768	8
53	9·655307	9·950330	9·704976	10·295024	9·669942	9·946404	9·723538	10·276462	7
54	9·655556	9·950266	9·705290	10·294710	9·670181	9·946337	9·723844	10·276156	6
55	9·655805	9·950202	9·705603	10·294397	9·670419	9·946270	9·724149	10·275851	5
56	9·656054	9.950138	9·705916	10·294084	9·670658	9·946203	9·724454	10·275546	4
57	9·656302	9·950074	9·706228	10·293772	9·670896	9·946136	9·724760	10·275240	3
58	9·656551	9·950010	9·706541	10·293459	9·671134	9·946069	9·725065	10·274935	2
59	9·656799	9·949945	9·706854	10·293146	9·671372	9·946002	9·725370	10·274630	1
60	9·657047	9·949881	9·707166	10·292834	9·671609	9·945935	9·725674	10·274326	0
	Cosine	Sine	Cotan.	Tan.	Cosine	Sine	Cotan.	Tan.	′
	63°				62°				

	28°				29°				
′	Sine	Cosine	Tan.	Cotan.	Sine	Cosine	Tan.	Cotan.	
0	9·671609	9·945935	9·725674	10·274326	9·685571	9·941819	9·743752	10·256248	60
1	9·671847	9·945868	9·725979	10·274021	9·685799	9·941749	9·744050	10·255950	59
2	9·672084	9·945800	9·726284	10·273716	9·686027	9·941679	9·744348	10·255652	58
3	9·672321	9·945733	9·726588	10·273412	9·686254	9·941609	9·744646	10·255355	57
4	9·672558	9·945666	9·726892	10·273108	9·686482	9·941539	9·744943	10·255057	56
5	9·672795	9·945598	9·727197	10·272803	9·686709	9·941469	9·745240	10·254760	55
6	9·673032	9·945531	9·727501	10·272499	9·686936	9·941398	9·745538	10·254462	54
7	9·673265	9·945464	9·727805	10·272195	9·687163	9·941328	9·745835	10·254165	53
8	9·673505	9·945396	9·728109	10·271891	9·687389	9·941258	9·746132	10·253868	52
9	9·673741	9·945328	9·728412	10·271588	9·687616	9·941187	9·746429	10·253571	51
10	9·673977	9·945261	9·728716	10·271284	9·687843	9·941117	9·746726	10·253274	50
11	9·674213	9·945193	9·729020	10·270980	9·688069	9·941046	9·747023	10·252977	49
12	9·674448	9·945125	9·729323	10·270677	9·688295	9·940975	9·747319	10·252681	48
13	9·674684	9·945058	9·729626	10·270374	9·688521	9·940905	9·747616	10·252384	47
14	9·674919	9·944990	9·729929	10·270071	9·688747	9·940834	9·747913	10·252087	46
15	9·675155	9·944922	9·730233	10·269767	9·688972	9·940763	9·748209	10·251791	45
16	9·675390	9·944854	9·730535	10·269465	9·689198	9·940693	9·748505	10·251495	44
17	9·675624	9·944786	9·730838	10·269162	9·689423	9·940622	9·748801	10·251199	43
18	9·675859	9·944718	9·731141	10·268859	9·689648	9·940551	9·749097	10·250903	42
19	9·676094	9·944650	9·731444	10·268556	9·689873	9·940480	9·749393	10·250607	41
20	9·676328	9·944582	9·731746	10·268254	9·690098	9·940409	9·749689	10·250311	40
21	9·676562	9·944514	9·732048	10·267952	9·690323	9·940338	9·749985	10·250015	39
22	9·676796	9·944446	9·732351	10·267649	9·690548	9·940267	9·750281	10·249719	38
23	9·677030	9·944377	9·732653	10·267347	9·690772	9·940196	9·750576	10·249424	37
24	9·677264	9·944309	9·732955	10·267045	9·690996	9·940125	9·750872	10·249128	36
25	9·677498	9·944241	9·733257	10·266743	9·691220	9·940054	9·751167	10·248833	35
26	9·677731	9·944172	9·733558	10·266442	9·691444	9·939982	9·751462	10·248538	34
27	9·677964	9·944104	9·733860	10·266140	9·691668	9·939911	9·751757	10·248243	33
28	9·678197	9·944036	9·734162	10·265838	9·691892	9·939840	9·752052	10·247948	32
29	9·678430	9·943967	9·734463	10·265537	9·692115	9·939768	9·752347	10·247653	31
30	9·678663	9·943899	9·734764	10·265236	9·692339	9·939697	9·752642	10·247358	30
31	9·678895	9·943830	9·735066	10·264934	9·692562	9·939625	9·752937	10·247063	29
32	9·679128	9·943761	9·735367	10·264633	9·692785	9·939554	9·753231	10·246769	28
33	9·679360	9·943693	9·735668	10·264332	9·693008	9·939482	9·753526	10·246474	27
34	9·679592	9·943624	9·735969	10·264031	9·693231	9·939410	9·753820	10·246180	26
35	9·679824	9·943555	9·736269	10·263731	9·693453	9·939339	9·754115	10·245885	25
36	9·680056	9·943486	9·736570	10·263430	9·693676	9·939267	9·754409	10·245591	24
37	9·680288	9·943417	9·736870	10·263130	9·693898	9·939195	9·754703	10·245297	23
38	9·680519	9·943348	9·737171	10·262829	9·694120	9·939123	9·754997	10·245003	22
39	9·680750	9·943279	9·737471	10·262529	9·694342	9·939052	9·755291	10·244709	21
40	9·680982	9·943210	9·737771	10·262229	9·694564	9·938980	9·755585	10·244415	20
41	9·681213	9·943141	9·738071	10·261929	9·694786	9·938908	9·755878	10·244122	19
42	9·681443	9·943072	9·738371	10·261629	9·695007	9·938836	9·756172	10·243828	18
43	9·681674	9·943003	9·738671	10·261329	9·695229	9·938763	9·756465	10·243535	17
44	9·681905	9·942934	9·738971	10·261029	9·695450	9·938691	9·756759	10·243241	16
45	9·682135	9·942864	9·739271	10·260729	9·695671	9·938619	9·757052	10·242948	15
46	9·682365	9·942795	9·739570	10·260430	9 695892	9·938547	9·757345	10·242655	14
47	9·682595	9·942726	9·739870	10·260130	9·696113	9·938475	9·757638	10·242362	13
48	9·682825	9·942656	9·740169	10·259831	9·996334	9·938402	9·757931	10·242069	12
49	9·683055	9·942587	9·740468	10·259532	9·696554	9·938330	9·758224	10·241776	11
50	9·683284	9·942517	9·740767	10·259233	9·696775	9·938258	9·758517	10·241483	10
51	9·683514	9·942448	9·741066	10·258934	9·696995	9·938185	9·758810	10·241190	9
52	9·683743	9·942378	9·741365	10·258635	9·697215	9·938113	9·759102	10·240898	8
53	9·683972	9·942308	9·741664	10·258336	9·697435	9·938040	9·759395	10·240605	7
54	9·684201	9·942239	9·741962	10·258038	9·697654	9·937967	9·759687	10·240313	6
55	9·684430	9·942169	9·742261	10·257739	9·697874	9·937895	9·759979	10·240021	5
56	9·684658	9·942099	9·742559	10·257441	9·698094	9·937822	9·760272	10·239728	4
57	9·684887	9·942029	9·742858	10·257142	9·698313	9·937749	9·760564	10·239436	3
58	9·685115	9·941959	9·743156	10·256844	9·698532	9·937676	9·760856	10·239144	2
59	9·685343	9·941889	9·743454	10·256546	9·698751	9·937604	9·761148	10·238852	1
60	9·685571	9·941819	9·743752	10·256248	9·698970	9·937531	9·761439	10·238561	0
	Cosine	Sine	Cotan.	Tan.	Cosine	Sine	Cotan.	Tan.	′
	61°				60°				

′	30° Sine	Cosine	Tan.	Cotan.	31° Sine	Cosine	Tan.	Cotan.	
0	9·698970	9·937531	9·761439	10·238561	9·711839	9·933066	9·778774	10·221226	60
1	9·699189	9·937458	9·761731	10·238269	9·712050	9·932990	9·779060	10·220940	59
2	9·699407	9·937385	9·762023	10·237977	9·712260	9·932914	9·779346	10·220654	58
3	9·699626	9·937312	9·762314	10·237686	9·712469	9·932838	9·779632	10·220368	57
4	9·699844	9·937238	9·762606	10·237394	9·712679	9·932762	9·779918	10·220082	56
5	9·700062	9·937165	9·762897	10·237103	9·712889	9·932685	9·780203	10·219797	55
6	9·700280	9·937092	9·763188	10·236812	9·713098	9·932609	9·780489	10·219511	54
7	9·700498	9·937019	9·763479	10·236521	9·713308	9·932533	9·780775	10·219225	53
8	9·700716	9·936946	9·763770	10·236230	9·713517	9·932457	9·781060	10·218940	52
9	9·700933	9·936872	9·764061	10·235939	9·713726	9·932380	9·781346	10·218654	51
10	9·701151	9·936799	9·764352	10·235648	9·713935	9·932304	9·781631	10·218369	50
11	9·701368	9·936725	9·764643	10·235357	9·714144	9·932228	9·781916	10·218084	49
12	9·701585	9·936652	9·764933	10·235067	9·714352	9·932151	9·782201	10·217799	48
13	9·701802	9·936578	9·765224	10·234776	9·714561	9·932075	9·782486	10·217514	47
14	9·702019	9·936505	9·765514	10·234486	9·714769	9·931998	9·782771	10·217229	46
15	9·702236	9·936431	9·765805	10·234195	9·714978	9·931921	9·783056	10·216944	45
16	9·702452	9·936357	9·766095	10·233905	9·715186	9·931845	9·783341	10·216659	44
17	9·702669	9·936284	9·766385	10·233615	9·715394	9·931768	9·783626	10·216374	43
18	9·702885	9·936210	9·766675	10·233325	9·715602	9·931691	9·783910	10·216090	42
19	9·703101	9·936136	9·766965	10·233035	9·715809	9·931614	9·784195	10·215805	41
20	9·703317	9·936062	9·767255	10·232745	9·716017	9·931537	9·784479	10·215521	40
21	9·703533	9·935988	9·767545	10·232455	9·716224	9·931460	9·784764	10·215236	39
22	9·703749	9·935914	9·767834	10·232166	9·716432	9·931383	9·785048	10·214952	38
23	9·703964	9·935840	9·768124	10·231876	9·716639	9·931306	9·785332	10·214668	37
24	9·704179	9·935766	9·768414	10·231586	9·716846	9·931229	9·785616	10·214384	36
25	9·704395	9·935692	9·768703	10·231297	9·717053	9·931152	9·785900	10·214100	35
26	9·704610	9·935618	9·768992	10·231008	9·717259	9·931075	9·786184	10·213816	34
27	9·704825	9·935543	9·769281	10·230719	9·717466	9·930998	9·786468	10·213532	33
28	9·705040	9·935469	9·769571	10·230429	9·717673	9·930921	9·786752	10·213248	32
29	9·705254	9·935395	9·769860	10·230140	9·717879	9·930843	9·787036	10·212964	31
30	9·705469	9·935320	9·770148	10·229852	9·718085	9·930766	9·787319	10·212681	30
31	9·705683	9·935246	9·770437	10·229563	9·718291	9·930688	9·787603	10·212397	29
32	9·705898	9·935171	9·770726	10·229274	9·718497	9·930611	9·787886	10·212114	28
33	9·706112	9·935097	9·771015	10·228985	9·718703	9·930533	9·788170	10·211830	27
34	9·706326	9·935022	9·771303	10·228697	9·718909	9·930456	9·788453	10·211547	26
35	9·706539	9·934948	9·771592	10·228408	9·719114	9·930378	9·788736	10·211264	25
36	9·706753	9·934873	9·771880	10·228120	9·719320	9·930300	9·789019	10·210981	24
37	9·706967	9·934798	9·772168	10·227832	9·719525	9·930223	9·789302	10·210698	23
38	9·707180	9·934723	9·772457	10·227543	9·719730	9·930145	9·789585	10·210415	22
39	9·707393	9·934649	9·772745	10·227255	9·719935	9·930067	9·789868	10·210132	21
40	9·707606	9·934574	9·773033	10·226967	9·720140	9·929989	9·790151	10·209849	20
41	9·707819	9·934499	9·773321	10·226679	9·720345	9·929911	9·790434	10·209566	19
42	9·708032	9·934424	9·773608	10·226392	9·720549	9·929833	9·790716	10·209284	18
43	9·708245	9·934349	9·773896	10·226104	9·720754	9·929755	9·790999	10·209001	17
44	9·708458	9·934274	9·774184	10·225816	9·720958	9·929677	9·791281	10·208719	16
45	9·708670	9·934199	9·774471	10·225529	9·721162	9·929599	9·791563	10·208437	15
46	9·708882	9·934123	9·774759	10·225241	9·721366	9·929521	9·791846	10·208154	14
47	9·709094	9·934048	9·775046	10·224954	9·721570	9·929442	9·792128	10·207872	13
48	9·709306	9·933973	9·775333	10·224667	9·721774	9·929364	9·792410	10·207590	12
49	9·709518	9·933898	9·775621	10·224379	9·721978	9·929286	9·792692	10·207308	11
50	9·709730	9·933822	9·775908	10·224092	9·722181	9·929207	9·792974	10·207026	10
51	9·709941	9·933747	9·776195	10·223805	9·722385	9·929129	9·793256	10·206744	9
52	9·710153	9·933671	9·776482	10·223518	9·722588	9·929050	9·793538	10·206462	8
53	9·710364	9·933596	9·776769	10·223232	9·722791	9·928972	9·793819	10·206181	7
54	9·710575	9·933520	9·777055	10·222945	9·722994	9·928853	9·794101	10·205899	6
55	9·710786	9·933445	9·777342	10·222658	9·723197	9·928815	9·794383	10·205617	5
56	9·710997	9·933369	9·777628	10·222372	9·723400	9·928736	9·794664	10·205336	4
57	9·711208	9·933293	9·777915	10·222085	9·723603	9·928657	9·794946	10·205054	3
58	9·711419	9·933217	9·778201	10·221799	9·723805	9·928578	9·795227	10·204773	2
59	9·711629	9·933141	9·778488	10·221512	9·724007	9·928499	9·795508	10·204492	1
60	9·711839	9·933066	9·778774	10·221226	9·724210	9·928420	9·795789	10·204211	0
	Cosine	Sine	Cotan.	Tan.	Cosine	Sine	Cotan.	Tan.	
	59°				58°				

	32°				33°				
′	Sine	Cosine	Tan.	Cotan.	Sine	Cosine	Tan.	Cotan.	
0	9·724210	9·928420	9·795789	10·204211	9·736109	9·923591	9·812517	10·187483	60
1	9·724412	9·928342	9·796070	10·203930	9·736303	9·923509	9·812794	10·187206	59
2	9·724614	9·928263	9·796351	10·203649	9·736498	9·923427	9·813070	10·186930	58
3	9·724816	9·928183	9·796632	10·203368	9·736692	9·923345	9·813347	10·186653	57
4	9·725017	9·928104	9·796913	10·203087	9·736886	9·923263	9·813623	10·186377	56
5	9·725219	9·928025	9·797194	10·202806	9·737080	9·923181	9·813899	10·186101	55
6	9·725420	9·927946	9·797474	10·202526	9·737274	9·923098	9·814176	10·185824	54
7	9·725622	9·927807	9·797755	10·202245	9·737467	9·923016	9·814452	10·185548	53
8	9·725823	9·927787	9·798036	10·201964	9·737661	9·922933	9·814728	10·185272	52
9	9·726024	9·927708	9·798316	10·201684	9·737855	9·922851	9·815004	10·184996	51
10	9·726225	9·927629	9·798596	10·201404	9·738048	9·922768	9·815280	10·184720	50
11	9·726426	9·927549	9·798877	10·201123	9·738241	9·922686	9·815555	10·184445	49
12	9·726626	9·927470	9·799157	10·200843	9·738434	9·922603	9·815831	10·184169	48
13	9·726827	9·927390	9·799437	10·200563	9·738627	9·922520	9·816107	10·183893	47
14	9·727027	9·927310	9·799717	10·200283	9·738820	9·922438	9·816382	10·183618	46
15	9·727228	9·927231	9·799997	10·200003	9·739013	9·922355	9·816658	10·183342	45
16	9·727428	9·927151	9·800277	10·199723	9·739206	9·922272	9·816933	10·183067	44
17	9·727628	9·927071	9·800557	10·199443	9·739398	9·922189	9·817209	10·182791	43
18	9·727828	9·926991	9·800836	10·199164	9·739590	9·922106	9·817484	10·182516	42
19	9·728027	9·926911	9·801116	10·198884	9·739783	9·922023	9·817759	10·182241	41
20	9·728227	9·926831	9·801396	10·198604	9·739975	9·921940	9·818035	10·181965	40
21	9·728427	9·926751	9·801675	10·198325	9·740167	9·921857	9·818310	10·181690	39
22	9·728626	9·926671	9·801955	10·198045	9·740359	9·921774	9·818585	10·181415	38
23	9·728825	9·926591	9·802234	10·197766	9·740550	9·921691	9·818860	10·181140	37
24	9·729024	9·926511	9·802513	10·197487	9·740742	9·921607	9·819135	10·180865	36
25	9·729223	9·926431	9·802792	10·197208	9·740934	9·921524	9·819410	10·180590	35
26	9·729422	9·926351	9·803073	10·196928	9·741125	9·921441	9·819684	10·180316	34
27	9·729621	9·926270	9·803351	10·196649	9·741316	9·921357	9·819959	10·180041	33
28	9·729820	9·926190	9·803630	10·196370	9·741508	9·921274	9·820234	10·179766	32
29	9·730018	9·926110	9·803909	10·196091	9·741699	9·921190	9·820508	10·179492	31
30	9·730217	9·926029	9·804187	10·195813	9·741889	9·921107	9·820783	10·179217	30
31	9·730415	9·925949	9·804466	10·195534	9·742080	9·921023	9·821057	10·178943	29
32	9·730613	9·925868	9·804745	10·195255	9·742274	9·920939	9·821332	10·178668	28
33	9·730811	9·925788	9·805023	10·194977	9·742462	9·920856	9·821606	10·178394	27
34	9·731009	9·925707	9·805302	10·194698	9·742652	9·920772	9·821880	10·178120	26
35	9·731206	9·925626	9·805580	10·194420	9·742842	9·920688	9·822154	10·177846	25
36	9·731404	9·925545	9·805859	10·194141	9·743033	9·920604	9·822429	10·177571	24
37	9·731602	9·925465	9·806137	10·193863	9·743223	9·920520	9·822703	10·177297	23
38	9·731799	9·925384	9·806415	10·193585	9·743413	9·920436	9·822977	10·177023	22
39	9·731996	9·925303	9·806693	10·193307	9·743602	9·920352	9·823251	10·176749	21
40	9·732493	9·925222	9·806971	10·193029	9·743792	9·920268	9·823524	10·176476	20
41	9·732390	9·925141	9·807249	10·192751	9·743982	9·920184	9·823798	10·176202	19
42	9·732587	9·925060	9·807527	10·192473	9·744171	9·920099	9·824072	10·175928	18
43	9·732784	9·924979	9·807805	10·192195	9·744361	9·920015	9·824345	10·175655	17
44	9·732980	9·924897	9·808083	10·191917	9·744550	9·919931	9·824619	10·175381	16
45	9·733177	9·924816	9·808361	10·191639	9·744739	9·919846	9·824893	10·175107	15
46	9·733373	9·924735	9·808638	10·191362	9·744928	9·919762	9·825166	10·174834	14
47	9·733569	9·924654	9·808916	10·191084	9·745117	9·919677	9·825439	10·174561	13
48	9·733765	9·924572	9·809193	10·190807	9·745306	9·919593	9·825713	10·174287	12
49	9·733961	9·924491	9·809471	10.190529	9·745494	9·919508	9·825986	10·174014	11
50	9·734157	9·924409	9·809748	10.190252	9·745683	9·919424	9·826259	10·173741	10
51	9·734353	9·924328	9·810025	10.189975	9·745871	9·919339	9·826532	10·173468	9
52	9·734549	9·924246	9·810302	10.189698	9·746060	9·919254	9·826805	10·173195	8
53	9·734744	9·924164	9·810580	10·189420	9·746218	9·919169	9·827078	10·172922	7
54	9·734939	9·924083	9·810857	10·189143	9·746436	9·919085	9·827351	10·172649	6
55	9·735135	9·924001	9·811134	10·188866	9·746624	9·919000	9·827624	10·172376	5
56	9·735330	9·923919	9·811410	10·188590	9·746812	9·918915	9·827897	10·172103	4
57	9·735525	9·923837	9·811687	10·188313	9·746999	9·918830	9·828170	10·171830	3
58	9·735719	9·923755	9·811964	10·188036	9·747187	9·918745	9·828442	10·171558	2
59	9·735914	9·923673	9·812241	10·187759	9·747374	9·918659	9·828715	10·171285	1
60	9·736109	9·923591	9·812517	10·187483	9·747562	9·918574	9·828987	10·171013	0
	Cosine	Sine	Cotan.	Tan.	Cosine	Sine	Cotan.	Tan.	′
	57°				56°				

	34°				35°				
′	Sine	Cosine	Tan	Cotan.	Sine	Cosine	Tan.	Cotan.	
0	9·747562	9·918574	9·828987	10·171013	9·758591	9·913365	9·845227	10·154773	60
1	9·747749	9·918489	9·829260	10·170740	9·758772	9·913276	9·845496	10·154504	59
2	9·747936	9·918404	9·829532	10·170468	9·758952	9·913187	9·845764	10·154236	58
3	9·748123	9·918318	9·829805	10·170195	9·759132	9·913099	9·846033	10·153967	57
4	9·748310	9·918233	9·830077	10·169923	9·759312	9·913010	9·846302	10·153698	56
5	9·748497	9·918147	9·830349	10·169651	9·759492	9·912922	9·846570	10·153430	55
6	9·748683	9·918062	9·830621	10·169379	9·759672	9·912833	9·846839	10·153161	54
7	9·748870	9·917976	9·830893	10·169107	9·759852	9·912744	9·847108	10·152892	53
8	9·749056	9·917891	9·831165	10·168835	9·760031	9·912655	9·847376	10·152624	52
9	9·749243	9·917805	9·831437	10·168563	9·760211	9·912566	9·847644	10·152356	51
10	9·749429	9·917719	9·831709	10·168291	9·760390	9·912477	9·847913	10·152087	50
11	9·749615	9·917634	9·831981	10·168019	9·760569	9·912388	9·848181	10·151819	49
12	9·749801	9·917548	9·832253	10·167747	9·760748	9·912299	9·848449	10·151551	48
13	9·749987	9·917462	9·832525	10·167475	9·760927	9·912210	9·848717	10·151283	47
14	9·750172	9·917376	9·832796	10·167204	9·761106	9·912121	9·848986	10·151014	46
15	9·750358	9·917290	9·833068	10·166932	9·761285	9·912031	9·849254	10·150746	45
16	9·750543	9·917204	9·833339	10·166661	9·761464	9·911942	9·849522	10·150478	44
17	9·750729	9·917118	9·833611	10·166389	9·761642	9·911853	9·849790	10·150210	43
18	9·750914	9·917032	9·833882	10·166118	9·761821	9·911763	9·850057	10·149943	42
19	9·751099	9·916946	9·834154	10·165846	9·761999	9·911674	9·850325	10·149675	41
20	9·751284	9·916859	9·834425	10·165575	9·762177	9·911584	9·850593	10·149407	40
21	9·751469	9·916773	9·834696	10·165304	9·762356	9·911495	9·850861	10·149139	39
22	9·751654	9·916687	9·834967	10·165033	9·762534	9·911405	9·851129	10·148871	38
23	9·751839	9·916600	9·835238	10·164762	9·762712	9·911315	9·851396	10·148604	37
24	9·752023	9·916514	9·835509	10·164491	9·762889	9·911226	9·851664	10·148336	36
25	9·752208	9·916427	9·835780	10·164220	9·763067	9·911136	9·851931	10·148069	35
26	9·752392	9·916341	9·836051	10·163949	9·763245	9·911046	9·852199	10·147801	34
27	9·752576	9·916254	9·836322	10·163678	9·763422	9·910956	9·852466	10·147534	33
28	9·752760	9·916167	9·836593	10·163407	9·763600	9·910866	9·852733	10·147267	32
29	9·752944	9·916081	9·836864	10·163136	9·763777	9·910776	9·853001	10·146999	31
30	9·753128	9·915994	9·837134	10·162866	9·763954	9·910686	9·853268	10·146732	30
31	9·753312	9·915907	9·837405	10·162595	9·764131	9·910596	9·853535	10·146465	29
32	9·753495	9·915820	9·837675	10·162325	9·764308	9·910506	9·853802	10·146198	28
33	9·753679	9·915733	9·837946	10·162054	9·764485	9·910415	9·854069	10·145931	27
34	9·753862	9·915646	9·838216	10·161784	9·764662	9·910325	9·854336	10·145664	26
35	9·754046	9·915559	9·838487	10·161513	9·764838	9·910235	9·854603	10·145397	25
36	9·754229	9·915472	9·838757	10·161243	9·765015	9·910144	9·854870	10·145130	24
37	9·754412	9·915385	9·839027	10·160973	9·765191	9·910054	9·855137	10·144863	23
38	9·754595	9·915297	9·839297	10·160703	9·765367	9·909963	9·855404	10·144596	22
39	9·754778	9·915210	9·839568	10·160432	9·765544	9·909873	9·855671	10·144329	21
40	9·754960	9·915123	9·839838	10·160162	9·765720	9·909782	9·855938	10·144062	20
41	9·755143	9·915035	9·840108	10·159892	9·765896	9·909691	9·856204	10·143796	19
42	9·755326	9·914948	9·840378	10·159622	9·766072	9·909601	9·856471	10·143529	18
43	9·755508	9·914860	9·840648	10·159352	9·766247	9·909510	9·856737	10·143263	17
44	9·755690	9·914773	9·840917	10·159083	9·766423	9·909419	9·857004	10·142996	16
45	9·755872	9·914685	9·841187	10·158813	9·766598	9·909328	9·857270	10·142730	15
46	9·756054	9·914598	9·841457	10·158543	9·766774	9·909237	9·857537	10·142463	14
47	9·756236	9·914510	9·841727	10·158273	9·766949	9·909146	9·857803	10·142197	13
48	9·756418	9·914422	9·841996	10·158004	9·767124	9·909055	9·858069	10·141931	12
49	9·756600	9·914334	9·842266	10·157734	9·767300	9·908964	9·858336	10·141664	11
50	9·756782	9·914246	9·842535	10·157465	9·767475	9·908873	9·858602	10·141398	10
51	9·756963	9·914158	9·842805	10·157195	9·767649	9·908781	9·858868	10·141132	9
52	9·757144	9·914070	9·843074	10·156926	9·767824	9·908690	9·859134	10·140866	8
53	9·757326	9·913982	9·843343	10·156657	9·767999	9·908599	9·859400	10·140600	7
54	9·757507	9·913894	9·843612	10·156388	9·768173	9·908507	9·859666	10·140334	6
55	9·757688	9·913806	9·843882	10·156118	9·768348	9·908416	9·859932	10·140068	5
56	9·757869	9·913718	9·844151	10·155849	9·768522	9·908324	9·860198	10·139802	4
57	9·758050	9·913630	9·844420	10·155580	9·768697	9·908233	9·860464	10·139536	3
58	9·758230	9·913541	9·844689	10·155311	9·768871	9·908141	9·860730	10·139270	2
59	9·758411	9·913453	9·844958	10·155042	9·769045	9·908049	9·860995	10·139005	1
60	9·758591	9·913365	9·845227	10·154773	9·769219	9·907958	9·861261	10·138739	0
	Cosine	Sine	Cotan.	Tan.	Cosine	Sine	Cotan.	Tan.	′
	55°				54°				

	36°				37°				
′	Sine	Cosine	Tan.	Cotan.	Sine	Cosine	Tan.	Cotan.	
0	9·769219	9·907958	9·861261	10·138749	9·779463	9·902359	9·874114	10·122886	60
1	9·769393	9·907866	9·861527	10·138473	9·779631	9·902253	9·877377	10·122623	59
2	9·769566	9·907774	9·861792	10·138208	9·779798	9·902158	9·877640	10·122360	58
3	9·769740	9·907682	9·862058	10·137942	9·779966	9·902063	9·877903	10·122097	57
4	9·769913	9·907590	9·862323	10·137677	9·780133	9·901967	9·878165	10·121835	56
5	9·770087	9·907498	9·862589	10·137411	9·780300	9·901872	9·878428	10·121572	55
6	9·770260	9·907406	9·862854	10·137146	9·780467	9·901776	9·878691	10·121309	54
7	9·770433	9·907314	9·863119	10·136881	9 780634	9·901681	9·878953	10·121047	53
8	9·770606	9·907222	9·863385	10·136615	9·780801	9·901585	9·879216	10·120784	52
9	9·770779	9·907129	9·863650	10·136350	9·780968	9·901490	9·879478	10·120522	51
10	9·770952	9·907037	9·863915	10·136085	9·781134	9·901394	9·879741	10·120259	50
11	9·771425	9·906945	9·864180	10·135820	9·781301	9·901298	9·880003	10·119997	49
12	9·771298	9·906852	9·864445	10·135555	9·781468	9·901202	9·880265	10·119735	48
13	9·771470	9·906760	9·864710	10·135290	9·781634	9·901106	9·880528	10·119472	47
14	9·771643	9·906667	9·864975	10·135025	9·781800	9·901010	9·880790	10·119210	46
15	9·771815	9·906575	9·865240	10·134760	9·781966	9·900914	9·881052	10·118918	45
16	9·771987	9·906482	9·865505	10·134495	9·782132	9·900818	9·881314	10·118686	44
17	9·772159	9·906389	9·865770	10·134230	9·782298	9·900722	9·881577	10·118423	43
18	9·772331	9·906296	9·866035	10·133965	9·782464	9·900626	9·881839	10·118161	42
19	9·772503	9·906204	9·866300	10·133700	9·782630	9·900529	9·882101	10·117899	41
20	9·772675	9·906111	9·866564	10·133436	9·782796	9·900433	9·882363	10·117637	40
21	9·772847	9·906018	9·866829	10·133171	9·782961	9·900337	9·882625	10·117375	39
22	9·773018	9·905925	9·867094	10·132906	9·783127	9·900240	9·882887	10·117113	38
23	9·773190	9·905832	9·867358	10·132642	9·783292	9·900144	9·883148	10·116852	37
24	9·773361	9·905739	9·867623	10·132377	9·783458	9·900047	9·883410	10·116590	36
25	9·773533	9·905645	9·867887	10·132113	9·783623	9·899951	9·883672	10·116328	35
26	9·773704	9·905552	9·868152	10·131848	9·783788	9·899854	9·883934	10·116066	34
27	9·773875	9·905459	9·868416	10·131584	9·783953	9·899757	9·884196	10·115804	33
28	9·774046	9·905366	9·868680	10·131320	9·784118	9·899660	9·884457	10·115543	32
29	9·774217	9·905272	9·868945	10·131055	9·784282	9·899564	9·884719	10·115281	31
30	9·774388	9·905179	9·869209	10·130791	9·784447	9·899467	9·884980	10·115020	30
31	9·774558	9·905085	9·869473	10·130527	9·784612	9·899370	9·885242	10·114758	29
32	9·774729	9·904992	9·869737	10·130263	9·784776	9·899273	9·885504	10·114496	28
33	9·774899	9·904898	9·870001	10·129999	9·784941	9·899176	9·885765	10·114235	27
34	9·775070	9·904804	9·870265	10·129735	9·785105	9·899078	9·886026	10·113974	26
35	9·775240	9·904711	9·870529	10·129471	9·785269	9·898981	9·886288	10·113712	25
36	9·775410	9·904617	9·870793	10·129207	9·785433	9·898884	9·886549	10·113451	24
37	9·775580	9·904523	9·871057	10·128943	9·785597	9·898787	9·886811	10·113189	23
38	9·775759	9·904429	9·871321	10·128679	9·785761	9·898689	9·887072	10·112928	22
39	9·775920	9·904335	9·871585	10·128415	9·785925	9·898592	9·887333	10·112667	21
40	9·776090	9·904241	9·871849	10·128151	9·786089	9·898494	9·887594	10·112406	20
41	9·776259	9·904147	9·872112	10·127888	9·786252	9·898397	9·887855	10·112145	19
42	9·776429	9·904053	9·872376	10·127624	9·786416	9·898299	9·888116	10·111884	18
43	9·776598	9·903959	9·872640	10·127360	9·786579	9·898202	9·888378	10·111622	17
44	9·776768	9·903864	9·872903	10·127097	9·786742	9·898104	9·888639	10·111361	16
45	9·776937	9·903770	9·873167	10·126833	9·786906	9·898006	9·888900	10·111100	15
46	9·777106	9·903676	9·873430	10·126570	9·787069	9·897908	9·889161	10·110839	14
47	9·777275	9·903581	9·873694	10·126306	9·787232	9·897810	9·889421	10·110579	13
48	9·777444	9·903487	9·873957	10·126043	9·787395	9·897712	9·889682	10·110318	12
49	9·777613	9·903392	9·874220	10·125780	9·787557	9·897614	9·889943	10·110057	11
50	9·777781	9·903298	9·874184	10·125516	9·787720	9·897516	9·890204	10·109796	10
51	9·777950	9·903203	9·874747	10·125253	9·787883	9·897148	9·890465	10·109535	9
52	9·778119	9·903108	9·875010	10·124990	9·788045	9·897320	9·890725	10·109275	8
53	9·778287	9·903014	9·875273	10·124727	9·788208	9·897222	9·890986	10·109014	7
54	9·778455	9·902919	9·875537	10·124463	9·788370	9·897123	9·891247	10·108753	6
55	9·778624	9·902824	9·8758 0	10·124200	9·788532	9·897025	9·891507	10·108493	5
56	9·778792	9·902729	9·876063	10·123937	9·788694	9·896926	9·891768	10·108232	4
57	9·778960	9·902634	9·876326	10·123674	9·788856	9·896828	9·892028	10·107972	3
58	9·779128	9·902539	9·876589	10·123411	9·789018	9·896729	9·892289	10·107711	2
59	9·779295	9·902444	9·876852	10·123148	9·789180	9·896631	9·892549	10·107451	1
60	9·779463	9·902349	9·877114	10·122886	9·789342	9·896532	9·892810	10·107190	0
	Cosine	Sine	Cotan.	Tan.	Cosine	Sine	Cotan.	Tan.	′
	53°				52°				

	38°				39°				
′	Sine	Cosine	Tan.	Cotan.	Sine	Cosine	Tan.	Cotan.	
0	9·789342	9·896532	9·892810	10·107190	9·798872	9·890503	9·908369	10·091631	60
1	9·789504	9·896433	9·893070	10·106930	9·799028	9·890400	9·908628	10·091372	59
2	9·789665	9·896335	9·893331	10·106669	9·799184	9·890298	9·908886	10·091114	58
3	9·789827	9·896236	9·893591	10·106409	9·799339	9·890195	9·909144	10·090856	57
4	9·789988	9·896137	9·893851	10·106149	9·799495	9·890093	9·909402	10.090598	56
5	9·790149	9·896038	9·894111	10·105889	9·799651	9·889990	9·909660	10·090340	55
6	9·790310	9·895939	9·894372	10·105368	9·799806	9·889888	9·909918	10·090082	54
7	9·790471	9·895840	9·894632	10·105628	9·799962	9·889785	9·910177	10·089823	53
8	9·790632	9·895741	9·894892	10·105108	9·800117	9·889682	9·910435	10·089565	52
9	9·790793	9·895641	9·895152	10·104848	9·800272	9·889579	9·910693	10·089307	51
10	9·790954	9·895542	9·895412	10·104588	9·800427	9·889477	9·910951	10·089049	50
11	9·791115	9·895443	9·895672	10·104328	9·800582	9·889374	9·911209	10·088791	49
12	9·791275	9·895343	9·895932	10·104068	9·800737	9·889271	9·911467	10·088533	48
13	9·791436	9·895244	9·896192	10·103808	9·800892	9·889168	9·911725	10·088275	47
14	9·791596	9·895145	9·896452	10·103548	9·801047	9·889064	9·911982	10·088018	46
15	9·791757	9·895045	9·896712	10·103288	9·801201	9·888961	9·912240	10·087760	45
16	9·791917	9·894945	9·896971	10·103029	9·801356	9·888858	9·912498	10·087502	44
17	9·792077	9·894846	9·897231	10·102769	9·801511	9·888755	9·912756	10·087244	43
18	9·792237	9·894746	9·897491	10·102509	9·801665	9·888651	9·913014	10·086986	42
19	9·792397	9·894646	9·897751	10·102249	9·801819	9·888548	9·913271	10·086729	41
20	9·792557	9·894546	9·898010	10·101990	9·801973	9·888444	9·913529	10·086471	40
21	9·792716	9·894446	9·898270	10·101730	9·802128	9·888341	9·913787	10·086213	39
22	9·792876	9·894346	9·898530	10·101470	9·802282	9·888237	9·914044	10·085956	38
23	9·793035	9·894246	9·898789	10·101211	9·802436	9·888134	9·914302	10·085698	37
24	9·793195	9·894146	9·899049	10·100951	9·802589	9·888030	9·914560	10·085440	36
25	9·793354	9·894046	9·899308	10·100692	9·802743	9·887926	9·914817	10·085183	35
26	9·793514	9·893946	9·899568	10·100432	9·802897	9·887822	9·915075	10·084925	34
27	9·793673	9·893846	9·899827	10·100173	9·803050	9·887718	9·915332	10·084668	33
28	9·793832	9·893745	9·900087	10·099913	9·803204	9·887614	9·915590	10·084410	32
29	9·793991	9·893645	9·900346	10·099654	9·803357	9·887510	9·915847	10·084153	31
30	9·794150	9·893544	9·900605	10·099395	9·803511	9·887406	9·916104	10·083896	30
31	9·794308	9·893444	9·900864	10·099136	9·803664	9·887302	9·916362	10·083638	29
32	9·794467	9·893343	9·901124	10·098876	9·803817	9·887198	9·916619	10·083381	28
33	9·794626	9·893243	9·901383	10·098617	9·803970	9·887093	9·916877	10·083123	27
34	9·794784	9·893142	9·901642	10·098358	9·804123	9·886989	9·917134	10·082866	26
35	9·794942	9·893041	9·901901	10·098099	9·804276	9·886885	9·917391	10·082609	25
36	9·795101	9·892940	9·902160	10·097840	9·804428	9·886780	9·917648	10·082352	24
37	9·795259	9.892839	9·902420	10·097580	9·804581	9·886676	9·917906	10·082094	23
38	9·795417	9·892739	9·902679	10·097321	9·804734	9·886571	9·918163	10·081837	22
39	9·795575	9·892638	9·902938	10·097062	9·804886	9·886466	9·918420	10·081580	21
40	9·795733	9·892536	9·903197	10·096803	9·805039	9·886362	9·918677	10·081323	20
41	9·795891	9·892435	9·903456	10·096544	9·805191	9·886257	9·918934	10·081066	19
42	9·796049	9·892334	9·903714	10·096286	9·805343	9·886152	9·919191	10·080809	18
43	9·796206	9·892233	9·903973	10·096027	9·805495	9·886047	9·919448	10·080552	17
44	9·796364	9·892132	9·904232	10·095768	9·805647	9·885942	9·919705	10·080295	16
45	9·796521	9·892030	9·904491	10·095509	9·805799	9·885837	9·919962	10·080038	15
46	9·796679	9·891929	9·904750	10·095250	9·805951	9·885732	9·920219	10·079781	14
47	9·796836	9·891827	9·905008	10·094992	9·806103	9·885627	9·920476	10·079524	13
48	9·796993	9·891726	9·905267	10·094733	9·806253	9·885522	9·920733	10·079267	12
49	9·797150	9·891624	9·905526	10·094474	9·806406	9·885416	9·920990	10·079010	11
50	9·797307	9·891523	9·905785	10·094215	9·806557	9·885311	9·921247	10·078753	10
51	9·797464	9·891421	9·906043	10·093957	9·806709	9·885205	9·921503	10·078497	9
52	9·797621	9·891319	9·906302	10·093698	9·806860	9·885100	9·921760	10·078240	8
53	9·797777	9·891217	9·906560	10·093440	9·807011	9·884994	9·922017	10·077983	7
54	9·797934	9·891115	9·906819	10·093181	9·807163	9·884889	9·922274	10·077726	6
55	9·798091	9·891013	9·907077	10·092923	9·807314	9·884783	9·922530	10·077470	5
56	9·798247	9·890911	9·907336	10·092664	9·807465	9·884677	9·922787	10·077213	4
57	9·798403	9·890809	9·907594	10·092406	9·807615	9·884572	9·923044	10·076956	3
58	9·798560	9·890707	9·907853	10·092147	9·807766	9·884466	9·923300	10·076700	2
59	9·798716	9·890605	9·908111	10·091889	9·807917	9·884360	9·923557	10·076443	1
60	9·798872	9·890503	9·908369	10·091631	9·808067	9·884254	9·923814	10·076186	0
	Cosine	Sine	Cotan.	Tan.	Cosine	Sine	Cotan.	Tan.	′
	51°				50°				

	40°				41°				
′	Sine	Cosine	Tan.	Cotan.	Sine	Cosine	Tan.	Cotan.	
0	9·808067	9·884254	9·923814	10·076186	9·816943	9·877780	9·939163	10·060837	60
1	9·808218	9·884148	9·924070	10·075930	9·817088	9·877670	9·939418	10·060582	59
2	9·808368	9·884042	9·924327	10·075673	9·817233	9·877560	9·939673	10·060327	58
3	9·808519	9·883936	9·924583	10·075417	9·817379	9·877450	9·939928	10·060072	57
4	9·808669	9·883829	9·924840	10·075160	9·817524	9·877340	9·940183	10·059817	56
5	9·808819	9·883723	9·925096	10·074904	9·817668	9·877230	9·940439	10·059561	55
6	9·808969	9·883617	9·925352	10·074648	9·817813	9·877120	9·940694	10·059306	54
7	9·809119	9·883510	9·925609	10·074391	9·817958	9·877010	9·940949	10·059051	53
8	9·809269	9·883404	9·925865	10·074135	9·818103	9·876899	9·941204	10·058796	52
9	9·809419	9·883297	9·926122	10·073878	9·818247	9·876789	9·941459	10·058541	51
10	9·809569	9·883191	9·926378	10·073622	9·818392	9·876678	9·941713	10·058287	50
11	9·809718	9·883084	9·926634	10·073366	9·818536	9·876568	9·941968	10·058032	49
12	9·809868	9·882977	9·926890	10·073110	9·818681	9·876457	9·942223	10·057777	48
13	9·810017	9·882871	9·927147	10·072853	9·818825	9·876347	9·942478	10·057522	47
14	9·810167	9·882764	9·927403	10·072597	9·818969	9·876236	9·942733	10·057267	46
15	9·810316	9·882657	9·927659	10·072341	9·819113	9·876125	9·942988	10·057012	45
16	9·810465	9·882550	9·927915	10·072085	9·819257	9·876014	9·943243	10·056757	44
17	9·810614	9·882443	9·928171	10·071829	9·819401	9·875904	9·943498	10·056502	43
18	9·810763	9·882336	9·928427	10·071573	9·819545	9·875793	9·943752	10·056248	42
19	9·810912	9·882229	9·928684	10·071316	9·819689	9·875682	9·944007	10·055993	41
20	9·811061	9·882121	9·928940	10·071060	9·819832	9·875571	9·944262	10·055738	40
21	9·811210	9·882014	9·929196	10·070804	9·819976	9·875459	9·944517	10·055483	39
22	9·811358	9·881907	9·929452	10·070548	9·820120	9·875348	9·944771	10·055229	38
23	9·811507	9·881799	9·929708	10·070292	9·820263	9·875237	9·945026	10·054974	37
24	9·811655	9·881692	9·929964	10·070036	9·820406	9·875126	9·945281	10·054719	36
25	9·811804	9·881584	9·930220	10·069780	9·820550	9·875014	9·945535	10·054465	35
26	9·811952	9·881477	9·930475	10·069525	9·820693	9·874903	9·945790	10·054210	34
27	9·812100	9·881369	9·930731	10·069269	9·820836	9·874791	9·946045	10·053955	33
28	9·812248	9·881261	9·930987	10·069013	9·820979	9·874680	9·946299	10·053701	32
29	9·812396	9·881153	9·931243	10·068757	9·821122	9·874568	9·946554	10·053446	31
30	9·812544	9·881046	9·931499	10·068501	9·821265	9·874456	9·946808	10·053192	30
31	9·812692	9·880938	9·931755	10·068245	9·821407	9·874344	9·947063	10·052937	29
32	9·812840	9·880830	9·932010	10·067990	9·821550	9·874232	9·947318	10·052682	28
33	9·812988	9·880722	9·932266	10·067734	9·821693	9·874121	9·947572	10·052428	27
34	9·813135	9·880613	9·932522	10·067478	9·821835	9·874009	9·947827	10·052173	26
35	9·813283	9·880505	9·932778	10·067222	9·821977	9·873896	9·948081	10·051919	25
36	9·813430	9·880397	9·933033	10·066967	9·822120	9·873784	9·948335	10·051665	24
37	9·813578	9·880289	9·933289	10·066711	9·822262	9·873672	9·948590	10·051410	23
38	9·813725	9·880180	9·933545	10·066455	9·822404	9·873560	9·948844	10·051156	22
39	9·813872	9·880072	9·933800	10·066200	9·822546	9·873448	9·949099	10·050901	21
40	9·814019	9·879963	9·934056	10·065944	9·822688	9·873335	9·949353	10·050647	20
41	9·814166	9·879855	9·934311	10·065689	9·822830	9·873223	9·949608	10·050392	19
42	9·814313	9·879746	9·934567	10·065433	9·822972	9·873110	9·949862	10·050138	18
43	9·814460	9·879637	9·934822	10·065178	9·823114	9·872998	9·950116	10·049884	17
44	9·814607	9·879529	9·935078	10·064922	9·823255	9·872885	9·950371	10·049629	16
45	9·814753	9·879420	9·935333	10·064667	9·823397	9·872772	9·950625	10·049375	15
46	9·814900	9·879311	9·935589	10·064411	9·823539	9·872659	9·950879	10·049121	14
47	9·815046	9·879202	9·935844	10·064156	9·823680	9·872547	9·951133	10·048867	13
48	9·815193	9·879093	9·936100	10·063900	9·823821	9·872434	9·951388	10·048612	12
49	9·815339	9·878984	9·936355	10·063645	9·823963	9·872321	9·951642	10·048358	11
50	9·815485	9·878875	9·936611	10·063389	9·824104	9·872208	9·951896	10·048104	10
51	9·815632	9·878766	9·936866	10·063134	9·824245	9·872095	9·952150	10·047850	9
52	9·815778	9·878656	9·937121	10·062879	9·824386	9·871981	9·952405	10·047595	8
53	9·815924	9·878547	9·937377	10·062623	9·824527	9·871868	9·952659	10·047341	7
54	9·816069	9·878438	9·937632	10·062368	9·824668	9·871755	9·952913	10·047087	6
55	9·816215	9·878328	9·937887	10·062113	9·824808	9·871641	9·953167	10·046832	5
56	9·816361	9·878219	9·938142	10·061858	9·824949	9·871528	9·953421	10·046579	4
57	9·816507	9·878109	9·938398	10·061602	9·825090	9·871414	9·953675	10·046325	3
58	9·816652	9·877999	9·938653	10·061347	9·825230	9·871301	9·953929	10·046071	2
59	9·816798	9·877890	9·938908	10·061092	9·825371	9·871187	9·954183	10·045817	1
60	9·816943	9·877780	9·939163	10·060837	9·825511	9·871073	9·954437	10·045563	0
	Cosine	Sine	Cotan.	Tan.	Cosine	Sine	Cotan.	Tan.	′
	49°				48°				

	42°				43°				
′	Sine	Cosine	Tan.	Cotan.	Sine	Cosine	Tan.	Cotan.	
0	9·825511	9·871073	9·954437	10·045563	9·833783	9·864127	9·969656	10·030344	60
1	9·825651	9·870960	9·954691	10·045309	9·833919	9·864010	9·969909	10·030091	59
2	9·825791	9·870846	9·954946	10·045054	9·834054	9·863892	9·970162	10·029838	58
3	9·825931	9·870732	9·955200	10·044800	9·834189	9·863774	9·970416	10·029584	57
4	9·826071	9·870618	9·955454	10·044546	9·834325	9·863656	9·970669	10·029331	56
5	9·826211	9·870504	9·955708	10·044292	9·834460	9·863538	9·970922	10·029078	55
6	9·826351	9·870390	9·955961	10·044039	9·834595	9·863419	9·971175	10·028825	54
7	9·826491	9·870276	9·956215	10·043785	9·834730	9·863301	9·971429	10·028571	53
8	9·826631	9·870161	9·956469	10·043531	9·834865	9·863183	9·971682	10·028318	52
9	9·826770	9·870047	9·956723	10·043277	9·834999	9·863064	9·971935	10·028065	51
10	9·826910	9·869933	9·956977	10·043023	9·835134	9·862946	9·972188	10·027812	50
11	9·827049	9·869818	9·957231	10·042769	9·835269	9·862827	9·972441	10·027559	49
12	9·827189	9·869704	9·957485	10·042515	9·835403	9·862709	9·972695	10·027305	48
13	9·827328	9·869589	9·957739	10·042261	9·835538	9·862590	9·972948	10·027052	47
14	9·827467	9·869474	9·957993	10·042007	9·835672	9·862471	9·973201	10·026799	46
15	9·827606	9·869360	9·958247	10·041753	9·835807	9·862353	9·973454	10·026546	45
16	9·827745	9·869245	9·958500	10·041500	9·835941	9·862234	9·973707	10·026293	44
17	9·827884	9·869130	9·958754	10·041246	9·836075	9·862115	9·973960	10·029040	43
18	9·828023	9·869015	9·959008	10·040992	9·836209	9·861996	9·974213	10 025787	42
19	9·828162	9·868900	9·959262	10·040738	9·836343	9·861877	9·974466	10·025534	41
20	9·828301	9·868785	9·959516	10·040484	9·836477	9·861758	9·974720	10·025280	40
21	9·828439	9·868670	9·959769	10·040231	9·836611	9·861638	9·974973	10·025027	39
22	9·828578	9·868555	9·960023	10·039977	9·836745	9·861519	9·975226	10·024774	38
23	9·828716	9·868440	9·960277	10·039723	9·836878	9·861400	9·975479	10·024521	37
24	9·828855	9·868324	9·960530	10·039470	9·837012	9·861280	9·975732	10·024268	36
25	9·828993	9·868209	9·960784	10·039216	9·837146	9·891161	9·975985	10·024015	35
26	9·829131	9·868093	9·961038	10·038962	9·837279	9·861041	9·976238	10·023762	34
27	9·829269	9·867978	9·961292	10·038708	9·837412	9·860922	9·976491	10·023509	33
28	9·829407	9·867862	9·961545	10·038455	9·837546	9·860802	9·976744	10·023256	32
29	9·829545	9·867747	9·961799	10·038201	9·837679	9·860682	9·976997	10·023003	31
30	9·829683	9·867631	9·962052	10·037948	9·837812	9·860562	9·977250	10·022750	30
31	9·829821	9·867515	9·962306	10·037694	9·837945	9·860442	9·977503	10·022497	29
32	9·829959	9·867399	9·962560	10·037440	9·838078	9·860322	9·977756	10·022244	28
33	9·830097	9·867283	9·962813	10·037187	9·838211	9·860202	9·978009	10 021991	27
34	9·830234	9·867167	9·963067	10·036933	9·838344	9·860082	9·978262	10·021738	26
35	9·830372	9·867051	9·963320	10·036680	9·838477	9·859962	9·978515	10·021485	25
36	9·830509	9·866935	9·963574	10·036426	9·838610	9·859842	9·978768	10·021232	24
37	9·830646	9·866819	9·963828	10·036172	9·838742	9·859721	9·979021	10·020979	23
38	9·830784	9·866703	9·964081	10·035919	9·838875	9·859601	9·979274	10·020726	22
39	9·830921	9·866586	9·964335	10·035665	9·839007	9·859480	9·979527	10·020473	21
40	9·831058	9·866470	9·964588	10·035412	9·839140	9·859360	9·979780	10·020220	20
41	9·831195	9·866353	9·964842	10·035158	9·839272	9 859239	9·980033	10·019967	19
42	9·831332	9·866237	9·965095	10·034905	9·839404	9·859119	9·980286	10·019714	18
43	9·831469	9·866120	9·965349	10·034651	9·839536	9·858998	9·980538	10·019462	17
44	9·831606	9·866004	9·965602	10·034398	9·839668	9·858877	9·980791	10·019209	16
45	9·831742	9·865887	9·965855	10·034145	9·839800	9·858756	9·981044	10·018956	15
46	9·831879	9·865770	9·966109	10·033891	9·839932	9·858635	9·981297	10·018703	14
47	9·832015	9·865653	9·966362	10·033638	9·840064	9·858514	9·981550	10·018450	13
48	9·832152	9·865536	9·966616	10·033384	9·840196	9·858393	9·981803	10·018197	12
49	9·832288	9·865419	9·966869	10·033131	9·840328	9·858272	9·982056	10·017944	11
50	9·832425	9·865302	9·967123	10·032877	9·840459	9·858151	9·982309	10·017691	10
51	9·832561	9·865185	9·967376	10·032624	9·840591	9·858029	9·982562	10·017438	9
52	9·832697	9·865068	9·967629	10·032371	9·840722	9·857908	9·982814	10·017186	8
53	9·832833	9·864950	9·967883	10·032117	9·840854	9·857786	9·983067	10·016933	7
54	9·832969	9·864833	9·968136	10·031864	9·840985	9·857665	9·983320	10·016680	6
55	9·833105	9 864716	9·968389	10·031611	9·841116	9·857543	9·983573	10·016427	5
56	9·833241	9·864598	9·968643	10·031357	9·841247	9·857422	9·983826	10·016174	4
57	9·833377	9·864481	9·968896	10·031104	9·841378	9·857300	9·984079	10·015921	3
58	9·833512	9·864363	9·969149	10·030851	9·841509	9·857178	9·984332	10·015668	2
59	9·833648	9·864245	9·969403	10·030597	9·841640	9·857056	9·984584	10·015416	1
60	9·833783	9·864127	9·969656	10·030344	9·841771	9·856934	9·984837	10·015163	0
	Cosine	Sine	Cotan.	Tan.	Cosine	Sine	Cotan.	Tan.	′
	47°				46°				

44°

′	Sine	Cosine	Tan.	Cotan.	
0	9·841771	9·856934	9·984837	10·015163	60
1	9·841902	9·856812	9·985090	10·014910	59
2	9·842033	9·856690	9·985343	10·014657	58
3	9·842163	9·856568	9·985596	10·014404	57
4	9·842294	9·856446	9·985848	10·014152	56
5	9·842424	9·856323	9·986101	10·013899	55
6	9·842555	9·856201	9·986354	10·013646	54
7	9·842685	9·856078	9·986607	10·013393	53
8	9·842815	9·855956	9·986860	10·013140	52
9	9·842946	9·855833	9·987112	10·012888	51
10	9·843076	9·855711	9·987365	10·012635	50
11	9·843206	9·855588	9·987618	10·012382	49
12	9·843336	9·855465	9·987871	10·012129	48
13	9·843466	9·855342	9·988123	10·011877	47
14	9·843595	9·855219	9·988376	10·011624	46
15	9·843725	9·855096	9·988629	10·011371	45
16	9·843855	9·854973	9·988882	10·011118	44
17	9·843984	9·854850	9·989134	10·010866	43
18	9·844114	9·854727	9·989387	10·010613	42
19	9·844243	9·854603	9·989640	10·010360	41
20	9·844372	9·954480	9·989893	10·010107	40
21	9·844502	9·854356	9·990145	10·009855	39
22	9·844631	9·854233	9·990398	10·009602	38
23	9·844760	9·85[illegible]09	9·990651	10·009349	37
24	9·844889	9·853986	9·990903	10·009097	36
25	9·845018	9·853862	9·991156	10·008844	35
26	9·845147	9·853738	9·991409	10·008591	34
27	9·845276	9·853614	9·991662	10·008338	33
28	9·845405	9·853490	9·991914	10·008086	32
29	9·845533	9·853366	9·992167	10·007833	31
30	9·845662	9·853242	9·992420	10·007580	30
31	9·845790	9·853118	9·992672	10·007328	29
32	9·845919	9·852994	9·992925	10·007075	28
33	9·846047	9·852869	9·993178	10·006822	27
34	9·846175	9·852745	9·993431	10·006569	26
35	9·846304	9·852620	9·993683	10·006317	25
36	9·846432	9·852496	9·993936	10·006064	24
37	9·846560	9·852371	9·994189	10·005811	23
38	9·846688	9·852247	9·994441	10·005559	22
39	9·846816	9·852122	9·994694	10·005306	21
40	9·846944	9·851997	9·994947	10·005053	20
41	9·847071	9·851872	9·995199	10·004801	19
42	9·847199	9·851747	9·995452	10·004548	18
43	9·847327	9·851622	9·995705	10·004295	17
44	9·847454	9·851497	9·995957	10·004043	16
45	9·847582	9·851372	9·996210	10·003790	15
46	9·847709	9·851246	9·996463	10·003537	14
47	9·847836	9·851121	9·996715	10·003285	13
48	9·847964	9·850996	9·996968	10·003032	12
49	9·848091	9·850870	9·997221	10.002779	11
50	9·848218	9·850745	9·997473	10.002527	10
51	9·848345	9·850619	9·997726	10.002274	9
52	9·848472	9·850493	9·997979	10.002021	8
53	9·848599	9·850368	9·998231	10·001769	7
54	9·848726	9·850242	9·998484	10·001516	6
55	9·848852	9·850116	9·998737	10·001263	5
56	9·848979	9·849990	9·998989	10·001011	4
57	9·849106	9·849864	9·999242	10·000758	3
58	9·849232	9·849738	9·999495	10·000505	2
59	9·849358	9·849611	9·999747	10·000253	1
60	9·849495	9·849485	10.00000	10·000000	0
	Cosine	Sine	Cotan.	Tan.	′

45°

TRAVERSE TABLES,

CALCULATED TO ANY NUMBER

OF

CHAINS, OR LINKS OF DISTANCE,

AND TO

THREE MINUTES OF THE ANGLE OF BEARING.*

* *The principle and method of using these tables will be found in the Third Part, page* **193.**

1		2		3		4		5		6		7		8		9		10		Chs.
at.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat	Dep	Lat.	Dep.	Lat.	Dep.	′
·00	·001	2·00	·002	3·00	003	4·00	·003	5·00	·004	6·00	·005	7·00	·006	8·00	·007	9·00	·008	10·00	·009	57
·00	·002	2·00	003	3·00	·005	4·00	·007	5·00	·008	6·00	·010	7·00	012	8·00	014	9·00	·016	10·00	·017	54
·00	·003	2·00	·005	3·00	·008	4·00	·010	5·00	·013	6·00	·016	7·00	·018	8·00	·020	9·00	·023	10·00	·026	51
·00	·003	2·00	·007	3·00	·010	4·00	014	5·00	·017	6·00	·021	7·00	·024	8·00	·028	9·00	031	10·00	·035	48
·00	·004	2·00	·009	3·00	·013	4·00	·017	5·00	·022	6·00	·026	7·00	·030	8·00	·035	9·00	039	10·00	·043	45
·00	005	2·00	010	3·00	016	4·00	021	5·00	·026	6·00	·031	7·00	·037	8·00	·042	9·00	·047	10·00	·052	42
·00	·006	2·00	·012	3·00	·018	4·00	·024	5·00	·030	6·00	·037	7·00	·043	8·00	·049	9·00	·055	10·00	·061	39
·00	·007	2·00	·014	3·00	·021	4·00	·028	5·00	·035	6·00	·042	7·00	049	8·00	·056	9·00	·061	10·00	·070	36
·00	·008	2·00	·016	3·00	·023	4·00	·031	5·00	039	6·00	·047	7·00	055	8·00	·063	9·00	073	10·00	·078	33
·00	·009	2·00	·018	3·00	·027	4·00	·036	5·00	045	6·00	·053	7·00	·062	8·00	·071	9·00	·079	10·00	·086	30
·00	·010	2·00	·019	3·00	·029	4·00	·038	5·00	048	6·00	·058	7·00	·067	8·00	·077	9·00	·086	10·00	·096	27
·00	·011	2·00	·021	3·00	·032	4·00	·042	5·00	055	6·00	·063	7·00	·074	8·00	·084	9·00	·095	10·00	·105	24
·00	·011	2·00	·023	3·00	·034	4·00	·045	5·00	·056	6·00	·068	7·00	·079	8·00	·090	9·00	·102	10·00	·113	21
·00	·012	2·00	·024	3·00	·036	4·00	·049	5·00	061	6·00	·073	7·00	·085	8·00	·098	9·00	·110	10·00	·122	18
·00	·013	2·00	·026	3·00	·039	4·00	·052	5·00	·065	6·00	·079	7·00	092	8·00	·105	9·00	·118	10·00	·131	15
·00	·014	2·00	·028	3·00	·042	4·00	·056	5·00	·070	6·00	·084	7·00	·098	8·00	·112	9·00	·126	10·00	·140	12
·00	·015	2·00	·030	3·00	·044	4·00	·059	5·00	·074	6·00	·089	7·00	104	8·00	·118	9·00	·133	10·00	·148	9
·00	·016	2·00	·031	3·00	·047	4·00	·063	5·00	·078	6·00	·094	7·00	109	8·00	·126	9·00	·141	10·00	·157	6
·00	·017	2·00	·033	3·00	·050	4·00	·066	5·00	·083	6·00	·100	7·00	·116	8·00	·123	9·00	·119	10·00	·166	3
·00	·017	2·00	·035	3·00	·052	4·00	·070	5·00	·087	6·00	·105	7·00	·122	8·00	·140	9·00	·157	10·00	·175	**89**
·00	·018	2·00	·036	3·00	·055	4·00	·073	5·00	·091	6·00	·110	7·00	·128	8·00	·146	9·00	·165	10·00	183	57
·00	·019	2·00	·038	3·00	·057	4·00	·076	5·00	·095	6·00	·115	7·00	·134	8·00	·153	9·00	·172	10·00	·191	54
·00	·020	2·00	·040	3·00	·060	4·00	·080	5·00	·100	6·00	·120	7·00	·140	8·00	·160	9·00	·180	10·00	·200	51
·00	·021	2·00	·042	3·00	·063	4·00	·084	5·00	·105	6·00	·126	7·00	·147	8·00	·168	9·00	·189	10·00	·210	48
·00	·022	2·00	·044	3·00	·065	4·00	·087	5·00	·109	6·00	·131	7·00	·153	8·00	·174	9·00	·196	10·00	·218	45
·00	·022	2·00	·045	3·00	·068	4·00	091	5·00	·113	6·00	·136	7·00	·150	8·00	·182	9·00	·204	10·00	227	42
·00	·023	2·00	·047	3·00	·070	4·00	·094	5·00	·117	6·00	·141	7·00	·164	8·00	·188	9·00	·211	10·00	·235	39
·00	·024	2·00	·049	3·00	·073	4·00	·097	5·00	·122	6·00	·146	7·00	·171	8·00	·195	9·00	·220	10·00	244	36
·00	·025	2·00	·051	3·00	·076	4·00	·101	5·00	·127	6·00	·152	7·00	·177	8·00	·202	9·00	·228	10·00	·253	33
·00	·026	2·00	·052	3·00	·079	4·00	·105	5·00	·131	6·00	·175	7·00	·183	8·00	·210	9·00	·236	10·00	·262	30
·00	·027	2·00	·054	8·00	·081	4·00	·108	5·00	·135	6·00	·162	7·00	·129	8·00	·216	9·00	·243	10·00	·270	27
·00	·028	2·00	·056	3·00	·084	4·00	·112	5·00	·140	6·00	·168	7·00	·190	8·00	·224	9·00	·252	10·00	·280	24
·00	·029	2·00	·058	3·00	·086	4·00	·115	5·00	·144	6·00	·173	7·00	·202	8·00	·230	9·00	·259	10·00	288	21
·00	·030	2·00	·059	3·00	·089	4·00	·119	5·00	·148	6·00	·178	7·00	·208	8·00	·238	9·00	·267	10·00	·297	18
·00	·031	2·00	·061	3·00	·092	4·00	·122	5·00	·153	6·00	·183	7·00	·214	8·00	·244	9·00	275	10·00	·305	15
·00	·031	2·00	·063	3·00	·094	4·00	·126	5·00	·157	6·00	·188	7·00	·220	8·00	·251	9·00	·283	10·00	314	12
99	·032	1·99	·065	2·99	·097	3·99	·129	4·99	·161	5·99	·194	6·99	·226	7·99	·258	8·99	·291	9·99	·323	9
99	·033	1·99	·066	2·99	·099	3·99	·132	4·99	·165	5·99	·199	6·99	·232	7·99	·265	8·99	298	9·99	·331	6
99	·034	1·99	·068	2·99	·102	3·99	·136	4·99	·170	5·99	·204	6·99	·238	7·99	·272	8·99	·306	9·99	·340	3
99	·035	1·99	·070	2·99	·105	3·99	·140	4·99	·175	5·99	·209	6·99	·244	7·99	·279	8·99	316	9·99	349	**88**
99	·036	1·99	·072	2·99	·107	3·99	·143	4·99	·179	5·99	215	6·99	·251	7·99	·286	8·99	322	9·99	358	57
99	·037	1·99	·073	2·99	·110	3·99	·146	4·99	·183	5·99	·220	6·99	·256	7·99	283	8·99	·329	9·99	366	54
99	·037	1·99	·075	2·99	·112	3·99	·150	4·99	·188	5·99	·225	6·99	·263	7·99	·300	8·99	·338	9·99	375	51
99	·038	1·99	·077	2·99	·115	3·99	·154	4·99	·192	5·99	·230	6·99	·269	7·99	·307	8·99	·346	9·99	384	48
99	·039	1·99	·078	2·99	·118	3·99	·157	4·99	·196	5·99	·235	6·99	·274	7·99	314	8·99	·353	9·99	·392	45
99	·040	1·99	·080	2·99	·120	3·99	·160	4·99	·201	5·99	241	6·99	·281	7·99	.321	8·99	·361	9·99	·401	42
99	·041	1·99	·082	2·99	·123	3·99	·164	4·99	·205	5·99	·246	6·99	·287	7·99	·328	8·99	·369	9·99	·410	39
99	·042	1·99	·084	2·99	·126	3·99	·168	4·99	·210	5·99	·251	6·99	·293	7·99	335	8·99	·377	9·99	419	36
99	·043	1·99	·085	2·99	·128	3·99	·171	4·99	·214	5·99	·256	6·99	·299	7·99	342	8·99	384	9·99	427	33
99	044	1·99	·087	2·99	·131	3·99	·174	4·99	·218	5·99	·262	6·99	305	7·99	349	8·99	392	9·99	·436	30
99	·045	1·99	·089	2·99	·134	3·99	·178	4·99	·223	5·99	·267	6·99	·312	7·99	·356	8·99	401	9·99	·445	27
99	·045	1·99	·091	2·99	·136	3·99	·182	4·99	·227	5·99	·272	6·99	·318	7·99	363	8·99	·409	9·99	454	24
99	·046	1·99	·092	2·99	·139	3·99	·185	4·99	·231	5·99	·277	6·99	·323	7·99	370	8·99	·416	9·99	462	21
99	·047	1·99	·094	2·99	141	3·99	·188	4·99	·236	5·99	·283	6·99	·330	7·99	·377	8·99	·424	9·99	471	18
99	·048	1·99	·096	2·99	·144	3·99	·192	4·99	·240	5·99	·288	6·99	·336	7·99	384	8·99	432	9·99	·480	15
98	·049	1·99	·098	2·99	·147	3·99	·196	4·99	·245	5·99	·293	6·99	·342	7·99	·391	8·99	·440	9·99	·489	12
98	·050	1·99	·099	2·99	·149	3·99	·199	4·99	·249	5·99	·298	6·99	·348	7·99	398	8·99	·447	9·99	497	9
98	·051	1·99	·101	2·99	·152	3·99	·202	4·99	·253	5·99	·304	6·99	·354	7·99	·405	8·99	·455	9·99	·506	6
98	·052	1·99	·103	2·99	·155	3·99	·206	4·99	·258	5·99	·309	6·99	·360	7·99	·412	8·99	·463	9·99	·515	3
ep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep	Lat.	Dep.	Lat.	Dep	Lat.	Dist.

Chs	1		2		3		4		5		6		7		8		9		10		Chs.
′	Lat.	Dep	Lat	Dep	Lat	Dep	Lat	Dep.	Lat.	Dep.	Lat.	Dep.	Lat	Dep.	Lat	Dep.	Lat	Dep.	Lat	Dep	′
3°	·998	·052	1·99	·105	2·99	·157	3·99	·209	4·99	·261	5·99	·314	6·99	·366	7·99	·418	8·99	·471	9·99	·523	**87°**
3	·998	·053	1·99	·106	2·99	·160	3·99	·213	4·99	·266	5·99	·319	6·99	·372	7·99	·426	8·99	·479	9·99	·532	57
6	·998	·054	1·99	·108	2·99	·162	3·99	·216	4·99	·271	5·99	·325	6·99	·379	7·99	·433	8·99	·487	9·99	·541	54
9	·998	·055	1·99	·110	2·99	·165	3·99	·220	4·99	·275	5·99	·330	6·99	·385	7·99	·440	8·99	·495	9·98	·550	51
12	·998	·056	1·99	·112	2·99	·167	3·99	·223	4·99	·279	5·99	·335	6·99	·391	7·99	·446	8·99	·502	9·98	·558	48
15	·998	·057	1·99	·113	2·99	·170	3·99	·227	4·99	·283	5·99	·340	6·99	·397	7·99	·454	8·99	·510	9·98	·567	45
18	·998	·058	1·99	·115	2·99	·173	3·99	·230	4·99	·288	5·99	·345	6·99	·403	7·99	·460	8·99	·518	9·98	·575	42
21	·998	·058	1·99	·117	2·99	·175	3·99	·234	4·99	·292	5·99	·350	6·99	·409	7·99	·467	8·98	·526	9·98	·584	39
24	·998	·059	1·99	·119	2·99	·178	3·99	·237	4·99	·297	5·99	·356	6·99	·415	7·99	·475	8·98	·534	9·98	·593	36
27	·998	·060	1·99	·120	2·99	·180	3·99	·240	4·99	·300	5·99	·361	6·99	·421	7·99	·481	8·98	·541	9·98	·601	33
30	·998	·061	1·99	·122	2·99	·183	3·99	·244	4·99	·305	5·99	·366	6·99	·427	7·99	·488	8·98	·549	9·98	·610	30
33	·998	·062	1·99	·124	2·99	·186	3·99	·248	4·99	·310	5·99	·371	6·99	·433	7·98	·495	8·98	·557	9·98	·619	27
36	·998	·063	1·99	·126	2·99	·188	3·99	·251	4·99	·314	5·99	·377	6·99	·440	7·98	·502	8·98	·565	9·98	·628	24
39	·998	·064	1·99	·127	2·99	·191	3·99	·254	4·99	·318	5·99	·382	6·99	·445	7·98	·509	8·98	·572	9·98	·636	21
42	·998	·064	1·99	·129	2·99	·193	3·99	·258	4·99	·323	5·99	·387	6·99	·452	7·98	·516	8·98	·581	9·98	·645	18
45	·998	·065	1·99	·131	2·99	·196	3·99	·262	4·99	·327	5·99	·392	6·99	·458	7·98	·523	8·98	·589	9·98	·654	15
48	·998	·066	1·99	·132	2·99	·199	3·99	·265	4·99	·331	5·99	·407	6·99	·463	7·98	·530	8·98	·596	9·98	·662	12
51	·998	·067	1·99	·134	2·99	·201	3·99	·268	4·99	·336	5·99	·403	6·98	·470	7·98	·537	8·98	·604	9·98	·671	9
54	·998	·068	1·99	·136	2·99	·204	3·99	·272	4·99	·340	5·99	·408	6·98	·476	7·98	·544	8·98	·612	9·98	·680	6
57	·998	·069	1·99	·138	2·99	·207	3·99	·276	4·99	·345	5·99	·413	6·98	·482	7·98	·551	8·98	·620	9·98	·689	3
4°	·998	·070	1·99	·139	2·99	·209	3·99	·279	4·99	·349	5·99	·418	6·98	·488	7·98	·558	8·98	·627	9·98	·697	**86°**
3	·998	·071	1·99	·141	2·99	·212	3·99	·282	4·99	·353	5·99	·424	6·98	·494	7·98	·565	8·98	·636	9·98	·706	57
6	·997	·071	1·99	·143	2·99	·214	3·99	·286	4·99	·357	5·99	·429	6·98	·500	7·98	·572	8·98	·643	9·97	·715	54
9	·997	·072	1·99	·145	2·99	·217	3·99	·289	4·99	·362	5·99	·434	6·98	·507	7·98	·579	8·98	·651	9·97	·724	51
12	·997	·073	1·99	·146	2·99	·220	3·99	·293	4·99	·366	5·99	·439	6·98	·513	7·98	·586	8·98	·659	9·97	·732	48
15	·997	·074	1·99	·148	2·99	·222	3·99	·296	4·99	·371	5·99	·445	6·98	·519	7·98	·593	8·98	·667	9·97	·741	45
18	·997	·075	1·99	·150	2·99	·225	3·99	·300	4·99	·375	5·99	·450	6·98	·525	7·98	·600	8·97	·675	9·97	·750	42
21	·997	·076	1·99	·152	2·99	·227	3·99	·303	4·99	·379	5·99	·455	6·98	·531	7·98	·607	8·97	·683	9·97	·758	39
24	·997	·077	1·99	·153	2·99	·230	3·99	·307	4·99	·384	5·99	·460	6·98	·537	7·98	·614	8·97	·690	9·97	·767	36
27	·997	·078	1·99	·155	2·99	·233	3·99	·310	4·98	·388	5·98	·466	6·98	·543	7·98	·621	8·97	·698	9·97	·776	33
30	·997	·078	1·99	·157	2·99	·235	3·99	·314	4·98	·392	5·98	·470	6·98	·549	7·98	·627	8·97	·706	9·97	·784	30
33	·997	·079	1·99	·159	2·99	·238	3·99	·317	4·98	·397	5·98	·476	6·98	·555	7·98	·634	8·97	·714	9·97	·793	27
36	·997	·080	1·99	·160	2·99	·241	3·99	·321	4·98	·401	5·98	·481	6·98	·561	7·98	·642	8·97	·722	9·97	·802	24
39	·997	·081	1·99	·162	2·99	·243	3·99	·324	4·98	·406	5·98	·487	6·98	·568	7·98	·649	8·97	·730	9·97	·811	21
42	·997	·082	1·99	·164	2·99	·246	3·99	·328	4·98	·410	5·98	·491	6·98	·573	7·97	·655	8·97	·737	9·97	·819	18
45	·997	·083	1·99	·166	2·99	·248	3·99	·331	4·98	·414	5·98	·497	6·98	·580	7·97	·662	8·97	·745	9·97	·828	15
48	·996	·084	1·99	·167	2·99	·251	3·99	·335	4·98	·419	5·98	·402	6·98	·586	7·97	·670	8·97	·753	9·96	·837	12
51	·996	·085	1·99	·169	2·99	·253	3·99	·338	4·98	·423	5·98	·407	6·98	·592	7·97	·676	8·97	·761	9·96	·845	9
54	·996	·085	1·99	·171	2·99	·256	3·99	·342	4·98	·427	5·98	·412	6·98	·598	7·97	·683	8·97	·766	9·96	·854	6
57	·996	·086	1·99	·172	2·99	·259	3·99	·345	4·98	·431	5·98	·417	6·98	·603	7·97	·690	8·97	·776	9·96	·862	3
5°	·996	·087	1·99	·174	2·99	·261	3·98	·349	4·98	·436	5·98	·523	6·97	·610	7·97	·697	8·96	·784	9·96	·872	**85°**
3	·996	·088	1·99	·176	2·99	·264	3·98	·352	4·98	·440	5·98	·528	6·97	·616	7·97	·704	8·96	·792	9·96	·880	57
6	·996	·089	1·99	·178	2·99	·267	3·98	·356	4·98	·445	5·98	·533	6·97	·622	7·97	·711	8·96	·800	9·96	·889	54
9	·996	·090	1·99	·179	2·99	·269	3·98	·359	4·98	·448	5·98	·538	6·97	·628	7·97	·718	8·96	·807	9·96	·897	51
12	·996	·091	1·99	·181	2·99	·272	3·98	·362	4·98	·453	5·97	·543	6·97	·634	7·97	·725	8·96	·815	9·96	·906	48
15	·996	·091	1·99	·183	2·99	·274	3·98	·366	4·98	·457	5·97	·549	6·97	·640	7·97	·732	8·96	·823	9·96	·915	45
18	·996	·092	1·99	·185	2·99	·277	3·98	·369	4·98	·462	5·97	·554	6·97	·647	7·97	·739	8·96	·831	9·96	·924	42
21	·996	·093	1·99	·186	2·99	·280	3·98	·373	4·98	·466	5·97	·559	6·97	·652	7·96	·745	8·96	·839	9·96	·932	39
24	·996	·094	1·99	·188	2·99	·282	3·98	·376	4·98	·470	5·97	·565	6·97	·659	7·96	·753	8·96	·847	9·96	·941	36
27	·996	·095	1·99	·190	2·99	·285	3·98	·380	4·98	·475	5·97	·570	6·97	·665	7·96	·760	8·96	·855	9·96	·950	33
30	·995	·096	1·99	·191	2·99	·287	3·98	·383	4·98	·479	5·97	·575	6·97	·671	7·96	·766	8·96	·862	9·95	·958	30
33	·995	·097	1·99	·193	2·99	·290	3·98	·387	4·98	·483	5·97	·580	6·97	·677	7·96	·774	8·96	·870	9·95	·967	27
36	·995	·098	1·99	·195	2·99	·293	3·98	·390	4·98	·488	5·97	·585	6·97	·683	7·96	·781	8·96	·878	9·95	·976	24
39	·995	·098	1·99	·197	2·99	·295	3·98	·393	4·97	·492	5·97	·591	6·97	·689	7·96	·787	8·96	·886	9·95	·984	21
42	·995	·099	1·99	·199	2·99	·298	3·98	·397	4·97	·497	5·97	·596	6·96	·695	7·96	·794	8·95	·894	9·95	·993	18
45	·995	·100	1·99	·200	2·98	·300	3·98	·400	4·97	·501	5·97	·601	6·96	·701	7·96	·801	8·95	·902	9·95	10·0	15
48	·995	·101	1·99	·202	2·98	·303	3·98	·404	4·97	·505	5·97	·606	6·96	·707	7·96	·808	8·95	·909	9·95	10·1	12
51	·995	·102	1·99	·204	2·98	·306	3·98	·408	4·97	·510	5·97	·612	6·96	·714	7·96	·816	8·95	·918	9·95	10·2	9
54	·995	·103	1·99	·205	2·98	·308	3·98	·411	4·97	·513	5·97	·616	6·96	·719	7·96	·822	8·95	·924	9·95	10·3	6
57	·995	·104	1·99	·207	2·98	·311	3·98	·415	4·97	·518	5·97	·622	6·96	·726	7·96	·830	8·95	·933	9·95	10·4	3
Dist.	Dep	Lat.	Dep	Lat.	Dep	Lat.	Dep.	Lat.	Dep.	Lat	Dep.	Lat	Dep	Lat	Dep	Lat	Dep	Lat.	Dep	Lat.	Dist.

Chs.	1		2		3		4		5		6		7		8		9		10		Chs.
′	Lat.	Dep	Lat.	Dep	Lat.	Dep	Lat.	Dep	Lat.	Dep	Lat.	Dep	Lat	Dep	Lat.	Dep	Lat.	Dep	Lat	Dep	′
6°	·995	·104	1·99	·209	2·98	·313	3·98	·418	4·97	·523	5·97	·627	6·96	·732	7·96	·836	8·95	·941	9·95	1·04	**84°**
3	·994	·105	1·99	·210	2·98	·316	3·98	·421	4·97	·526	5·97	·632	6·96	·737	7·96	·842	8·95	·948	9·94	1·05	57
6	·994	·106	1·99	·213	2·98	·319	3·98	·425	4·97	·531	5·97	·638	6·96	·744	7·95	·850	8·95	·957	9·94	1·06	54
9	·994	·107	1·99	·214	2·98	·321	3·98	·428	4·97	·535	5·97	·643	6·96	·750	7·95	·857	8·95	·964	9·94	1·07	51
12	·994	·108	1·99	·216	2·98	·324	3·98	·432	4·97	·540	5·96	·648	6·96	·756	7·95	·864	8·95	·972	9·94	1·08	48
15	·994	·109	1·99	·218	2·98	·327	3·98	·436	4·97	·544	5·96	·653	6·96	·762	7·95	·871	8·95	·980	9·94	1·09	45
18	·994	·110	1·99	·219	2·98	·329	3·98	·439	4·97	·548	5·96	·658	6·96	·768	7·95	·878	8·95	·987	9·94	1·10	42
21	·994	·111	1·99	·221	2·98	·332	3·98	·442	4·97	·553	5·96	·663	6·96	·774	7·95	·885	8·95	·995	9·94	1·11	39
24	·994	·111	1·99	·223	2·98	·334	3·98	·446	4·97	·557	5·96	·669	6·96	·780	7·95	·892	8·94	1·00	9·94	1·11	36
27	·994	·112	1·99	·224	2·98	·337	3·98	·449	4·97	·561	5·96	·674	6·96	·786	7·95	·898	8·94	1·01	9·94	1·12	33
30	·994	·113	1·99	·226	2·98	·339	3·97	·453	4·97	·566	5·96	·679	6·95	·792	7·95	·905	8·94	1·02	9·94	1·13	30
33	·993	·114	1·99	·228	2·98	·342	3·97	·456	4·97	·570	5·96	·685	6·95	·799	7·95	·913	8·94	1·03	9·93	1·14	27
36	·993	·115	1·99	·230	2·98	·345	3·97	·460	4·97	·575	5·96	·689	6·95	·804	7·95	·919	8·94	1·03	9·93	1·15	24
39	·993	·116	1·99	·232	2·98	·347	3·97	·463	4·97	·579	5·96	·695	6·95	·810	7·95	·926	8·94	1·04	9·93	1·16	21
42	·993	·117	1·99	·233	2·98	·350	3·97	·467	4·97	·583	5·96	·700	6·95	·817	7·95	·934	8·94	1·05	9·93	1·17	18
45	·993	·117	1·99	·235	2·98	·352	3·97	·470	4·96	·587	5·96	·705	6·95	·822	7·94	·940	8·94	1·06	9·93	1·17	15
48	·993	·118	1·99	·237	2·98	·355	3·97	·473	4·96	·592	5·96	·710	6·95	·829	7·94	·947	8·94	1·07	9·93	1·18	12
51	·993	·119	1·99	·238	2·98	·358	3·97	·477	4·96	·596	5·96	·716	6·95	·835	7·94	·954	8·94	1·07	9·93	1·19	9
54	·993	·120	1·99	·240	2·98	·360	3·97	·480	4·96	·600	5·96	·721	6·95	·841	7·94	·961	8·93	1·08	9·93	1·20	6
57	·993	·121	1·99	·242	2·98	·363	3·97	·484	4·96	·605	5·96	·726	6·95	·847	7·94	·968	8·93	1·09	9·93	1·21	3
7	·993	·122	1·99	·244	2·98	·366	3·97	·488	4·96	·609	5·96	·731	6·95	·853	7·94	·975	8·93	1·10	9·93	1·22	**83°**
3	·992	·123	1·98	·245	2·98	·368	3·97	·491	4·96	·613	5·95	·736	6·95	·859	7·94	·981	8·93	1·10	9·92	1·23	57
6	·992	·124	1·98	·247	2·98	·371	3·97	·494	4·96	·618	5·95	·741	6·95	·865	7·94	·989	8·93	1·11	9·92	1·24	54
9	·992	·124	1·98	·249	2·98	·373	3·97	·498	4·96	·622	5·95	·747	6·95	·871	7·94	·996	8·93	1·12	9·92	1·24	51
12	·992	·125	1·98	·251	2·98	·376	3·97	·501	4·96	·626	5·95	·752	6·94	·877	7·94	1·00	8·93	1·13	9·92	1·25	48
15	·992	·126	1·98	·252	2·98	·378	3·97	·505	4·96	·631	5·95	·757	6·94	·883	7·94	1·01	8·93	1·13	9·92	1·26	45
18	·992	·127	1·98	·254	2·98	·381	3·97	·508	4·96	·635	5·95	·762	6·94	·889	7·94	1·01	8·93	1·14	9·92	1·27	42
21	·992	·128	1·98	·256	2·98	·384	3·97	·512	4·96	·640	5·95	·768	6·94	·896	7·93	1·02	8·93	1·15	9·92	1·28	39
24	·992	·129	1·98	·258	2·98	·386	3·97	·515	4·96	·644	5·95	·773	6·94	·902	7·93	1·03	8·93	1·16	9·92	1·29	36
27	·992	·130	1·98	·259	2·98	·389	3·97	·519	4·96	·648	5·95	·778	6·94	·908	7·93	1·04	8·92	1·17	9·92	1·30	33
30	·991	·130	1·98	·261	2·97	·391	3·97	·522	4·96	·652	5·95	·783	6·94	·913	7·93	1·04	8·92	1·17	9·91	1·30	30
33	·991	·131	1·98	·263	2·97	·394	3·97	·525	4·96	·657	5·95	·788	6·94	·920	7·93	1·05	8·92	1·18	9·91	1·31	27
36	·991	·132	1·98	·264	2·97	·397	3·96	·529	4·96	·661	5·95	·793	6·94	·925	7·93	1·06	8·92	1·19	9·91	1·32	24
39	·991	·133	1·98	·266	2·97	·399	3·96	·532	4·96	·665	5·95	·798	6·94	·932	7·93	1·06	8·92	1·20	9·91	1·33	21
42	·991	·134	1·98	·268	2·97	·402	3·96	·536	4·95	·670	5·95	·804	6·94	·938	7·93	1·07	8·92	1·21	9·91	1·34	18
45	·991	·135	1·98	·270	2·97	·405	3·96	·540	4·95	·675	5·95	·809	6·94	·944	7·93	1·08	8·92	1·21	9·91	1·35	15
48	·991	·136	1·98	·271	2·97	·407	3·96	·543	4·95	·678	5·94	·814	6·93	·950	7·93	1·09	8·92	1·22	9·91	1·36	12
51	·991	·137	1·98	·273	2·97	·410	3·96	·546	4·95	·683	5·94	·819	6·93	·956	7·92	1·09	8·92	1·23	9·91	1·37	9
54	·991	·137	1·98	·275	2·97	·412	3·96	·549	4·95	·687	5·94	·824	6·93	·962	7·92	1·10	8·91	1·24	9·90	1·37	6
57	·990	·138	1·98	·276	2·97	·415	3·96	·553	4·95	·691	5·94	·830	6·93	·968	7·92	1·11	8·91	1·24	9·90	1·38	3
8	·990	·139	1·98	·278	2·97	·417	3·96	·557	4·95	·696	5·94	·835	6·93	·974	7·92	1·11	8·91	1·25	9·90	1·39	**82°**
3	·990	·140	1·98	·280	2·97	·420	3·96	·560	4·95	·700	5·94	·840	6·93	·980	7·92	1·12	8·91	1·26	9·90	1·40	57
6	·990	·141	1·98	·282	2·97	·423	3·96	·564	4·95	·705	5·94	·846	6·93	·987	7·92	1·13	8·91	1·27	9·90	1·41	54
9	·990	·142	1·98	·284	2·97	·425	3·96	·567	4·95	·709	5·94	·851	6·93	·992	7·92	1·13	8·91	1·27	9·90	1·42	51
12	·990	·143	1·98	·285	2·97	·428	3·96	·570	4·95	·713	5·94	·855	6·93	·998	7·92	1·14	8·91	1·28	9·90	1·43	48
15	·990	·143	1·98	·287	2·97	·430	3·96	·574	4·95	·717	5·94	·861	6·93	1·00	7·92	1·15	8·91	1·29	9·90	1·43	45
18	·989	·144	1·98	·288	2·97	·433	3·96	·577	4·95	·721	5·94	·866	6·93	1·01	7·92	1·15	8·91	1·30	9·90	1·44	42
21	·989	·145	1·98	·290	2·97	·435	3·96	·581	4·95	·726	5·94	·871	6·93	1·02	7·91	1·16	8·90	1·31	9·89	1·45	39
24	·989	·146	1·98	·292	2·97	·438	3·96	·584	4·95	·730	5·94	·876	6·92	1·02	7·91	1·17	8·90	1·31	9·89	1·46	36
27	·989	·147	1·98	·294	2·97	·441	3·96	·588	4·94	·734	5·93	·881	6·92	1·03	7·91	1·17	8·90	1·32	9·89	1·47	33
30	·989	·148	1·98	·296	2·97	·443	3·96	·591	4·94	·739	5·93	·887	6·92	1·03	7·91	1·18	8·90	1·33	9·89	1.48	30
33	·989	·149	1·98	·298	2·96	·446	3·95	·595	4·94	·743	5·93	·892	6·92	1·04	7·91	1·19	8·90	1·34	9·89	1·49	27
36	·989	·149	1·98	·299	2·96	·448	3·95	·598	4·94	·747	5·93	·897	6·92	1·05	7·91	1·20	8·90	1·34	9·89	1·49	24
39	·989	·150	1·98	·301	2·96	·451	3·95	·601	4·94	·752	5·93	·902	6·92	1·05	7·91	1·20	8·90	1·35	9·89	1·50	21
42	·988	·151	1·98	·302	2·96	·454	3·95	·605	4·94	·756	5·93	·908	6·92	1·06	7·91	1·21	8·90	1·36	9·88	1·51	18
45	·988	·152	1·98	·304	2·96	·456	3·95	·608	4·94	·760	5·93	·913	6·92	1·06	7·91	1·22	8·89	1·37	9·88	1·52	15
48	·988	·153	1·98	·306	2·96	·459	3·95	·612	4·94	·765	5·93	·918	6·92	1·07	7·90	1·22	8·89	1·38	9·88	1·53	12
51	·988	·154	1·98	·308	2·96	·461	3·95	·615	4·94	·769	5·93	·923	6·92	1·08	7·90	1·23	8·89	1·38	9·88	1·54	9
54	·988	·155	1·98	·309	2·96	·464	3·95	·619	4·94	·773	5·93	·928	6·91	1·08	7·90	1·24	8·89	1·39	9·88	1·55	6
57	·988	·156	1·97	·311	2·96	·467	3·95	·622	4·94	·778	5·93	·933	6·91	1·09	7·90	1·24	8·89	1·40	9·88	1·56	3
Dist.	Dep	Lat	Dep	Lat.	Dep	Lat	Dep	Lat.	Dep	Lat.	Dep	Lat	Dep	Lat	Dep	Lat.	Dep	Lat.	Dep	Lat.	Dist.

Chs.	1		2		3		4		5		6		7		8		9		10		Chs
′	Lat.	Dep	Lat.	Dep	Lat.	Dep	Lat	Dep	Lat	Dep	Lat.	Dep	Lat	Dep	Lat	Dep	Lat	Dep	Lat.	Dep	′
9°	·988	·156	1·97	·313	2·96	·469	3·95	·626	4·94	·782	5·93	·938	6·91	1·09	7·90	1·25	8·89	1·41	9·88	1·56	**81°**
3	·988	·157	1·97	·314	2·96	·472	3·95	·629	4·94	·786	5·93	·944	6·91	1·10	7·90	1·26	8·89	1·41	9·88	1·57	57
6	·987	·158	1·97	·316	2·96	·474	3·95	·632	4·94	·790	5·92	·948	6·91	1·11	7·90	1·26	8·89	1·42	9·87	1·58	54
9	·987	·159	1·97	·318	2·96	·477	3·95	·636	4·94	·795	5·92	·954	6·91	1·11	7·90	1·27	8·89	1·43	9·87	1·59	51
12	·987	·160	1·97	·320	2·96	·480	3·95	·640	4·94	·800	5·92	·960	6·91	1·12	7·90	1·28	8·88	1·44	9·87	1·60	48
15	·987	·161	1·97	·321	2·96	·482	3·95	·643	4·93	·803	5·92	·964	6·91	1·12	7·90	1·28	8·88	1·45	9·87	1·61	45
18	·987	·162	1·97	·323	2·96	·485	3·95	·646	4·93	·808	5·92	·969	6·91	1·13	7·89	1·29	8·88	1·45	9·87	1·62	42
21	·987	·162	1·97	·325	2·96	·487	3·95	·650	4·93	·812	5·92	·975	6·91	1·14	7·89	1·30	8·88	1·46	9·87	1·62	39
24	·987	·163	1·97	·326	2·96	·490	3·95	·653	4·93	·816	5·92	·980	6·91	1·14	7·89	1·31	8·88	1·47	9·87	1·63	36
27	·986	·164	1·97	·328	2·96	·492	3·95	·657	4·93	·821	5·92	·985	6·90	1·15	7·89	1·31	8·88	1·48	9·86	1·64	33
30	·986	·165	1·97	·330	2·96	·495	3·94	·660	4·93	·825	5·92	·990	6·90	1·15	7·89	1·32	8·88	1·48	9·86	1·65	30
33	·986	·166	1·97	·332	2·96	·498	3·94	·664	4·93	·830	5·92	·996	6·90	1·16	7·89	1·33	8·87	1·49	9·86	1·66	27
36	·986	·167	1·97	·334	2·96	·500	3·94	·667	4·93	·834	5·92	1·00	6·90	1·17	7·89	1·33	8·87	1·50	9·86	1·67	24
39	·986	·168	1·97	·335	2·96	·503	3·94	·670	4·93	·838	5·91	1·00	6·90	1·17	7·89	1·34	8·87	1·51	9·86	1·68	21
42	·986	·168	1·97	·337	2·96	·505	3·94	·674	4·93	·842	5·91	1·01	6·90	1·18	7·89	1·35	8·87	1·52	9·86	1·68	18
45	·986	·169	1·97	·338	2·96	·508	3·94	·677	4·93	·846	5·91	1·02	6·90	1·18	7·88	1·35	8·87	1·52	9·86	1·69	15
48	·985	·170	1·97	·340	2·96	·510	3·94	·681	4·93	·851	5·91	1·02	6·90	1·19	7·88	1·36	8·87	1·53	9·85	1·70	12
51	·985	·171	1·97	·342	2·96	·513	3·94	·684	4·93	·855	5·91	1·03	6·90	1·20	7·88	1·37	8·87	1·54	9·85	1·71	9
54	·985	·172	1·97	·344	2·95	·516	3·94	·688	4·92	·860	5·91	1·03	6·90	1·20	7·88	1·37	8·87	1·55	9·85	1·72	6
57	·985	·173	1·97	·346	2·95	·518	3·94	·691	4·92	·864	5·91	1·04	6·89	1·21	7·88	1·38	8·86	1·55	9·85	1·73	3
10°	·985	·174	1·97	·347	2·95	·521	3·94	·694	4·92	·868	5·91	1·04	6·89	1·21	7·88	1·39	8·86	1·56	9·85	1·74	**80°**
3	·985	·174	1·97	·349	2·95	·523	3·94	·698	4·92	·872	5·91	1·05	6·89	1·22	7·88	1·40	8·86	1·57	9·85	1·74	57
6	·985	·175	1·97	·351	2·95	·526	3·94	·701	4·92	·877	5·91	1·05	6·89	1·23	7·88	1·40	8·86	1·58	9·85	1·75	54
9	·984	·176	1·97	·352	2·95	·528	3·94	·705	4·92	·881	5·91	1·06	6·89	1·23	7·87	1·41	8·86	1·58	9·84	1·76	51
12	·984	·177	1·97	·354	2·95	·531	3·94	·708	4·92	·885	5·90	1·06	6·89	1·24	7·87	1·42	8·86	1·59	9·84	1·77	48
15	·984	·178	1·97	·356	2·95	·534	3·94	·712	4·92	·890	5·90	1·07	6·89	1·25	7·87	1·42	8·86	1·60	9·84	1·78	45
18	·984	·179	1·97	·358	2·95	·536	3·94	·715	4·92	·894	5·90	1·07	6·89	1·25	7·87	1·43	8·85	1·61	9·84	1·79	42
21	·984	·180	1·97	·359	2·95	·539	3·93	·719	4·92	·898	5·90	1·08	6·89	1·26	7·87	1·44	8·85	1·61	9·84	1·80	39
24	·984	·180	1·97	·361	2·95	·541	3·93	·722	4·92	·902	5·90	1·08	6·88	1·26	7·87	1·44	8·85	1·62	9·84	1·80	36
27	·983	·181	1·97	·363	2·95	·544	3·93	·725	4·92	·907	5·90	1·09	6·88	1·27	7·87	1·45	8·85	1·63	9·83	1·81	33
30	·983	·182	1·97	·364	2·95	·546	3·93	·729	4·92	·911	5·90	1·09	6·88	1·27	7·87	1·46	8·85	1·64	9·83	1·82	30
33	·983	·183	1·97	·366	2·95	·549	3·93	·732	4·91	·915	5·90	1·10	6·88	1·28	7·86	1·46	8·85	1·65	9·83	1·83	27
36	·983	·184	1·97	·368	2·95	·552	3·93	·736	4·91	·920	5·90	1·10	6·88	1·29	7·86	1·47	8·85	1·65	9·83	1·84	24
39	·983	·185	1·97	·370	2·95	·554	3·93	·739	4·91	·924	5·90	1·11	6·88	1·29	7·86	1·48	8·84	1·66	9·83	1·85	21
42	·983	·186	1·96	·371	2·95	·557	3·93	·743	4·91	·928	5·90	1·11	6·88	1·30	7·86	1·48	8·84	1·67	9·83	1·86	18
45	·982	·186	1·96	·373	2·95	·559	3·93	·746	4·91	·932	5·89	1·12	6·88	1·30	7·86	1·49	8·84	1·68	9·82	1·86	15
48	·982	·187	1·96	·375	2·95	·562	3·93	·749	4·91	·937	5·89	1·12	6·88	1·31	7·86	1·50	8·84	1·68	9·82	1·87	12
51	·982	·188	1·96	·376	2·95	·564	3·93	·753	4·91	·941	5·89	1·13	6·87	1·32	7·86	1·50	8·84	1·69	9·82	1·88	9
54	·982	·189	1·96	·378	2·95	·567	3·93	·756	4·91	·945	5·89	1·13	6·87	1·32	7·86	1·51	8·84	1·70	9·82	1·89	6
57	·982	·190	1·96	·380	2·94	·570	3·93	·760	4·91	·950	5·89	1·14	6·87	1·33	7·85	1·52	8·84	1·71	9·82	1·90	3
11°	·982	·191	1·96	·382	2·94	·572	3·93	·763	4·91	·954	5·89	1·14	6·87	1·33	7·85	1·53	8·83	1·72	9·82	1·91	**79°**
3	·982	·192	1·96	·383	2·94	·575	3·93	·767	4·91	·958	5·89	1·15	6·87	1·34	7·85	1·53	8·83	1·72	9·82	1·92	57
6	·981	·192	1·96	·385	2·94	·577	3·93	·770	4·91	·962	5·89	1·15	6·87	1·35	7·85	1·54	8·83	1·73	9·81	1·92	54
9	·981	·193	1·96	·387	2·94	·580	3·92	·773	4·91	·967	5·89	1·16	6·87	1·35	7·85	1·55	8·83	1·74	9·81	1·93	51
12	·981	·194	1·96	·388	2·94	·582	3·92	·777	4·90	·971	5·88	1·16	6·87	1·36	7·84	1·55	8·83	1·75	9·81	1·94	48
15	·981	·195	1·96	·390	2·94	·585	3·92	·780	4·90	·975	5·88	1·17	6·87	1·36	7·84	1·56	8·83	1·75	9·81	1·95	45
18	·981	·196	1·96	·392	2·94	·588	3·92	·784	4·90	·280	5·88	1·17	6·86	1·37	7·84	1·57	8·83	1·76	9·81	1·96	42
21	·980	·197	1·96	·394	2·94	·590	3·92	·787	4·90	·984	5·88	1·18	6·86	1·38	7·84	1·57	8·82	1·77	9·80	1·97	39
24	·980	·198	1·96	·395	2·94	·593	3·92	·790	4·90	·988	5·88	1·18	6·86	1·38	7·84	1·58	8·82	1·78	9·80	1·98	36
27	·980	·198	1·96	·397	2·94	·595	3·92	·794	4·90	·992	5·88	1·19	6·86	1·39	7·84	1·59	8·82	1·79	9·80	1·98	33
30	·980	·199	1·96	·399	2·94	·598	3·92	·797	4·90	·997	5·88	1·20	6·86	1·39	7·84	1·59	8·82	1·79	9·80	1·99	30
33	·980	·200	1·96	·400	2·94	·600	3·92	·801	4·90	1·00	5·88	1·20	6·86	1·40	7·84	1·60	8·82	1·80	9·80	2·00	27
36	·980	·201	1·96	·402	2·94	·603	3·92	·804	4·90	1·00	5·88	1·20	6·86	1·41	7·84	1·60	8·82	1·81	9·80	2·01	24
39	·979	·202	1·96	·404	2·94	·606	3·92	·808	4·90	1·01	5·88	1·21	6·86	1·41	7·84	1·61	8·81	1·82	9·79	2·02	21
42	·979	·203	1·96	·406	2·94	·608	3·92	·811	4·90	1·01	5·88	1·22	6·85	1·42	7·83	1·62	8·81	1·82	9·79	2·03	18
45	·979	·204	1·96	·407	2·94	·611	3·92	·814	4·90	1·02	5·87	1·22	6·85	1·43	7·83	1·63	8·81	1·83	9·79	2·04	15
48	·979	·204	1·96	·409	2·94	·613	3·91	·818	4·89	1·02	5·87	1·23	6·85	1·43	7·83	1·64	8·81	1·84	9·79	2·04	12
51	·979	·205	1·96	·410	2·94	·616	3·91	·821	4·89	1·02	5·87	1·23	6·85	1·44	7·83	1·64	8·81	1·85	9·79	2·05	9
54	·979	·206	1·96	·412	2·93	·618	3·91	·825	4·89	1·03	5·87	1·24	6·85	1·44	7·83	1·65	8·81	1·85	9·79	2·06	6
57	·978	·207	1·96	·414	2·93	·621	3·91	·828	4·89	1·03	5·87	1·24	6·85	1·45	7·83	1·65	8·80	1·86	9·78	2·07	3
Dist.	Dep	Lat.	Dep	Lat.	Dep	Lat.	Dep	Lat.	Dep	Lat.	Dep	Lat.	Dep	Lat.	Dep	Lat.	Dep	Lat.	Dep	Lat.	Dist.

Chs.	1		2		3		4		5		6		7		8		9		10		Chs.
′	Lat.	Dep	Lat.	Dep	Lat.	Dep	Lat	Dep	Lat.	Dep	Lat.	Dep	Lat.	Dep	Lat.	Dep	Lat.	Dep	Lat.	Dep	′
12°	·978	·208	1·96	·416	2·93	·624	3·91	·832	4·89	1·04	5·87	1·25	6·85	1·46	7·82	1·66	8·80	1·87	9·78	2·08	**78°**
3	·978	·209	1·96	·418	2·93	·626	3·91	·835	4·89	1·04	5·87	1·25	6·85	1·46	7·82	1·67	8·80	1·88	9·78	2·09	57
6	·978	·210	1·96	·419	2·93	·629	3·91	·838	4·89	1·05	5·87	1·26	6·84	1·47	7·82	1·68	8·80	1·89	9·78	2·10	54
9	·978	·210	1·96	·421	2·93	·633	3·91	·842	4·89	1·05	5·87	1·26	6·84	1·47	7·82	1·68	8·80	1·90	9·78	2·10	51
12	·977	·211	1·95	·421	2·93	·634	3·91	·845	4·89	1·06	5·86	1·27	6·84	1·48	7·82	1·69	8·80	1·90	9·77	2·11	48
15	·977	·212	1·95	·424	2·93	·637	3·91	·849	4·89	1·06	5·86	1·27	6·84	1·49	7·82	1·70	8·80	1·91	9·77	2·12	45
18	·977	·213	1·95	·426	2·93	·639	3·91	·852	4·89	1·07	5·86	1·28	6·84	1·49	7·82	1·70	8·79	1·92	9·77	2·13	42
21	·977	·214	1·95	·428	2·93	·642	3·91	·856	4·88	1·07	5·86	1·28	6·84	1·50	7·81	1·71	8·79	1·92	9·77	2·14	39
24	·977	·215	1·95	·429	2·93	·644	3·91	·859	4·88	1·07	5·86	1·29	6·84	1·50	7·81	1·72	8·79	1·93	9·77	2·15	36
27	·977	·216	1·95	·431	2·93	·647	3·91	·862	4·88	1·08	5·86	1·29	6·84	1·51	7·81	1·72	8·79	1·94	9·76	2·16	33
30	·976	·216	1·95	·433	2·93	·649	3·91	·866	4·88	1·08	5·86	1·30	6·83	1·52	7·81	1·73	8·79	1·95	9·76	2·16	30
33	·976	·217	1·95	·435	2·93	·651	3·90	·869	4·88	1·09	5·86	1·30	6·83	1·52	7·81	1·74	8·78	1·96	9·76	2·17	27
36	·976	·218	1·95	·436	2·93	·654	3·90	·873	4·88	1·09	5·86	1·31	6·83	1·53	7·81	1·75	8·78	1·96	9·76	2·18	24
39	·976	·219	1·95	·438	2·93	·657	3·90	·876	4·88	1·09	5·85	1·31	6·83	1·53	7·81	1·75	8·78	1·97	9·76	2·19	21
42	·976	·220	1·95	·440	2·93	·660	3·90	·879	4·88	1·10	5·85	1·32	6·83	1·54	7·80	1·76	8·78	1·98	9·76	2·20	18
45	·975	·221	1·95	·441	2·93	·662	3·90	·883	4·88	1·10	5·85	1·32	6·83	1·54	7·80	1·77	8·78	1·99	9·75	2·21	15
48	·975	·222	1·95	·443	2·93	·665	3·90	·886	4·88	1·11	5·85	1·33	6·83	1·55	7·80	1·77	8·78	1·99	9·75	2·22	12
51	·975	·222	1·95	·445	2·92	·667	3·90	·890	4·87	1·11	5·85	1·33	6·82	1·56	7·80	1·78	8·77	2·00	9·75	2·22	9
54	·975	·223	1·95	·417	2·92	·670	3·90	·893	4·87	1·12	5·85	1·34	6·82	1·56	7·80	1·79	8·77	2·01	9·75	2·23	6
57	·975	·224	1·95	·448	2·92	·672	3·90	·896	4·87	1·12	5·85	1·34	6·82	1·57	7·80	1·79	8·77	2·02	9·75	2·24	3
13°	·974	·226	1·95	·450	2·92	·675	3·90	·900	4·87	1·12	5·85	1·35	6·82	1·57	7·80	1·80	8·77	2·02	9·74	2·25	**77°**
3	·974	·226	1·95	·452	2·92	·677	3·90	·903	4·87	1·13	5·84	1·35	6·82	1·58	7·79	1·81	8·77	2·03	9·74	2·26	57
6	·974	·227	1·95	·453	2·92	·680	3·90	·907	4·87	1·13	5·84	1·36	6·82	1·59	7·79	1·81	8·76	2·04	9·74	2·27	54
9	·974	·227	1·95	·455	2·92	·682	3·90	·910	4·87	1·14	5·84	1·36	6·82	1·59	7·79	1·82	8·76	2·05	9·74	2·27	51
12	·974	·228	1·95	·457	2·92	·685	3·89	·913	4·87	1·14	5·84	1·37	6·81	1·60	7·79	1·83	8·76	2·06	9·74	2·28	48
15	·974	·229	1·95	·458	2·92	·688	3·89	·917	4·87	1·15	5·84	1·38	6·81	1·60	7·79	1·83	8·76	2·06	9·73	2·29	45
18	·973	·230	1·95	·460	2·92	·690	3·89	·920	4·87	1·15	5·84	1·38	6·81	1·61	7·78	1·83	8·76	2·07	9·73	2·30	42
21	·973	·231	1·95	·462	2·92	·693	3·89	·924	4·86	1·15	5·84	1·38	6·81	1·62	7·78	1·85	8·76	2·08	9·73	2·31	39
24	.973	·232	1·95	·463	2·92	·695	3·89	·927	4·86	1·16	5·81	1·38	6·81	1·62	7·78	1·85	8·75	2·08	9·73	2·32	36
27	·973	·233	1·94	·466	2·92	·699	3·89	·931	4·86	1·16	5·83	1·40	6·81	1·63	7·78	1·86	8·75	2·10	9·73	2·33	33
30	·972	·233	1·94	·467	2·92	·700	3·89	·934	4·86	1·17	5·83	1·40	6·81	1·63	7·78	1·86	8·75	2·10	9·72	2·33	30
33	·972	·234	1·94	·468	2·92	·702	3·89	·937	4·86	1·17	5·83	1·41	6·81	1·64	7·78	1·87	8·75	2·11	9·72	2·34	27
36	·972	·235	1·94	·470	2·92	·705	3·89	·940	4·86	1·18	5·83	1·41	6·80	1·65	7·78	1·88	8·75	2·12	9·72	2·35	24
39	·972	·236	1·94	·472	2·91	·708	3·89	·944	4·86	1·18	5·83	1·42	6·80	1·65	7·77	1·89	8·75	2·12	9·72	2·36	21
42	·972	·237	1·94	·474	2·91	·710	3·89	·947	4·86	1·18	5·83	1·42	6·80	1·66	7·77	1·89	8·74	2·13	9·72	2·37	18
45	·971	·238	1·94	·475	2·91	·713	3·89	·951	4·86	1·19	5·83	1·43	6·80	1·66	7·77	1·90	8·74	2·14	9·71	2·38	15
48	·971	·239	1·94	·478	2·91	·716	3·88	·955	4·86	1·19	5·83	1·43	6·80	1·67	7·77	1·91	8·74	2·15	9·71	2·39	12
51	·971	·239	1·94	·479	2·91	·718	3·88	·957	4·85	1·20	5·82	1·44	6·80	1·67	7·77	1·91	8·74	2·15	9·71	2·39	9
54	·971	·240	1·94	·480	2·91	·721	3·88	·960	4·85	1·20	5·82	1·44	6·79	1·68	7·77	1·92	8·74	2·16	9·71	2·40	6
57	·971	·241	1·94	·482	2·91	·723	3·88	·964	4·85	1·21	5·82	1·45	6·79	1·69	7·76	1·93	8·73	2·17	9·70	2·41	3
14°	·970	·242	1·94	·484	2·91	·726	3·88	·968	4·85	1·21	5·82	1·45	6·79	1·69	7·76	1·93	8·73	2·18	9·70	2·42	**76°**
3	·970	·243	1·94	·485	2·91	·728	3·88	·971	4·85	1·21	5·82	1·46	6·79	1·70	7·76	1·91	8·73	2·18	9·70	2·43	57
6	·970	·243	1·94	·487	2·91	·730	3·88	·974	4·85	1·22	5·82	1·46	6·79	1·70	7·76	1·95	8·73	2·19	9·70	2·44	54
9	·970	·244	1·94	·489	2·91	·733	3·88	·978	4·85	1·22	5·82	1·47	6·79	1·71	7·76	1·96	8·73	2·20	9·70	2·44	51
12	·970	·245	1·94	·491	2·91	·736	3·88	·981	4·85	1·23	5·82	1·47	6·79	1·72	7·76	1·96	8·73	2·21	9·69	2·45	48
15	·969	·246	1·94	·492	2·91	·738	3·88	·985	4·85	1·23	5·82	1·48	6·78	1·72	7·75	1·97	8·72	2·22	9·69	2·46	45
18	·969	·247	1·94	·494	2·91	·741	3·88	·988	4·85	1·23	5·81	1·48	6·78	1·73	7·75	1·97	8·72	2·22	9·69	2·47	42
21	·969	·248	1·94	·496	2·91	·743	3·87	·991	4·84	1·24	5·81	1·49	6·78	1·73	7·75	1·98	8·72	2·23	9·69	2·48	39
24	·969	·249	1·94	·497	2·91	·746	3·87	·995	4·84	1·24	5·81	1·49	6·78	1·74	7·75	1·99	8·72	2·24	9·69	2·49	36
27	·968	·250	1·94	·499	2·90	·749	3·87	·998	4·84	1·25	5·81	1·50	6·78	1·75	7·75	2·00	8·72	2·25	9·68	2·50	33
30	·968	·250	1·94	·500	2·90	·751	3·87	·100	4·84	1·25	5·81	1·50	6·78	1·75	7·75	2·00	8·71	2·25	9·68	2·50	30
33	·968	·251	1·94	·502	2·90	·751	3·87	·100	4·84	1·26	6·81	1·51	6·78	1·76	7·74	2·01	8·71	2·26	9·68	2·51	27
36	·968	·252	1·94	·504	2·90	·756	3·87	·101	4·84	1·26	5·81	1·51	6·77	1·76	7·74	2·02	8·71	2·27	9·68	2·52	24
39	·967	·253	1·94	·506	2·90	·759	3·87	·101	4·84	1·26	5·80	1·52	6·77	1·77	7·74	2·02	8·71	2·28	9·67	2·53	21
42	·967	·254	1·93	·507	2·90	·761	3·87	·101	4·84	1·27	5·80	1·52	6·77	1·78	7·74	2·03	8·71	2·28	9·67	2·54	18
45	·967	·255	1·93	·509	2·90	·761	3·87	·102	4·84	1·27	5·80	1·53	6·77	1·78	7·74	2·04	8·70	2·29	9·67	2·55	15
48	·967	·255	1·93	·511	2·90	·766	3·87	·102	4·83	1·28	5·80	1·53	6·77	1·79	7·73	2·04	8·70	2·30	9·67	2·55	12
51	·967	·256	1·93	·513	2·90	·769	3·87	·103	4·83	1·28	5·80	1·54	6·77	1·79	7·73	2·05	8·70	2·31	9·67	2·56	9
54	·966	·257	1·93	·514	2·90	·771	3·87	·103	4·83	1·29	5·80	1·54	6·76	1·80	7·73	2·06	8·70	2·31	9·67	2·57	6
57	·966	·258	1·93	·516	2·90	·774	3·86	·103	4·83	1·29	5·80	1·55	6·76	1·81	7·73	2·06	8·70	2·32	9·66	2·58	3
Dist.	Dep	Lat.	Dep	Lat.	Dep	Lat	Dep	Lat.	Dep	Lat.	Dep	Lat.	Dep	Lat.	Dep	Lat.	Dep	Lat.	Dep	Lat.	Dist.

Chs.	1		2		3		4		5		6		7		8		9		10		Chs.
′	Lat.	Dep	Lat.	Dep	Lat.	Dep	Lat	Dep	Lat	Dep	Lat.	Dep	Lat.	Dep	Lat.	Dep	Lat	Dep	Lat	Dep	′
15°	·966	·259	1·93	·518	2·90	·776	3·86	1·04	4·83	1·29	5·80	1·55	6·76	1·81	7·73	2·07	8·69	2·33	9·66	2·59	75°
3	·966	·260	1·93	·519	2·90	·779	3·86	1·04	4·83	1·30	5·79	1·56	6·76	1·82	7·73	2·08	8·69	2·34	9·66	2·60	57
6	·965	·260	1·93	·521	2·90	·781	3·86	1·04	4·83	1·30	5·79	1·56	6·76	1·82	7·72	2·08	8·69	2·34	9·65	2·60	54
9	·965	·261	1·93	·523	2·90	·784	3·86	1·05	4·83	1·31	5·79	1·57	6·76	1·83	7·72	2·09	8·69	2·35	9·66	2·61	51
12	·965	·262	1·93	·524	2·89	·787	3·86	1·05	4·82	1·31	5·79	1·57	6·75	1·83	7·72	2·10	8·68	2·36	9·65	2·62	48
15	·965	·263	1·93	·526	2·89	·789	3·86	1·05	4·82	1·32	5·79	1·58	6·75	1·81	7·72	2·10	8·68	2·37	9·65	2·63	45
18	·965	·264	1·93	·528	2·89	·792	3·86	1·06	4·82	1·32	5·79	1·58	6·75	1·85	7·72	2·11	8·68	2·37	9·65	2·64	42
21	·964	·265	1·93	·529	2·89	·791	3·86	1·06	4·82	1·32	5·79	1·59	6·75	1·85	7·71	2·12	8·68	2·38	9·64	2·65	39
24	·964	·266	1·93	·531	2·89	·797	3·86	1·06	4·82	1·33	5·78	1·59	6·75	1·85	7·71	2·12	8·68	2·39	9·64	2·66	36
27	·964	·266	1·93	·533	2·89	·799	3·86	1·07	4·82	1·33	5·78	1·60	6·75	1·86	7·71	2·13	8·67	2·40	9·64	2·66	33
30	·964	·267	1·93	·534	2·89	·802	3·85	1·07	4·82	1·34	5·78	1·60	6·75	1·87	7·71	2·14	8·67	2·41	9·64	2·67	30
33	·963	·268	1·93	·536	2·89	·801	3·85	1·07	4·82	1·34	5·78	1·61	6·74	1·88	7·71	2·14	8·67	2·41	9·63	2·68	27
36	·963	·279	1·93	·538	2·89	·807	3·85	1·08	4·82	1·34	5·78	1·61	6·74	1·88	7·71	2·15	8·67	2·42	9·63	2·69	21
39	·963	·270	1·93	·540	2·89	·809	3·85	1·08	4·81	1·35	5·78	1·62	6·74	1·89	7·70	2·16	8·67	2·43	9·63	2·70	21
42	·963	·271	1·93	·541	2·89	·812	3·85	1·08	4·81	1·35	5·78	1·62	6·74	1·89	7·70	2·16	8·66	2·44	9·63	2·71	18
45	·962	·271	1·92	·543	2·89	·814	3·85	1·09	4·81	1·36	5·77	1·63	6·74	1·90	7·70	2·17	8·66	2·44	9·62	2·71	15
48	·962	·272	1·92	·545	2·89	·817	3·85	1·09	4·81	1·36	5·77	1·63	6·74	1·91	7·70	2·18	8·66	2·45	9·62	2·72	12
51	·962	·273	1·92	·546	2·89	·819	3·85	1·09	4·81	1·37	5·77	1·64	6·73	1·98	7·70	2·18	8·66	2·46	9·62	2·73	9
54	·962	·274	1·92	·548	2·89	·822	3·85	1·10	4·81	1·37	5·77	1·64	6·73	1·92	7·69	2·19	8·66	2·47	9·62	2·74	6
57	·961	·275	1·92	·550	2·88	·824	3·85	1·10	4·81	1·37	5·77	1·65	6·73	1·92	7·69	2·20	8·65	2·47	9·61	2·75	3
16°	·961	·276	1·92	·551	2·88	·827	3·85	1·10	4·81	1·38	5·77	1·65	6·73	1·93	7·69	2·20	8·65	2·48	9·61	2·76	74°
3	·961	·276	1·92	·552	2·88	·829	3·84	1·10	4·80	1·38	5·77	1·66	6·73	1·94	7·69	2·21	8·65	2·49	9·61	2·76	57
6	·961	·277	1·92	·554	2·88	·831	3·84	1·11	4·80	1·39	5·76	1·69	6·73	1·94	7·69	2·22	8·65	2·50	9·61	2·77	54
9	·961	·278	1·92	·556	2·88	·834	3·84	1·11	4·80	1·39	5·76	1·67	6·72	1·95	7·68	2·22	8·64	2·50	9·61	2·78	51
12	·960	·279	1·92	·558	2·88	·837	3·84	1·12	4·80	1·39	5·76	1·67	6·72	1·95	7·68	2·23	8·64	2·51	9·60	2·79	48
15	·960	·280	1·92	·560	2·88	·839	3·84	1·12	4·80	1·40	5·76	1·68	6·72	1·96	7·68	2·24	8·64	2·52	9·60	2·80	45
18	·960	·281	1·92	·561	2·88	·842	3·84	1·12	4·80	1·40	5·76	1·68	6·72	1·96	7·68	2·24	8·64	2·53	9·60	2·81	42
21	·960	·281	1·92	·563	2·88	·844	3·84	1·13	4·80	1·41	5·76	1·69	6·72	1·97	7·68	2·25	8·64	2·53	9·60	2·81	39
24	·959	·282	1·92	·565	2·88	·847	3·84	1·13	4·80	1·41	5·76	1·59	6·72	1·98	7·67	2·26	8·63	2·54	9·59	2·82	36
27	·959	·283	1·92	·566	2·88	·850	3·84	1·13	4·80	1·42	5·75	1·70	6·71	1·98	7·67	2·26	8·63	2·55	9·59	2·83	33
30	·959	·284	1·92	·568	2·88	·852	3·84	1·14	4·79	1·42	5·75	1·70	6·71	1·99	7·67	2·27	8·63	2·56	9·59	2·84	30
33	·959	·285	1·92	·570	2·88	·854	3·83	1·14	4·79	1·42	5·75	1·71	6·71	1·99	7·67	2·28	8·63	2·56	9·59	2·85	27
36	·958	·286	1·92	·571	2·87	·857	3·83	1·14	4·79	1·43	5·75	1·71	6·71	2·00	7·67	2·28	8·62	2·57	9·58	2·86	24
39	·958	·287	1·92	·573	2·87	·860	3·83	1·15	4·79	1·43	5·75	1·72	6·71	2·01	7·66	2·30	8·62	2·58	9·58	2·87	21
42	·958	·287	1·92	·575	2·87	·862	3·83	1·15	4·79	1·44	5·75	1·72	6·70	2·01	7·66	2·30	8·62	2·59	9·58	2·87	18
45	·958	·288	1·92	·576	2·87	·865	3·83	1·15	4·79	1·44	5·75	1·73	6·70	2·02	7·66	2·31	8·62	2·59	9·58	2·88	15
48	·957	·289	1·91	·578	2·87	·867	3·83	1·16	4·79	1·44	5·74	1·73	6·70	2·02	7·66	2·31	8·62	2·60	9·57	2·89	12
51	·957	·290	1·91	·580	2·87	·870	3·83	1·16	4·79	1·45	5·74	1·74	6·70	2·03	7·66	2·32	8·61	2·61	9·57	2·90	9
54	·957	·291	1·91	·581	2·87	·872	3·83	1·16	4·78	1·45	5·74	1·74	6·70	2·03	7·65	2·32	8·61	2·62	9·57	2·91	6
57	·957	·291	1·91	·583	2·87	·875	3·83	1·17	4·78	1·46	5·74	1·75	6·70	2·04	7·65	2·33	8·61	2·62	9·57	2·91	3
17°	·956	·292	1·91	·585	2·87	·877	3·83	1·17	4·78	1·46	5·74	1·75	6·69	2·05	7·65	2·34	8·61	2·63	9·56	2·92	73°
3	·956	·293	1·91	·586	2·87	·880	3·82	1·17	4·78	1·47	5·73	1·76	6·70	2·05	7·65	2·35	8·60	2·64	9·56	2·93	57
6	·956	·294	1·91	·588	2·87	·882	3·82	1·18	4·78	1·47	5·73	1·76	6·69	2·06	7·65	2·35	8·60	2·65	9·56	2·94	54
9	·956	·295	1·91	·590	2·87	·885	3·82	1·18	4·78	1·47	5·73	1·77	6·69	2·06	7·64	2·36	8·60	2·65	9·56	2·95	51
12	·955	·296	1·91	·591	2·87	·887	3·82	1·18	4·78	1·48	5·73	1·77	6·69	2·07	7·64	2·37	8·60	2·66	9·55	2·96	48
15	·955	·297	1·91	·593	2·87	·890	3·82	1·19	4·78	1·48	5·73	1·78	6·69	2·08	7·64	2·37	8·60	2·67	9·55	2·97	45
18	·955	·297	1·91	·595	2·86	·892	3·82	1·19	4·77	1·49	5·73	1·78	6·68	2·08	7·64	2·38	8·59	2·68	9·55	2·97	42
21	·955	·298	1·91	·596	2·86	·895	3·82	1·19	4·77	1·49	5·73	1·79	6·68	2·09	7·64	2·39	8·59	2·68	9·55	2·98	39
24	·954	·299	1·91	·598	2·86	·897	3·82	1·20	4·77	1·49	5·73	1·79	6·68	2·09	7·63	2·39	8·59	2·69	9·54	2·99	36
27	·954	·300	1·91	·600	2·86	·900	3·82	1·20	4·77	1·50	5·72	1·80	6·68	2·10	7·63	2·40	8·59	2·70	9·54	3·00	33
30	·954	·301	1·91	·601	2·86	·902	3·81	1·20	4·77	1·50	5·72	1·80	6·68	2·10	7·63	2·40	8·58	2·71	9·54	3·01	30
33	·953	·301	1·91	·603	2·86	·905	3·81	1·21	4·77	1·51	5·72	1·81	6·67	2·11	7·63	2·41	8·58	2·71	9·53	3·01	27
36	·953	·302	1·91	·605	2·86	·905	3·81	1·21	4·77	1·51	5·72	1·81	6·67	2·12	7·63	2·42	8·58	2·72	9·53	3·02	24
39	·953	·303	1·91	·606	2·86	·910	3·81	1·21	4·76	1·52	5·72	1·82	6·67	2·12	7·62	2·43	8·58	2·73	9·53	3·03	21
42	·953	·304	1·91	·608	2·86	·912	3·81	1·22	4·76	1·52	5·71	1·82	6·67	2·13	7·62	2·43	8·57	2·74	9·52	3·05	18
45	·952	·305	1·91	·610	2·86	·915	3·81	1·22	4·76	1·52	5·71	1·83	6·67	2·13	7·62	2·44	8·57	2·74	9·52	3·06	15
48	·952	·306	1·90	·611	2·86	·917	3·81	1·22	4·76	1·53	5·71	1·83	6·66	2·14	7·61	2·44	8·57	2·75	9·52	3·06	12
51	·952	·306	1·90	·613	2·86	·920	3·81	1·23	4·76	1·53	5·71	1·84	6·66	2·15	7·61	2·45	8·57	2·76	9·52	3·07	9
54	·952	·307	1·90	·615	2·85	·922	3·81	1·23	4·76	1·54	5·71	1·84	6·66	2·15	7·61	2·46	8·56	2·77	9·52	3·08	6
57	·951	·308	1·90	·616	2·85	·925	3·81	1·23	4·76	1·54	5·71	1·85	6·66	2·16	7·61	2·46	8·56	2·77	9·51	3·09	3
Dist.	Dep	Lat.	Dep	Lat.	Dep	Lat.	Dep	Lat.	Dep	Lat.	Dep	Lat.	Dep	Lat.	Dep	Lat.	Dep	Lat.	Dep	Lat	Dist.

Chs.	1		2		3		4		5		6		7		8		9		10		Chs.
′	Lat	Dep	Lat.	Dep	Lat	Dep	Lat	Dep	Lat	Dep	Lat	Dep	Lat	Dep	Lat	Dep	Lat	Dep	Lat	Dep	′
18°	·951	·309	1·90	·618	2·85	·927	3·81	1·24	4·76	1·55	5·71	1·85	6·66	2·16	7·61	2·46	8·56	2·78	9·51	3·09	72°
3	·951	·310	1·90	·620	2·85	·930	3·80	1·24	4·75	1·55	5·70	1·86	6·66	2·17	7·61	2·48	8·56	2·79	9·51	3·10	57
6	·951	·311	1·90	·621	2·85	·932	3·80	1·24	4·75	1·55	5·70	1·86	6·65	2·17	7·60	2·48	8·55	2·80	9·51	3·11	54
9	·950	·312	1·90	·623	2·85	·935	3·80	1·25	4·75	1·56	5·70	1·87	6·65	2·18	7·60	2·49	8·55	2·80	9·50	3·12	51
12	·950	·312	1·90	·625	2·85	·937	3·80	1·25	4·75	1·56	5·70	1·87	6·65	2·19	7·60	2·50	8·55	2·81	9·50	3·12	48
15	·950	·313	1·90	·626	2·85	·939	3·80	1·25	4·75	1·57	5·70	1·88	6·65	2·19	7·60	2·51	8·55	2·82	9·50	3·13	45
18	·949	·314	1·90	·628	2·85	·942	3·80	1·26	4·75	1·57	5·70	1·88	6·65	2·20	7·60	2·51	8·54	2·83	9·49	3·14	42
21	·949	·315	1·90	·630	2·85	·944	3·80	1·26	4·75	1·57	5·69	1·89	6·64	2·20	7·59	2·52	8·54	2·83	9·49	3·15	39
24	·949	·316	1·90	·631	2·85	·947	3·80	1·26	4·74	1·58	5·69	1·89	6·64	2·21	7·59	2·52	8·54	2·84	9·49	3·16	36
27	·949	·316	1·90	·633	2·85	·949	3·79	1·27	4·74	1·58	5·69	1·90	6·64	2·22	7·59	2·53	8·54	2·85	9·49	3·16	33
30	·948	·317	1·90	·635	2·84	·952	3·79	1·27	4·74	1·59	5·69	1·90	6·64	2·22	7·59	2·54	8·53	2·86	9·48	3·17	30
33	·948	·318	1·90	·636	2·84	·954	3·79	1·27	4·74	1·59	5·69	1·91	6·64	2·23	7·58	2·55	8·53	2·86	9·48	3·18	27
36	·948	·319	1·90	·638	2·84	·957	3·79	1·28	4·74	1·59	5·69	1·91	6·63	2·23	7·58	2·55	8·53	2·87	9·48	3·19	24
39	·947	·320	1·89	·640	2·84	·959	3·79	1·28	4·74	1·60	5·68	1·92	6·63	2·24	7·58	2·56	8·53	2·88	9·47	3·20	21
42	·947	·321	1·89	·641	2·84	·962	3·79	1·28	4·74	1·60	5·68	1·92	6·63	2·24	7·58	2·56	8·52	2·88	9·47	3·21	18
45	·947	·321	1·89	·643	2·84	·964	3·79	1·29	4·73	1·61	5·68	1·93	6·63	2·25	7·58	2·57	8·52	2·89	9·47	3·21	15
48	·947	·322	1·89	·645	2·84	·967	3·79	1·19	4·73	1·61	5·68	1·93	6·63	2·26	7·57	2·58	8·52	2·90	9·47	3·22	12
51	·946	·323	1·89	·646	2·84	·969	3·78	1·29	4·73	1·62	5·68	1·94	6·62	2·26	7·57	2·58	8·52	2·91	9·46	3·23	9
54	·946	·324	1·89	·648	2·84	·972	3·78	1·30	4·73	1·62	5·68	1·94	6·62	2·26	7·57	2·59	8·51	2·91	9·46	3·24	6
57	·946	·325	1·89	·649	2·84	·974	3·78	1·30	4·73	1·62	5·67	1·95	6·62	2·27	7·57	2·60	8·51	2·92	9·46	3·25	3
19	·946	·326	1·89	·651	2·84	·977	3·78	1·30	4·73	1·63	5·67	1·95	6·62	2·28	7·56	2·60	8·51	2·93	9·46	3·26	71°
3	·945	·326	1·89	·653	2·84	·979	3·78	1·31	4·73	1·63	5·67	1·96	6·62	2·28	7·56	2·61	8·51	2·94	9·45	3·26	57
6	·945	·327	1·89	·654	2·83	·982	3·78	1·31	4·72	1·64	5·67	1·96	6·61	2·29	7·56	2·62	8·50	2·95	9·45	3·27	54
9	·945	·328	1·89	·656	2·83	·984	3·78	1·31	4·72	1·64	5·67	1·97	6·61	2·30	7·56	2·62	8·50	2·95	9·45	3·28	51
12	·944	·329	1·89	·658	2·83	·987	3·78	1·31	4·72	1·64	5·67	1·97	6·61	2·30	7·56	2·63	8·50	2·96	9·44	3·29	48
15	·944	·330	1·89	·659	2·83	·989	3·78	1·32	4·72	1·65	5·66	1·98	6·60	2·31	7·55	2·64	8·50	2·97	9·44	3·30	45
18	·944	·330	1·89	·661	2·83	·992	3·78	1·32	4·72	1·65	5·66	1·98	6·61	2·31	7·55	2·64	8·49	2·97	9·44	3·30	42
21	·943	·331	1·89	·663	2·83	·994	3·77	1·32	4·72	1·66	5·66	1·99	6·60	2·32	7·55	2·65	8·49	2·98	9·43	3·31	39
24	·943	·332	1·89	·664	2·83	·996	3·77	1·33	4·72	1·66	5·66	1·99	6·60	2·33	7·55	2·66	8·49	2·99	9·43	3·32	36
27	·943	·333	1·89	·666	2·83	·999	3·77	1·33	4·71	1·66	5·66	2·00	6·60	2·33	7·54	2·66	8·49	3·00	9·43	3·33	33
30	·943	·334	1·89	·668	2·83	·100	3·77	1·34	4·71	1·67	5·66	2·00	6·60	2·34	7·54	2·67	8·48	3·00	9·43	3·34	30
33	·942	·335	1·88	·669	2·83	·100	3·77	1·34	4·71	1·67	5·65	2·01	6·60	2·34	7·54	2·68	8·48	3·01	9·42	3·35	27
36	·942	·335	1·88	·671	2·83	·101	3·77	1·34	4·71	1·68	5·65	2·01	6·59	2·35	7·54	2·68	8·48	3·02	9·42	3·35	24
39	·942	·336	1·88	·973	2·83	·101	3·77	1·35	4·71	1·68	5·65	2·02	6·59	2·35	7·53	2·69	8·48	3·03	9·42	3·36	21
42	·941	·337	1·88	·674	2·82	·101	3·77	1·35	4·71	1·68	5·65	2·02	6·59	2·36	7·53	2·70	8·47	3·03	9·41	3·37	18
45	·941	·338	1·88	·676	2·82	·101	3·76	1·35	4·71	1·69	5·65	2·03	6·59	2·37	7·53	2·70	8·47	3·04	9·41	3·38	15
48	·941	·339	1·88	·677	2·82	·102	3·76	1·35	4·70	1·69	5·64	2·03	6·59	2·37	7·53	2·71	8·47	3·05	9·41	3·39	12
51	·941	·340	1·88	·679	2·82	·102	3·76	1·36	4·70	1·70	5·64	2·04	6·58	2·38	7·52	2·72	8·47	3·06	9·41	3·40	9
54	·940	·340	1·88	·681	2·82	·102	3·76	1·36	4·70	1·70	5·64	2·04	6·58	2·38	7·52	2·72	8·46	3·06	9·40	3·40	6
57	·940	·341	1·88	·682	2·82	·102	3·76	1·36	4·70	1·71	5·64	2·05	6·58	2·39	7·52	2·73	8·46	3·07	9·40	3·41	3
20	·940	·342	1·88	·684	2·82	·103	3·76	1·37	4·70	1·71	5·64	2·05	6·58	2·39	7·52	2·74	8·46	3·08	9·40	3·42	70°
3	·939	·343	1·88	·686	2·82	·103	3·76	1·37	4·70	1·71	5·64	2·06	6·58	2·40	7·52	2·74	8·45	3·09	9·39	3·43	57
6	·939	·344	1·88	·687	2·82	·103	3·76	1·37	4·70	1·72	5·63	2·06	6·57	2·41	7·51	2·75	8·45	3·09	9·39	3·44	54
9	·939	·344	1·88	·689	2·82	·103	3·76	1·38	4·69	1·72	5·63	2·07	6·57	2·41	7·51	2·76	8·45	3·10	9·39	3·44	51
12	·938	·345	1·88	·691	2·82	·104	3·75	1·38	4·69	1·73	5·63	2·07	6·57	2·42	7·51	2·76	8·45	3·11	9·38	3·45	48
15	·938	·346	1·88	·692	2·81	·104	3·75	1·38	4·69	1·73	5·63	2·08	6·57	2·42	7·51	2·77	8·44	3·12	9·38	3·46	45
18	·938	·347	1·88	·694	2·81	·104	3·75	1·39	4·69	1·73	5·63	2·08	6·57	2·43	7·50	2·77	8·44	3·12	9·38	3·46	42
21	·938	·348	1·88	·695	2·81	·104	3·75	1·39	4·69	1·74	5·63	2·09	6·56	2·43	7·50	2·78	8·44	3·13	9·37	3·48	39
24	·937	·349	1·87	·697	2·81	·105	3·75	1·39	4·69	1·74	5·62	2·09	6·56	2·44	7·50	2·79	8·44	3·14	9·37	3·49	36
27	·937	·349	1·87	·699	2·81	·105	3·75	1·40	4·68	1·75	5·62	2·10	6·56	2·45	7·50	2·80	8·43	3·14	9·37	3·49	33
30	·937	·350	1·87	·700	2·81	·105	3·75	1·40	4·68	1·75	5·62	2·10	6·56	2·45	7·49	2·80	8·43	3·15	9·37	3·50	30
33	·036	·351	1·87	·702	2·81	·105	3·75	1·40	4·68	1·75	5·62	2·11	6·55	2·46	7·49	2·81	8·43	3·16	9·36	3·51	27
36	·936	·352	1·87	·704	2·81	·106	3·74	1·41	4·68	1·76	5·62	2·11	6·55	2·46	7·49	2·81	8·42	3·17	9·36	3·52	24
39	·936	·353	1·87	·705	2·81	·106	3·74	1·41	4·68	1·76	5·61	2·12	6·55	2·47	7·49	2·82	8·42	3·17	9·36	3·53	21
42	·935	·353	1·87	·707	2·81	·106	3·74	1·41	4·68	1·77	5·61	2·12	6·55	2·47	7·48	2·83	8·42	3·18	9·35	3·53	18
45	·935	·354	1·87	·709	2·81	·106	3·74	1·42	4·68	1·77	5·61	2·13	6·55	2·48	7·48	2·83	8·42	3·19	9·35	3·54	15
48	·935	·355	1·87	·710	2·80	·107	3·74	1·42	4·67	1·78	5·61	2·13	6·54	2·48	7·48	2·84	8·41	3·20	9·35	3·55	12
51	·935	·356	1·87	·712	2·80	·107	3·74	1·42	4·67	1·78	5·61	2·14	6·54	2·49	7·48	2·85	8·41	3·20	9·35	3·56	9
54	·934	·357	1·87	·713	2·80	·107	3·74	1·43	4·67	1·78	5·61	2·14	6·54	2·50	7·47	2·85	8·41	3·21	9·34	3·57	6
57	·934	·358	1·87	·715	2·80	·107	3·74	1·43	4·67	1·79	5·60	2·15	6·54	2·50	7·47	2·86	8·41	3·22	9·34	3·58	3
Dist.	Dep	Lat.	Dep	Lat	Dep	Lat.	Dep	Lat.	Dep	Lat.	Dep	Lat	Dep	Lat.	Dep	Lat	Dep	Lat.	Dep	Lat.	Dist.

Chs.	1		2		3		4		5		6		7		8		9		10		Chs.
′	Lat.	Dep	Lat.	Dep	Lat.	Dep	Lat.	Dep	Lat	Dep	Lat.	Dep	Lat.	Dep	Lat.	Dep	Lat.	Dep	Lat	Dep	′
21	·934	·358	1·87	·717	2·80	1·07	3·73	1·43	1·67	1·79	5·60	2·15	6·54	2·51	7·47	2·87	8·40	3·23	9·34	3·58	**69**
3	·933	·359	1·87	·718	2·80	1·08	3·73	1·44	4·67	1·80	5·60	2·16	6·53	2·51	7·47	2·87	8·40	3·23	9·33	3·59	57
6	·933	·360	1·87	·720	2·80	1·08	3·73	1·44	4·66	1·80	5·60	2·16	6·53	2·52	7·46	2·88	8·40	3·24	9·33	3·60	54
9	·933	·361	1·87	·722	2·80	1·08	3·73	1·44	4·66	1·80	5·60	2·16	6·53	2·53	7·46	2·89	8·39	3·25	9·33	3·61	51
12	·932	·361	1·86	·723	2·80	1·08	3·73	1·45	4·66	1·81	5·59	2·17	6·53	2·53	7·46	2·89	8·39	3·25	9·32	3·61	48
15	·932	·362	1·86	·725	2·80	1·09	3·73	1·45	4·66	1·81	5·59	2·17	6·52	2·54	7·46	2·90	8·39	3·26	9·32	3·62	45
18	·932	·363	1·86	·726	2·80	1·09	3·73	1·45	4·66	1·82	5·59	2·18	6·52	2·54	7·45	2·91	8·39	3·27	9·32	3·63	42
21	·931	·364	1·86	·728	2·79	1·09	3·73	1·46	4·66	1·82	5·59	2·18	6·52	2·55	7·45	2·91	8·38	3·28	9·31	3·64	39
24	·931	·365	1·86	·730	2·79	1·09	3·72	1·46	4·66	1·82	5·59	2·19	6·52	2·55	7·45	2·92	8·38	3·28	9·31	3·65	36
27	·931	·366	1·86	·731	2·79	1·10	3·72	1·46	4·65	1·83	5·58	2·19	6·51	2·56	7·45	2·93	8·38	3·29	9·31	3·66	33
30	·930	·366	1·86	·733	2·79	1·10	3·72	1·47	4·65	1·83	5·58	2·20	6·51	2·57	7·44	2·93	8·37	3·30	9·30	3·66	30
33	·930	·367	1·86	·735	2·79	1·10	3·72	1·47	4·65	1·84	5·58	2·20	6·51	2·57	7·44	2·94	8·37	3·31	9·30	3·67	27
36	·930	·368	1·86	·736	2·79	1·10	3·72	1·47	4·65	1·84	5·58	2·21	6·51	2·58	7·44	2·94	8·37	3·31	9·30	3·68	24
39	·929	·369	1·86	·738	2·79	1·11	3·72	1·48	4·65	1·84	5·58	2·21	6·51	2·58	7·44	2·95	8·37	3·32	9·29	3·69	21
42	·929	·370	1·86	·739	2·79	1·11	3·72	1·48	4·65	1·85	5·57	2·22	6·50	2·59	7·43	2·96	8·37	3·33	9·29	3·70	18
45	·929	·371	1·86	·741	2·79	1·11	3·72	1·48	4·64	1·85	5·57	2·22	6·50	2·59	7·43	2·96	8·36	3·33	9·29	3·71	15
48	·928	·371	1·86	·743	2·79	1·11	3·71	1·49	4·64	1·86	5·57	2·23	6·50	2·60	7·43	2·97	8·36	3·34	9·28	3·71	12
51	·928	·372	1·86	·744	2·78	1·12	3·71	1·49	4·64	1·86	5·67	2·23	6·50	2·61	7·43	2·98	8·35	3·35	9·28	3·72	9
54	·928	·373	1·86	·746	2·78	1·12	3·71	1·49	4·64	1·86	5·87	2·24	6·49	2·61	7·42	2·98	8·35	3·36	9·28	3·73	6
57	·928	·374	1·86	·748	2·78	1·12	3·71	1·50	4·64	1·87	5·57	2·24	6·49	2·62	7·42	2·99	8·35	3·36	9·28	3·74	3
22°	·927	·375	1·85	·749	2·78	1·12	3·71	1·50	1·64	1·87	5·56	2·25	6·49	2·62	7·42	3·00	8·34	3·37	9·27	3·75	**68°**
3	·927	·375	1·85	·751	2·78	1·13	3·71	1·50	4·63	1·88	5·56	2·25	6·49	2·63	7·41	3·00	8·34	3·38	9·27	3·75	57
6	·927	·376	1·85	·752	2·78	1·13	3·71	1·50	4·63	1·88	5·56	2·26	6·49	2·63	7·41	3·01	8·34	3·39	9·27	3·76	54
9	·926	·377	1·85	·754	2·78	1·13	3·70	1·51	4·63	1·89	5·56	2·26	6·48	2·64	7·41	3·02	8·31	3·39	9·26	3·77	51
12	·926	·378	1·85	·756	2·78	1·13	3·70	1·51	4·63	1·89	5·56	2·27	6·48	2·64	7·41	3·02	8·33	3·40	9·26	3·78	48
15	·926	·379	1·85	·757	2·78	1·14	3·70	1·51	4·63	1·89	5·55	2·27	6·48	2·65	7·40	3·03	8·33	3·41	9·26	3·79	45
18	·925	·379	1·85	·759	2·78	1·14	3·70	1·52	4·63	1·90	5·55	2·28	6·48	2·66	7·40	3·04	8·33	3·42	9·25	3·79	42
21	·925	·380	1·85	·761	2·77	1·14	3·70	1·52	4·62	1·90	5·55	2·28	6·17	2·66	7·40	3·04	8·32	3·42	9·25	3·80	39
24	·925	·381	1·85	·762	2·77	1·14	3·70	1·52	4·62	1·91	5·55	2·29	6·47	2·67	7·40	3·05	8·32	3·43	9·25	3·81	36
27	·924	·382	1·85	·764	2·77	1·15	3·70	1·53	4·62	1·91	5·55	2·29	6·47	2·67	7·40	3·06	8·32	3·44	9·24	3·82	33
30	·924	·383	1·85	·765	2·77	1·15	3·70	1·53	4·62	1·91	5·54	2·30	6·47	2·68	7·39	3·06	8·32	3·44	9·24	3·83	30
33	·924	·383	1·85	·767	2·77	1·15	3·69	1·53	4·62	1·92	5·54	2·30	6·46	2·68	7·39	3·07	8·31	3·45	9·24	3·83	27
36	·923	·384	1·85	·769	2·77	1·15	3·69	1·54	4·62	1·92	5·54	2·31	6·46	2·69	7·39	3·07	8·31	3·46	9·23	3·84	24
39	·923	·385	1·85	·770	2·77	1·16	3·69	1·54	4·61	1·93	5·54	2·31	6·46	2·70	7·38	3·08	8·31	3·47	9·23	3·85	21
42	·923	·386	1·85	·772	2·77	1·16	3·69	1·54	4·61	1·93	5·54	2·32	6·46	2·70	7·38	3·09	8·30	3·47	9·23	3·86	18
45	·922	·387	1·84	·773	2·77	1·16	3·69	1·55	4·61	1·93	5·53	2·32	6·46	2·71	7·38	3·09	8·30	3·48	9·22	3·87	15
48	·922	·388	1·84	·775	2·77	1·16	3·69	1·55	4·61	1·94	5·53	2·33	6·45	2·71	7·37	3·10	8·30	3·49	9·22	3·88	12
51	·922	·388	1·84	·777	2·76	1·17	3·69	1·55	4·61	1·94	5·53	2·33	6·45	2·72	7·37	3·11	8·29	3·49	9·22	3·88	9
54	·921	·389	1·84	·778	2·76	1·17	3·68	1·56	4·61	1·95	5·53	2·33	6·45	2·72	7·37	3·11	8·29	3·50	9·21	3·89	6
57	·921	·390	1·84	·780	2·76	1·17	3·68	1·56	4·60	1·95	5·52	2·34	6·45	2·73	7·37	3·12	8·29	3·51	9·21	3·90	3
23°	·921	·391	1·84	·781	2.76	1·17	3·68	1·56	4·60	1·95	5·52	2·34	6·44	2·74	7·36	3·13	8·28	3·52	9·21	3·91	**67°**
3	·920	·392	1·84	·783	2·76	1·17	3·68	1·57	4·60	1·96	5·52	2·35	6·44	2·74	7·36	3·13	8·28	3·52	9·20	3·92	57
6	·920	·392	1·84	·785	2·76	1·18	3·68	1·57	4·60	1·96	5·52	2·35	6·44	2·75	7·36	3·14	8·28	3·53	9·20	3·92	54
9	·919	·393	1·84	·786	2·76	1·18	3·68	1·57	4·60	1·97	5·52	2·36	6·44	2·75	7·36	3·15	8·28	3·54	9·19	3·93	51
12	·919	·394	1·84	·788	2·76	1·18	3·68	1·58	4·60	1·97	5·51	2·36	6·43	2·76	7·35	3·15	8·27	3·55	9·19	3·94	48
15	·919	·395	1·84	·790	2·76	1·18	3·68	1·58	4·59	1·97	5·51	2·37	6·43	2·76	7·35	3·16	8·27	3·55	9·19	3·95	45
18	·918	·396	1·84	·791	2·76	1·19	3·67	1·58	4·59	1·98	5·51	2·37	6·43	2·77	7·35	3·16	8·27	3·56	9·18	3·96	42
21	·918	·396	1·84	·793	2·75	1·19	3·67	1·59	4·59	1·98	5·51	2·38	6·43	2·77	7·34	3·17	8·26	3·57	9·18	3·96	39
24	·918	·397	1·84	·794	2·75	1·19	3·67	1·59	4·59	1·99	5·51	2·38	6·42	2·78	7·34	3·18	8·26	3·57	9·18	3·97	36
27	·917	·398	1·83	·796	2·75	1·19	3·67	1·59	4·59	1·99	5·50	2·39	6·42	2·79	7·34	3·18	8·26	3·58	9·17	3·98	33
30	·917	·399	1·83	·797	2·75	1·20	3·67	1·59	4·58	1·99	5·50	2·39	6·42	2·79	7·34	3·19	8·25	3·59	9·17	3·99	30
33	·917	·400	1·83	·800	2·75	1·20	3·67	1·60	4·58	2·00	5·50	2·40	6·42	2·80	7·33	[illegible]	8·25	3·60	9·17	4·00	27
36	·916	·400	1·83	·801	2·75	1·20	3·67	1·60	4·58	2·00	5·50	2·40	6·41	2·80	7·33	3·20	8·25	3·60	9·16	4·00	24
39	·916	·401	1·83	·802	2·75	1·20	3·66	1·60	4·58	2·01	5·50	2·41	6·41	2·81	7·33	3·21	8·24	3·61	9·16	4·01	21
42	·916	·402	1·83	·804	2·75	1·21	3·66	1·61	4·58	2·01	5·49	2·41	6·41	2·81	7·33	3·22	8·24	3·62	9·16	4·02	18
45	·915	·403	1·83	·805	2·75	1·21	3·66	1·61	4·58	2·01	5·49	2·42	6·41	2·82	7·32	3·22	8·24	3·62	9·15	4·03	15
48	·915	·404	1·83	·807	2·74	1·21	3·66	1·61	4·57	2·02	5·49	2·42	6·40	2·82	7·32	3·23	8·23	3·63	9·15	4·04	12
51	·915	·404	1·83	·809	2·74	1·21	3·66	1·61	4·57	2·02	5·49	2·43	6·40	·83	7·32	3·23	8·23	3·64	9·15	4·04	9
54	·914	·405	1·83	·810	2·74	1·22	3·66	1·62	4·57	2·03	5·49	2·43	6·40	2·84	7·31	3·24	8·23	3·65	9·14	4·05	6
57	·914	·406	1·83	·812	2·74	1·22	3·66	1·62	4·57	2·03	5·48	2·44	6·40	2·84	7·31	3·25	8·23	3·65	9·14	4·06	3
Dist.	Dep	Lat.	Dep	Lat.	Dep	Lat.	Dep	Lat.	Dep	Lat.	Dep	Lat.	Dep	Lat.	Dep	Lat.	Dep	Lat.	Dep	Lat.	Dist.

Chs.	1		2		3		4		5		6		7		8		9		10		Chs.
′	Lat.	Dep	Lat.	Dep	Lat.	Dep	Lat.	Dep	Lat.	Dep	Lat.	Dep	Lat.	Dep	Lat.	Dep	Lat.	Dep	Lat	Dep	′
24°	·914	·407	1·83	·813	2·74	1·22	3·65	1·63	4·57	2·03	5·48	2·44	6·40	2·85	7·31	3·25	8·22	3·66	9·14	4·07	**66°**
3	·913	·408	1·83	·815	2·74	1·22	3·65	1·63	4·57	2·04	5·48	2·45	6·39	2·85	7·31	3·26	8·22	3·67	9·13	4·08	57
6	·913	·408	1·83	·817	2·74	1·22	3·65	1·63	4·56	2·04	5·48	2·45	6·39	2·86	7·30	3·27	8·22	3·67	9·13	4·08	54
9	·912	·409	1·82	·818	2·74	1·23	3·65	1·64	4·56	2·05	5·47	2·45	6·39	2·86	7·30	3·27	8·21	3·68	9·12	4·09	51
12	·912	·410	1·82	·820	2·74	1·23	3·65	1·64	4·56	2·05	5·47	2·46	6·38	2·87	7·30	3·28	8·21	3·69	9·12	4·10	48
15	·912	·411	1·82	·821	2·74	1·23	3·65	1·64	4·56	2·05	5·47	2·46	6·38	2·87	7·29	3·29	8·21	3·70	9·12	4·11	45
18	·911	·412	1·82	·823	2·73	1·23	3·65	1·65	4·56	2·06	5·47	2·47	6·38	2·88	7·29	3·29	8·20	3·70	9·11	4·12	42
21	·911	·412	1·82	·825	2·73	1·24	3·64	1·65	4·56	2·06	5·47	2·47	6·38	2·89	7·29	3·30	8·20	3·71	9·11	4·12	39
24	·911	·413	1·82	·826	2·73	1·24	3·64	1·65	4·55	2·07	5·46	2·48	6·37	2·89	7·29	3·30	8·20	3·72	9·11	4·13	36
27	·910	·414	1·82	·828	2·73	1·24	3·64	1·66	4·55	2·07	5·46	2·48	6·37	2·90	7·28	3·31	8·19	3·72	9·10	4·14	33
30	·910	·415	1·82	·829	2·73	1·24	3·64	1·66	4·55	2·07	5·46	2·49	6·37	2·90	7·28	3·32	8·19	3·73	9·10	4·15	30
33	·910	·415	1·82	·831	2·73	1·25	3·64	1·66	4·55	2·08	5·46	2·49	6·37	2·91	7·28	3·32	8·19	3·74	9·10	4·15	27
36	·909	·416	1·82	·833	2·73	1·25	3·64	1·66	4·55	2·08	5·46	2·50	6·36	2·91	7·27	3·33	8·18	3·75	9·09	4·16	24
39	·909	·417	1·82	·834	2·73	1·25	3·64	1·67	4·54	2·09	5·45	2·50	6·36	2·92	7·27	3·34	8·18	3·75	9·09	4·17	21
42	·909	·418	1·82	·836	2·73	1·25	3·63	1·67	4·54	2·09	5·45	2·51	6·36	2·93	7·27	3·34	8·18	3·76	9·09	4·18	18
45	·908	·419	1·82	·837	2·72	1·26	3·63	1·67	4·54	2·09	5·45	2·51	6·36	2·93	7·27	3·35	8·17	3·77	9·08	4·19	15
48	·908	·419	1·82	·839	2·72	1·26	3·63	1·68	4·54	2·10	5·45	2·52	6·35	2·94	7·26	3·36	8·17	3·78	9·08	4·19	12
51	·907	·420	1·81	·840	2·72	1·26	3·63	1·68	4·54	2·10	5·44	2·52	6·35	2·94	7·26	3·36	8·17	3·78	9·07	4·20	9
54	·907	·421	1·81	·842	2·72	1·26	3·63	1·68	4·54	2·11	5·44	2·53	6·35	2·95	7·26	3·37	8·16	3·79	9·07	4·21	6
57	·907	·422	1·81	·844	2·72	1·27	3·63	1·69	4·53	2·11	5·44	2·53	6·35	2·95	7·25	3·37	8·16	3·80	9·07	4·22	3
25°	·906	·423	1·81	·845	2·72	1·27	3·63	1·69	4·53	2·11	5·44	2·54	6·34	2·96	7·25	3·38	8·16	3·80	9·06	4·23	**65°**
3	·906	·423	1·81	·847	2·72	1·27	3·62	1·69	4·53	2·11	5·44	2·54	6·34	2·96	7·25	3·39	8·15	3·81	9·06	4·23	57
6	·906	·424	1·81	·848	2·72	1·27	3·62	1·70	4·53	2·12	5·43	2·55	6·34	2·97	7·24	3·39	8·15	3·82	9·06	4·24	54
9	·905	·425	1·81	·850	2·72	1·27	3·62	1·70	4·53	2·12	5·43	2·55	6·34	2·97	7·24	3·40	8·15	3·82	9·05	4·25	51
12	·905	·426	1·81	·852	2·71	1·28	3·62	1·70	4·52	2·13	5·43	2·55	6·33	2·98	7·24	3·41	8·14	3·83	9·05	4·26	48
15	·905	·427	1·81	·853	2·71	1·28	3·62	1·71	4·52	2·13	5·43	2·56	6·33	2·99	7·24	3·41	8·14	3·84	9·05	4·27	45
18	·904	·427	1·81	·855	2·71	1·28	3·62	1·71	4·52	2·14	5·42	2·56	6·33	2·99	7·23	3·42	8·14	3·85	9·04	4·27	42
21	·904	·428	1·81	·856	2·71	1·28	3·61	1·71	4·52	2·14	5·42	2·57	6·33	3·00	7·23	3·43	8·13	3·85	9·04	4·28	39
24	·903	·429	1·81	·858	2·71	1·29	3·61	1·72	4·52	2·14	5·42	2·57	6·32	3·00	7·23	3·43	8·13	3·86	9·03	4·29	36
27	·903	·430	1·81	·860	2·71	1·29	3·61	1·72	4·51	2·15	5·42	2·58	6·32	3·01	7·22	3·44	8·13	3·87	9·03	4·30	33
30	·903	·430	1·81	·861	2·71	1·29	3·61	1·72	4·51	2·15	5·42	2·58	6·32	3·01	7·22	3·44	8·12	3·87	9·03	4·30	30
33	·902	·431	1·80	·863	2·71	1·29	3·61	1·73	4·51	2·16	5·41	2·59	6·32	3·02	7·22	3·45	8·12	3·88	9·02	4·31	27
36	·902	·432	1·80	·864	2·71	1·30	3·61	1·73	4·51	2·16	5·41	2·59	6·31	3·02	7·21	3·46	8·12	3·89	9·02	4·32	24
39	·901	·433	1·80	·866	2·70	1·30	3·61	1·73	4·51	2·16	5·41	2·60	6·31	3·03	7·21	3·46	8·11	3·90	9·02	4·33	21
42	·901	·434	1·80	·867	2·70	1·30	3·60	1·73	4·51	2·17	5·41	2·60	6·31	3·04	7·21	3·47	8·11	3·90	9·02	4·34	18
45	·901	·434	1·80	·869	2·70	1·30	3·60	1·74	4·50	2·17	5·40	2·61	6·30	3·04	7·21	3·48	8·11	3·91	9·02	4·34	15
48	·900	·435	1·80	·870	2·70	1·31	3·60	1·74	4·50	2·18	5·40	2·61	6·30	3·05	7·20	3·48	8·10	3·92	9·00	4·35	12
51	·900	·436	1·80	·872	2·70	1·31	3·60	1·74	4·50	2·18	5·49	2·62	6·30	3·05	7·20	3·49	8·10	3·92	9·00	4·36	9
54	·900	·437	1·80	·874	2·70	1·31	3·60	1·75	4·50	2·18	5·40	2·62	6·30	3·06	7·20	3·49	8·10	3·93	9·00	4·37	6
57	·899	·438	1·80	·875	2·70	1·31	3·60	1·75	4·50	2·19	5·40	2·63	6·29	3·06	7·19	3·50	8·09	3·94	8·99	4·38	3
26°	·899	·438	1·80	·877	2·70	1·32	3·60	1·75	4·49	2·19	5·39	2·63	6·29	3·07	7·19	3·51	8·09	3·95	8·99	4·38	**64°**
3	·898	·439	1·80	·878	2·70	1·32	3·59	1·76	4·49	2·20	5·39	2·63	6·29	3·07	7·19	3·51	8·09	3·95	8·98	4·39	57
6	·898	·440	1·80	·880	2·69	1·32	3·59	1·76	4·49	2·20	5·39	2·64	6·29	3·08	7·18	3·52	8·08	3·96	8·98	4·40	54
9	·898	·441	1·80	·881	2·69	1·32	3·59	1·76	4·49	2·20	5·39	2·64	6·28	3·08	7·18	3·53	8·08	3·97	8·98	4·41	51
12	·897	·441	1·79	·883	2·69	1·32	3·59	1·77	4·49	2·21	5·38	2·65	6·28	3·09	7·18	3·53	8·08	3·97	8·97	4·41	48
15	·897	·442	1·79	·885	2·69	1·33	3·59	1·77	4·48	2·21	5·38	2·65	6·28	3·10	7·17	3·54	8·07	3·98	8·97	4·42	45
18	·896	·443	1·79	·886	2·69	1·33	3·59	1·77	4·48	2·22	5·38	2·66	6·28	3·10	7·17	3·54	8·07	3·99	8·96	4·43	42
21	·896	·444	1·79	·888	2·69	1·33	3·58	1·78	4·48	2·22	5·38	2·66	6·27	3·11	7·17	3·55	8·06	3·99	8·96	4·44	39
24	·896	·447	1·79	·889	2·69	1·33	3·58	1·78	4·48	2·22	5·37	2·67	6·27	3·11	7·17	3·56	8·06	4·00	8·96	4·46	36
27	·895	·445	1·79	·891	2·69	1·34	3·58	1·78	4·48	2·23	5·37	2·67	6·27	3·12	7·16	3·56	8·06	4·01	8·95	4·45	33
30	·895	·446	1·79	·892	2·68	1·34	3·58	1·78	4·47	2·23	5·37	2·68	6·26	3·12	7·16	3·57	8·05	4·02	8·95	4·46	30
33	·895	·447	1·79	·894	2·68	1·34	3·58	1·79	4·47	2·23	5·37	2·68	6·26	3·13	7·16	3·58	8·05	4·02	8·95	4·47	27
36	·894	·448	1·79	·896	2·68	1·34	3·58	1·79	4·47	2·24	5·36	2·69	6·26	3·13	7·15	3·58	8·05	4·03	8·94	4·48	24
39	·894	·449	1·79	·897	2·68	1·35	3·58	1·79	4·47	2·24	5·36	2·69	6·26	3·14	7·15	3·59	8·04	4·04	8·94	4·49	21
42	·893	·449	1·79	·899	2·68	1·35	3·57	1·80	4·47	2·25	5·36	2·70	6·25	3·15	7·15	3·59	8·04	4·04	8·93	4·49	18
45	·893	·450	1·79	·900	2·68	1·35	3·57	1·80	4·46	2·25	5·36	2·70	6·25	3·15	7·14	3·60	8·04	4·05	8·93	4·50	15
48	·893	·451	1·79	·902	2·68	1·35	3·57	1·80	4·46	2·25	5·36	2·71	6·25	3·16	7·14	3·61	8·03	4·06	8·93	4·51	12
51	·892	·452	1·78	·903	2·68	1·35	3·57	1·81	4·46	2·26	5·35	2·71	6·25	3·16	7·14	3·61	8·03	4·06	8·92	4·52	9
54	·892	·452	1·78	·905	2·68	1·36	3·57	1·81	4·46	2·26	5·35	2·71	6·24	3·17	7·14	3·62	8·03	4·07	8·92	4·52	6
57	·891	·453	1·78	·906	2·67	1·36	3·57	1·81	4·46	2·27	5·35	2·72	6·24	3·17	7·13	3·63	8·02	4·08	8·91	4·53	3
Dist.	Dep	Lat	Dep	Lat.	Dep	Lat.	Dep	Lat.	Dep	Lat.	Dep	Lat.	Dep	Lat.	Dep	Lat.	Dep	Lat.	Dep	Lat	Dist.

Chs.	1		2		3		4		5		6		7		8		9		10		Chs
′	Lat.	Dep	Lat.	Dep	Lat	Dep	Lat	Dep	Lat	Dep	Lat	Dep	Lat	Dep	Lat.	Dep	Lat.	Dep	Lat	Dep	′
27°	·891	·454	1·78	·908	2·67	1·36	3·56	1·82	4·46	2·27	5·35	2·72	6·24	3·18	7·13	3·63	6·02	4·09	8·91	4·54	63°
3	·891	·455	1·78	·910	2·67	1·36	3·56	1·82	4·45	2·27	5·34	2·73	6·23	3·18	7·12	3·64	8·02	4·09	8·91	4·55	57
6	·890	·456	1·78	·911	2·67	1·37	3·56	1·82	4·45	2·28	5·34	2·73	6·23	3·19	7·12	3·64	8·01	4·10	8·90	4·56	54
9	·890	·456	1·78	·913	2·67	1·37	3·56	1·83	4·45	2·28	5·34	2·74	6·23	3·19	7·12	3·65	8·01	4·11	8·90	4·56	51
12	·889	·457	1·78	·914	2·67	1·37	3·56	1·83	4·45	2·29	5·34	2·74	6·23	3·20	7·12	3·66	8·00	4·11	8·89	4·57	48
15	·889	·458	1·78	·916	2·67	1·37	3·56	1·83	4·44	2·29	5·33	2·75	6·22	3·21	7·11	3·66	8·00	4·12	8·89	4·58	45
18	·889	·459	1·78	·917	2·67	1·38	3·55	1·83	4·44	2·29	5·33	2·75	6·22	3·21	7·11	3·67	8·00	4·13	8·89	4·59	42
21	·888	·459	1·78	·919	2·66	1·38	3·55	1·84	4·44	2·30	5·33	2·76	6·22	3·22	7·11	3·68	8·00	4·13	8·88	4·59	39
24	·888	·460	1·78	·920	2·66	1·38	3·55	1·84	4·44	2·30	5·33	2·76	6·21	3·22	7·10	3·68	7·99	4·14	8·88	4·60	36
27	·887	·461	1·77	·922	2·66	1·38	3·55	1·84	4·44	2·30	5·32	2·77	6·21	3·23	7·10	3·69	7·99	4·15	8·87	4·61	33
30	·887	·462	1·77	·923	2·66	1·38	3·55	1·85	4·44	2·31	5·32	2·77	6·21	3·23	7·10	3·69	7·98	4·16	8·87	4·62	30
33	·887	·463	1·77	·925	2·66	1·39	3·55	1·85	4·43	2·31	5·32	2·78	6·21	3·24	7·10	3·70	7·98	4·16	8·87	4·63	27
36	·886	·463	1·77	·927	2·66	1·39	3·54	1·85	4·43	2·32	5·32	2·78	6·20	3·24	7·09	3·71	7·98	4·17	8·86	4·63	24
39	·886	·464	1·77	·928	2·66	1·39	3·54	1·86	4·43	2·32	5·31	2·78	6·20	3·25	7·09	3·71	7·97	4·18	8·86	4·64	21
42	·885	·465	1·77	·930	2·66	1·39	3·54	1·86	4·43	2·32	5·31	2·79	6·20	3·25	7·08	3·72	7·97	4·18	8·85	4·65	18
45	·885	·466	1·77	·931	2·65	1·40	3·54	1·86	4·42	2·33	5·31	2·79	6·19	3·26	7·08	3·72	7·96	4·19	8·85	4·66	15
48	·885	·466	1·77	·933	2·65	1·40	3·54	1·87	4·42	2·33	5·31	2·80	6·19	3·26	7·08	3·73	7·96	4·20	8·85	4·66	12
51	·884	·467	1·77	·934	2·65	1·40	3·54	1·87	4·42	2·34	5·31	2·80	6·19	3·27	7·07	3·74	7·96	4·20	8·84	4·67	9
54	·884	·468	1·77	·936	2·65	1·40	3·54	1·87	4·42	2·34	5·30	2·81	6·19	3·28	7·07	3·74	7·95	4·21	8·84	4·68	6
57	·883	·469	1·77	·937	2·65	1·41	3·53	1·87	4·42	2·34	5·30	2·81	6·18	3·28	7·07	3·75	7·95	4·22	8·83	4·69	3
28°	·883	·469	1·77	·939	2·65	1·41	3·53	1·88	4·41	2·35	5·30	2·82	6·18	3·29	7·06	3·76	7·95	4·22	8·83	4·69	62°
3	·883	·470	1·77	·940	2·65	1·41	3·53	1·88	4·41	2·35	5·30	2·82	6·18	3·29	7·06	3·76	7·94	4·23	8·83	4·70	57
6	·882	·471	1·76	·942	2·65	1·41	3·53	1·88	4·41	2·36	5·29	2·83	6·17	3·30	7·06	3·77	7·94	4·24	8·82	4·71	54
9	·882	·472	1·76	·944	2·65	1·42	3·53	1·89	4·41	2·36	5·29	2·83	6·17	3·30	7·05	3·77	7·94	4·25	8·82	4·72	51
12	·881	·473	1·76	·945	2·64	1·42	3·53	1·89	4·41	2·36	5·29	2·84	6·17	3·31	7·05	3·78	7·93	4·25	8·81	4·73	48
15	·881	·473	1·76	·947	2·64	1·42	3·52	1·89	4·40	2·37	5·29	2·84	6·17	3·31	7·05	3·79	7·93	4·26	8·81	4·73	45
18	·880	·474	1·76	·948	2·64	1·42	3·52	1·90	4·40	2·37	5·28	2·84	6·16	3·32	7·04	3·79	7·92	4·27	8·80	4·74	42
21	·880	·475	1·76	·950	2·64	1·42	3·52	1·90	4·40	2·37	5·28	2·85	9·16	3·32	7·04	3·80	7·92	4·27	8·80	4·75	39
24	·880	·476	1·76	·951	2·64	1·43	3·52	1·90	4·40	2·38	5·28	2·85	6·16	3·33	7·04	3·80	7·92	4·28	8·80	4·76	36
27	·879	·476	1·76	·953	2·64	1·43	3·52	1·91	4·40	2·38	5·28	2·86	6·15	3·33	7·03	3·81	7·91	4·29	8·79	4·76	33
30	·879	·477	1·76	·954	2·64	1·43	3·52	1·91	4·39	2·39	5·27	2·86	6·15	3·34	7·03	3·82	7·91	4·29	8·79	4·77	30
33	·878	·478	1·76	·956	2·64	1·43	3·51	1·91	4·39	2·39	5·27	2·87	6·15	3·35	7·03	3·82	7·91	4·30	8·78	4·78	27
36	·878	·479	1·76	·957	2·63	1·44	3·51	1·91	4·39	2·39	5·27	2·87	6·15	3·35	7·02	3·83	7·90	4·31	8·78	4·79	24
39	·878	·479	1·76	·959	2·63	1·44	3·51	1·92	4·39	2·40	5·27	2·88	6·14	3·36	7·02	3·84	7·90	4·32	8·78	4·79	21
42	·877	·480	1·75	·960	2·63	1·44	3·51	1·92	4·39	2·40	5·26	2·88	6·14	3·36	7·02	3·84	7·89	4·32	8·77	4·80	18
45	·877	·481	1·75	·962	2·63	1·44	3·51	1·92	4·38	2·40	5·26	2·89	6·14	3·37	7·01	3·85	7·89	4·33	8·77	4·81	15
48	·876	·482	1·75	·964	2·63	1·45	3·51	1·93	4·38	2·41	5·26	2·89	6·14	3·37	7·01	3·85	7·89	4·34	8·76	4·82	12
51	·876	·483	1·75	·965	2·63	1·45	3·50	1·93	4·38	2·41	5·26	2·90	6·13	3·38	7·01	3·86	7·88	4·34	8·76	4·83	9
54	·875	·483	1·75	·967	2·63	1·45	3·50	1·93	4·38	2·42	5·25	2·90	6·13	3·38	7·00	3·87	7·88	4·35	8·75	4·83	6
57	·875	·484	1·75	·968	2·63	1·45	3·50	1·94	4·38	2·42	5·25	2·90	6·13	3·39	7·00	3·87	7·88	4·36	8·75	4·84	3
29°	·875	·485	1·75	·970	2·62	1·45	3·50	1·94	4·37	2·42	5·25	2·91	6·12	3·39	7·00	3·88	7·87	4·36	8·75	4·85	61
3	·874	·486	1·75	·971	2·62	1·46	3·50	1·94	4·37	2·43	5·25	2·91	6·12	3·40	6·99	3·88	7·87	4·37	8·74	4·86	57
6	·874	·486	1·75	·973	2·62	1·46	3·50	1·95	4·37	2·43	5·24	2·92	6·12	3·40	6·99	3·89	7·86	4·38	8·74	4·86	54
9	·873	·487	1·75	·974	2·62	1·46	3·49	1·95	4·37	2·44	5·24	2·92	6·11	3·41	6·99	3·90	7·86	4·38	8·73	4·87	51
12	·873	·488	1·75	·976	2·62	1·46	3·49	1·95	4·36	2·44	5·24	2·93	6·11	3·42	6·98	3·90	7·86	4·39	8·73	4·88	48
15	·872	·489	1·74	·977	2·62	1·47	3·49	1·95	4·36	2·44	5·23	2·93	6·11	3·42	6·98	3·91	7·85	4·40	8·72	4·89	45
18	·872	·489	1·74	·979	2·62	1·47	3·49	1·96	4·36	2·45	5·23	2·94	6·10	3·43	6·98	3·92	7·85	4·40	8·72	4·89	42
21	·872	·490	1·74	·980	2·61	1·47	3·49	1·96	4·36	2·45	5·23	2·94	6·10	3·43	6·97	3·92	7·84	4·41	8·72	4·90	39
24	·871	·491	1·74	·982	2·61	1·47	3·48	1·96	4·36	2·45	5·23	2·95	6·10	3·44	6·97	3·93	7·84	4·42	8·71	4·91	36
27	·871	·492	1·74	·983	2·61	1·47	3·48	1·97	4·35	2·46	5·22	2·95	6·10	3·44	6·97	3·93	7·84	4·42	8·71	4·92	33
30	·870	·492	1·74	·985	2·61	1·48	3·48	1·97	4·35	2·46	5·22	2·95	6·09	3·45	6·96	3·94	7·83	4·43	8·70	4·92	30
33	·870	·493	1·74	·986	2·61	1·48	3·48	1·97	4·35	2·47	5·22	2·96	6·09	3·45	6·96	3·95	7·83	4·44	8·70	4·93	27
36	·869	·494	1·74	·988	2·61	1·48	3·48	1·98	4·35	2·47	5·22	2·96	6·09	3·46	6·96	3·95	7·83	4·45	8·69	4·94	24
39	·869	·495	1·74	·989	2·61	1·48	3·48	1·98	4·35	2·47	5·21	2·97	6·08	3·46	6·95	3·96	7·82	4·45	8·69	4·95	21
42	·869	·495	1·74	·991	2·61	1·49	3·47	1·98	4·34	2·48	5·21	2·97	6·08	3·47	6·95	3·96	7·82	4·46	8·69	4·95	18
45	·868	·496	1·74	·992	2·60	1·49	3·47	1·98	4·34	2·48	5·21	2·98	6·08	3·47	6·95	3·97	7·81	4·47	8·68	4·96	15
48	·868	·497	1·74	·994	2·60	1·49	3·47	1·99	4·34	2·48	5·21	2·98	6·07	3·48	6·94	3·98	7·81	4·47	8·68	4·97	12
51	·867	·498	1·73	·995	2·60	1·49	3·47	1·99	4·34	2·49	5·20	2·99	6·07	3·48	6·94	3·98	7·81	4·48	8·67	4·98	9
54	·867	·498	1·73	·997	2·60	1·50	3·47	1·99	4·33	2·49	5·20	2·99	6·07	3·49	6·94	3·99	7·80	4·49	8·67	4·98	6
57	·866	·499	1·73	·998	2·60	1·50	3·47	2·00	4·33	2·50	5·20	3·00	6·07	3·49	6·93	3·99	7·80	4·49	8·66	4·99	3
Dist.	Dep	Lat.	Dep	Lat	Dep	Lat.	Dep	Lat.	Dep	Lat.	Dep	Lat.	Dep	Lat	Dep	Lat.	Dep	Lat.	Dep	Lat	Dist

Chs.	1		2		3		4		5		6		7		8		9		10		Chs.
′	Lat.	Dep	Lat.	Dep	Lat.	Dep	Lat	Dep	Lat	Dep	Lat.	Dep	Lat.	Dep	Lat	Dep	Lat.	Dep	Lat	Dep	′
30°	·866	·500	1·73	1·00	2·60	1·50	3·46	2·00	4·33	2·50	5·20	3·00	6·06	3·50	6·93	4·00	7·79	4·50	8·66	5·00	60°
3	·866	·501	1·73	1·00	2·60	1·50	3·46	2·00	4·33	2·50	5·19	3·00	6·06	3·51	6·92	4·01	7·79	4·51	8·66	5·01	57
6	·865	·502	1·73	1·00	2·60	1·50	3·46	2·01	4·33	2·51	5·19	3·01	6·06	3·51	6·92	4·01	7·79	4·51	8·65	5·02	54
9	·865	·502	1·73	1·00	2·59	1·51	3·46	2·01	4·33	2·51	5·19	3·01	6·05	3·52	6·92	4·02	7·78	4·52	8·65	5·02	51
12	·865	·503	1·73	1·01	2·59	1·51	3·46	2·01	4·32	2·52	5·19	3·02	6·05	3·52	6·91	4·02	7·78	4·53	8·65	5·03	48
15	·864	·504	1·73	1·01	2·59	1·51	3·46	2·02	4·32	2·52	5·18	3·02	6·05	3·53	6·91	4·03	7·77	4·53	8·64	5·04	45
18	·863	·505	1·73	1·01	2·59	1·51	3·45	2·02	4·32	2·52	5·18	3·03	6·04	3·53	6·91	4·04	7·77	4·54	8·64	5·05	42
21	·863	·505	1·73	1·01	2·59	1·52	3·45	2·02	4·31	2·53	5·18	3·03	6·04	3·54	6·90	4·04	7·77	4·55	8·63	5·05	39
24	·862	·506	1·73	1·01	2·59	1·52	3·45	2·02	4·31	2·53	5·18	3·04	6·04	3·54	6·90	4·05	7·76	4·55	8·62	5·06	36
27	·862	·507	1·72	1·01	2·59	1·52	3·45	2·03	4·31	2·53	5·17	3·04	6·03	3·55	6·90	4·05	7·76	4·56	8·62	5·07	33
30	·862	·508	1·72	1·02	2·58	1·52	3·45	2·03	4·31	2·54	5·17	3·05	6·03	3·55	6·89	4·06	7·75	4·57	8·62	5·08	30
33	·861	·508	1·72	1·02	2·58	1·52	3·44	2·03	4·31	2·54	5·17	3·05	6·03	3·56	6·89	4·07	7·75	4·57	8·61	5·08	27
36	·861	·509	1·72	1·02	2·58	1·53	3·44	2·04	4·30	2·55	5·16	3·05	6·03	3·56	6·89	4·07	7·75	4·58	8·61	5·09	24
39	·860	·510	1·72	1·02	2·58	1·53	3·44	2·04	4·30	2·55	5·16	3·06	6·02	3·57	6·88	4·08	7·74	4·59	8·60	5·10	21
42	·860	·511	1·72	1·02	2·58	1·53	3·44	2·04	4·30	2·55	5·16	3·06	6·02	3·57	6·88	4·08	7·74	4·59	8·60	5·11	18
45	·860	·511	1·72	1·02	2·58	1·53	3·44	2·05	4·29	2·56	5·16	3·07	6·02	3·58	6·87	4·09	7·73	4·60	8·59	5·11	15
48	·859	·512	1·72	1·02	2·58	1·53	3·44	2·05	4·29	2·56	5·15	3·07	6·01	3·58	6·87	4·10	7·73	4·61	8·59	5·12	12
51	·859	·513	1·72	1·03	2·58	1·54	3·43	2·05	4·29	2·56	5·15	3·08	6·01	3·59	6·87	4·10	7·73	4·62	8·59	5·13	9
54	·858	·514	1·72	1·03	2·57	1·54	3·43	2·05	4·29	2·57	5·15	3·08	6·01	3·59	6·86	4·11	7·72	4·62	8·58	5·14	6
57	·858	·514	1·72	1·03	2·57	1·54	3·43	2·06	4·29	2·57	5·15	3·09	6·00	3·60	6·86	4·11	7·72	4·63	8·58	5·14	3
31°	·857	·515	1·71	1·03	2·57	1·55	3·43	2·06	4·29	2·58	5·14	3·09	6·00	3·61	6·86	4·12	7·71	4·64	8·57	5·15	59°
3	·857	·516	1·71	1·03	2·57	1·55	3·43	2·06	4·28	2·58	5·14	3·09	6·00	3·61	6·85	4·13	7·71	4·64	8·57	5·16	57
6	·856	·517	1·71	1·03	2·57	1·55	3·43	2·07	4·28	2·58	5·14	3·10	5·99	3·62	6·85	4·13	7·71	4·65	8·56	5·17	54
9	·856	·517	1·71	1·03	2·57	1·55	3·42	2·07	4·28	2·59	5·13	3·10	5·99	3·62	6·85	4·14	7·70	4·66	8·56	5·17	51
12	·855	·518	1·71	1·04	2·57	1·55	3·42	2·07	4·28	2·59	5·13	3·11	5·99	3·63	6·84	4·14	7·70	4·66	8·55	5·18	48
15	·855	·519	1·71	1·04	2·56	1·56	3·42	2·08	4·27	2·59	5·13	3·11	5·98	3·63	6·84	4·15	7·69	4·67	8·55	5·19	45
18	·854	·520	1·71	1·04	2·56	1·56	3·42	2·08	4·27	2·60	5·13	3·12	5·98	3·64	6·84	4·16	7·69	4·68	8·54	5·20	42
21	·854	·520	1·71	1·04	2·56	1·56	3·42	2·08	4·27	2·60	5·12	3·12	5·98	3·64	6·83	4·16	7·69	4·68	8·54	5·20	39
24	·854	·521	1·71	1·04	2·56	1·56	3·41	2·08	4·27	2·61	5·12	3·13	5·97	3·65	6·83	4·17	7·68	4·69	8·54	5·21	36
27	·853	·522	1·71	1·04	2·56	1·57	3·41	2·09	4·27	2·61	5·12	3·13	5·97	3·65	6·82	4·17	7·68	4·70	8·53	5·22	33
30	·853	·522	1·71	1·04	2·56	1·57	3·41	2·09	4·26	2·61	5·12	3·13	5·97	3·66	6·82	4·18	7·67	4·70	8·53	5·22	30
33	·852	·523	1·70	1·05	2·56	1·57	3·41	2·09	4·26	2·62	5·11	3·14	5·97	3·66	6·82	4·19	7·67	4·71	8·52	5·23	27
36	·852	·524	1·70	1·05	2·56	1·57	3·41	2·10	4·26	2·62	5·11	3·14	5·96	3·66	6·81	4·19	7·67	4·72	8·52	5·24	24
39	·851	·525	1·70	1·05	2·55	1·57	3·41	2·10	4·26	2·62	5·11	3·15	5·96	3·67	6·81	4·20	7·66	4·72	8·51	5·25	21
42	·851	·525	1·70	1·05	2·55	1·58	3·40	2·10	4·95	2·63	5·10	3·15	5·96	3·68	6·81	4·20	7·66	4·73	8·51	5·25	18
45	·850	·526	1·70	1·05	2·55	1·58	3·40	2·10	4·25	2·63	5·10	3·16	5·95	3·68	6·80	4·21	7·65	4·74	8·50	5·26	15
48	·850	·527	1·70	1·06	2·55	1·58	3·40	2·11	4·25	2·63	5·10	3·16	5·95	3·69	6·80	4·22	7·65	4·74	8·50	5·27	12
51	·849	·528	1·70	1·06	2·55	1·58	3·40	2·11	4·25	2·64	5·10	3·17	5·95	3·69	6·80	4·22	7·64	4·75	8·49	5·28	9
54	·849	·528	1·70	1·06	2·55	1·59	3·40	2·11	4·24	2·64	5·10	3·17	5·94	3·70	6·79	4·23	7·64	4·76	8·49	5·28	6
57	·849	·529	1·70	1·06	2·55	1·59	3·39	2·12	4·24	2·65	5·09	3·18	5·94	3·70	6·79	4·23	7·64	4·76	8·49	5·29	3
32°	·848	·530	1·70	1·06	2·54	1·59	3·39	2·12	4·24	2·65	5·09	3·18	5·94	3·71	6·78	4·24	7·63	4·77	8·48	5·30	58°
3	·848	·531	1·70	1·06	2·54	1·59	3·39	2·12	4·24	2·65	5·09	3·18	5·93	3·71	6·78	4·25	7·63	4·78	8·48	5·31	57
6	·847	·531	1·69	1·06	2·54	1·59	3·39	2·13	4·24	2·66	5·08	3·19	5·93	3·72	6·78	4·25	7·62	4·78	8·47	5·31	54
9	·847	·532	1·69	1·06	2·54	1·60	3·39	2·13	4·23	2·66	5·08	3·19	5·93	3·72	6·77	4·26	7·62	4·79	8·47	5·32	51
12	·846	·533	1·69	1·07	2·54	1·60	3·38	2·13	4·23	2·66	5·08	3·20	5·92	3·73	6·77	4·26	7·62	4·80	8·46	5·33	48
15	·846	·534	1·69	1·07	2·54	1·60	3·38	2·13	4·23	2·67	5·07	3·20	5·92	3·74	6·77	4·27	7·61	4·80	8·46	5·34	45
18	·845	·534	1·69	1·07	2·54	1·60	3·38	2·14	4·23	2·67	5·07	3·21	5·92	3·74	6·76	4·27	7·61	4·81	8·45	5·34	42
21	·845	·535	1·69	1·07	2·53	1·61	3·38	2·14	4·22	2·68	5·07	3·21	5·91	3·74	6·76	4·28	7·60	4·82	8·45	5·35	39
24	·844	·536	1·69	1·07	2·53	1·61	3·38	2·14	4·22	2·68	5·07	3·21	5·91	3·75	6·75	4·29	7·60	4·82	8·44	5·36	36
27	·844	·537	1·69	1·07	2·53	1·61	3·38	2·15	4·22	2·68	5·06	3·22	5·91	3·76	6·75	4·29	7·59	4·83	8·44	5·37	33
30	·843	·537	1·69	1·07	2·53	1·61	3·37	2·16	4·22	2·69	5·06	3·22	5·90	3·76	6·75	4·30	7·59	4·84	8·43	5·37	30
33	·843	·538	1·69	1·08	2·53	1·61	3·37	2·15	4·21	2·69	5·06	3·23	5·90	3·77	6·74	4·30	7·59	4·84	8·43	5·38	27
36	·842	·539	1·68	1·08	2·53	1·62	3·87	2·16	4·21	2·69	5·05	3·23	5·90	3·77	6·74	4·31	7·58	4·85	8·42	5·39	24
39	·842	·540	1·68	1·08	2·53	1·62	3·87	2·16	4·21	2·70	5·05	3·24	5·89	3·78	6·74	4·32	7·58	4·86	8·42	5·40	21
42	·842	·540	1·68	1·08	2·52	1·62	3·87	2·16	4·21	2·70	5·05	3·24	5·89	3·78	6·73	4·32	7·57	4·86	8·42	5·40	18
45	·841	·541	1·68	1·08	2·52	1·62	3·36	2·16	4·21	2·70	5·05	3·25	5·89	3·79	6·73	4·33	7·57	4·87	8·41	5·41	15
48	·841	·542	1·68	1·08	2·52	1·63	3·36	2·17	4·20	2·71	5·04	3·25	5·88	3·79	6·72	4·33	7·57	4·88	8·41	5·42	12
51	·840	·542	1·68	1·08	2·52	1·63	3·36	2·17	4·20	2·71	5·04	3·25	5·88	3·80	6·72	4·34	7·56	4·88	8·40	5·42	9
54	·840	·543	1·68	1·09	2·52	1·63	3·36	2·17	4·20	2·72	5·04	3·26	5·88	3·80	6·72	4·35	7·56	4·89	8·40	5·43	6
57	·839	·544	1·68	1·09	2·52	1·63	3·36	2·18	4·20	2·72	5·03	3·26	5·87	3·81	6·71	4·35	7·55	4·90	8·39	5·44	3
Dist.	Dep	Lat.	Dep	Lat	Dep	Lat.	Dep	Lat.	Dep	Lat.	Dep	Lat.	Dep	Lat.	Dep	Lat.	Dep	Lat	Dep	Lat	Dist.

Chs.	1		2		3		4		5		6		7		8		9		10		Chs.
′	Lat.	Dep	Lat.	Dep	Lat.	Dep	Lat.	Dep	Lat	Dep	Lat.	Dep	Lat.	Dep	Lat.	Dep	Lat.	Dep	Lat.	D.	′
33°	·839	·545	1·68	1·09	2·52	1·63	3·35	2·18	4·19	2·72	5·03	3·27	5·87	3·81	6·71	4·36	7·55	4·90	8·39	5·45	57°
3	·839	·545	1·68	1·09	2·51	1·64	3·35	2·18	4·19	2·73	5·03	3·27	5·87	3·82	6·71	4·36	7·54	4·91	8·38	5·45	57
6	·838	·546	1·68	1·09	2·51	1·64	3·35	2·18	4·19	2·73	5·03	3·28	5·86	3·82	6·70	4·37	7·54	4·91	8·38	5·46	54
9	·837	·547	1·67	1·09	2·51	1·64	3·35	2·19	4·19	2·73	5·02	3·28	5·86	3·83	6·70	4·37	7·54	4·92	8·37	5·47	51
12	·837	·548	1·67	1·10	2·51	1·64	3·35	2·19	4·18	2·74	5·02	3·29	5·86	3·83	6·69	4·38	7·53	4·93	8·37	5·48	48
15	·836	·548	1·67	1·10	2·51	1·64	3·35	2·19	4·18	2·74	5·02	3·29	5·85	3·84	6·69	4·39	7·53	4·93	8·36	5·48	45
18	·836	·549	1·67	1·10	2·51	1·65	3·34	2·20	4·18	2·75	5·01	3·29	5·85	3·84	6·69	4·39	7·52	4·94	8·36	5·49	42
21	·835	·550	1·67	1·10	2·51	1·65	3·34	2·20	4·18	2·75	5·01	3·30	5·85	3·85	6·68	4·40	7·52	4·95	8·35	5·50	39
24	·835	·550	1·67	1·10	2·50	1·65	3·34	2·20	4·17	2·75	5·01	3·30	5·84	3·85	6·68	4·40	7·51	4·95	8·35	5·50	36
27	·834	·551	1·67	1·10	2·50	1·65	3·34	2·20	4·17	2·76	5·01	3·31	5·84	3·86	6·67	4·41	7·51	4·96	8·34	5·51	33
30	·834	·552	1·67	1·10	2·50	1·66	3·34	2·21	4·17	2·76	5·00	3·31	5·84	3·86	6·67	4·42	7·50	4·97	8·34	5·52	30
33	·833	·553	1·67	1·11	2·50	1·66	3·33	2·21	4·17	2·76	5·00	3·32	5·83	3·87	6·67	4·42	7·50	4·97	8·33	5·53	27
36	·833	·553	1·67	1·11	2·50	1·66	3·33	2·21	4·16	2·77	5·00	3·32	5·83	3·87	6·66	4·43	7·50	4·98	8·33	5·53	24
39	·832	·554	1·66	1·11	2·50	1·66	3·33	2·22	4·16	2·77	4·99	3·32	5·83	3·88	6·66	4·43	7·49	4·99	8·32	5·54	21
42	·832	·555	1·66	1·11	2·50	1·66	3·33	2·22	4·16	2·77	4·99	3·33	5·82	3·88	6·66	4·44	7·49	4·99	8·32	5·55	18
45	·831	·556	1·66	1·11	2·49	1·67	3·33	2·22	4·16	2·78	4·99	3·33	5·82	3·89	6·65	4·44	7·48	5·00	8·31	5·56	15
48	·831	·556	1·66	1·11	2·49	1·67	3·32	2·22	4·15	2·78	4·99	3·34	5·82	3·89	6·65	4·45	7·48	5·01	8·31	5·56	12
51	·830	·557	1·66	1·11	2·49	1·67	3·32	2·23	4·15	2·79	4·98	3·34	5·81	3·90	6·64	4·46	7·47	5·01	8·30	5·57	9
54	·830	·558	1·66	1·12	2·49	1·67	3·32	2·23	4·15	2·79	4·98	3·35	5·81	3·90	6·64	4·46	7·47	5·02	8·30	5·58	6
57	·830	·558	1·66	1·12	2·49	1·68	3·32	2·23	4·15	2·79	4·98	3·35	5·81	3·91	6·64	4·47	7·47	5·03	8·30	5·58	3
34°	·829	·559	1·66	1·12	2·49	1·68	3·32	2·24	4·15	2·80	4·97	3·36	5·80	3·91	6·63	4·47	7·46	5·03	8·29	5·59	56°
3	·829	·560	1·66	1·12	2·49	1·68	3·31	2·24	4·14	2·80	4·97	3·36	5·80	3·92	6·63	4·48	7·46	5·04	8·29	5·60	57
6	·828	·561	1·66	1·12	2·48	1·68	3·31	2·24	4·14	2·80	4·97	3·36	5·80	3·92	6·62	4·49	7·45	5·05	8·28	5·61	54
9	·828	·561	1·66	1·12	2·48	1·68	3·31	2·25	4·14	2·81	4·97	3·37	5·79	3·93	6·62	4·49	7·45	5·05	8·28	5·61	51
12	·827	·562	1·65	1·12	2·48	1·69	3·31	2·25	4·14	2·81	4·96	3·37	5·79	3·93	6·62	4·50	7·44	5·06	8·27	5·62	48
15	·827	·563	1·65	1·13	2·48	1·69	3·31	2·25	4·13	2·81	4·96	3·38	5·79	3·94	6·61	4·50	7·44	5·07	8·27	5·63	45
18	·826	·564	1·65	1·13	2·48	1·69	3·30	2·25	4·13	2·82	4·96	3·38	5·78	3·94	6·61	4·51	7·43	5·07	8·26	5·64	42
21	·826	·764	1·65	1·13	2·48	1·69	3·30	2·26	4·13	2·82	4·95	3·39	5·78	3·95	6·60	4·51	7·43	5·08	8·26	5·64	39
24	·825	·565	1·65	1·13	2·48	1·69	3·30	2·26	4·13	2·82	4·95	3·39	5·78	3·95	6·60	4·52	7·43	5·08	8·25	5·65	36
27	·825	·566	1·65	1·13	2·47	1·70	3·30	2·26	4·12	2·83	4·95	3·39	5·77	3·96	6·60	4·53	7·42	5·09	8·25	5·66	33
30	·824	·566	1·65	1·13	2·47	1·70	3·30	2·27	4·12	2·83	4·94	3·40	5·77	3·96	6·59	4·53	7·42	5·10	8·24	5·66	30
33	·824	·567	1·65	1·13	2·47	1·70	3·29	2·27	4·12	2·84	4·94	3·40	5·77	3·97	6·59	4·54	7·41	5·10	8·24	5·67	27
36	·823	·568	1·65	1·14	2·47	1·70	3·29	2·27	4·12	2·84	4·94	3·41	5·76	3·97	6·59	4·54	7·41	5·11	8·23	5·68	24
39	·823	·569	1·65	1·14	2·47	1·71	3·29	2·27	4·11	2·84	4·94	3·41	5·76	3·98	6·58	4·55	7·40	5·12	8·23	5·69	21
42	·822	·569	1·64	1·14	2·47	1·71	3·29	2·28	4·11	2·85	1·93	3·42	5·75	3·98	6·58	4·55	7·40	5·12	8·22	5·69	18
45	·822	·570	1·64	1·14	2·46	1·71	3·29	2·28	4·11	2·85	4·93	3·42	5·75	3·99	6·57	4·56	7·39	5·13	8·22	5·70	15
48	·821	·571	1·64	1·14	2·46	1·71	3·28	2·28	4·11	2·85	4·93	3·42	5·75	3·99	6·57	4·57	7·39	5·14	8·21	5·71	12
51	·821	·571	1·64	1·14	2·46	1·71	3·28	2·29	4·10	2·86	4·92	3·43	5·74	4·00	6·57	4·57	7·39	5·14	8·21	5·71	9
54	·820	·572	1·64	1·14	2·46	1·72	3·28	2·29	4·10	2·86	4·92	3·43	5·74	4·01	6·56	4·58	7·38	5·15	8·20	5·72	6
57	·820	·573	1·64	1·15	2·46	1·72	3·28	2·29	4·10	2·86	4·92	3·44	5·74	4·01	6·56	4·58	7·38	5·16	8·20	5·73	3
35°	·819	·574	1·64	1·15	2·46	1·72	3·28	2·29	4·10	2·87	4·91	3·44	5·73	4·02	6·55	4·59	7·37	5·16	8·19	5·74	55°
3	·819	·574	1·64	1·15	2·46	1·72	3·27	2·30	4·09	2·87	4·91	3·45	5·73	4·02	6·55	4·59	7·37	5·17	8·19	5·74	57
6	·818	·575	1·64	1·15	2·45	1·73	3·27	2·30	4·09	2·88	4·91	3·45	5·73	4·03	6·55	4·60	7·36	5·18	8·18	5·75	54
9	·818	·576	1·64	1·15	2·45	1·73	3·27	2·30	4·09	2·88	4·91	3·45	5·72	4·03	6·54	4·61	7·36	5·18	8·18	5·76	51
12	·817	·576	1·63	1·15	2·45	1·73	3·27	2·31	1·09	2·88	4·90	3·46	5·72	4·04	6·54	4·61	7·35	5·19	8·17	5·76	48
15	·817	·577	1·63	1·15	2·45	1·73	3·27	2·31	4·08	2·89	4·90	3·46	5·72	4·04	6·53	4·62	7·35	5·19	8·17	5·77	45
18	·816	·578	1·63	1·16	2·45	1·73	3·26	2·31	1·08	2·89	4·90	3·47	5·71	4·04	6·53	4·62	7·35	5·20	8·16	5·78	42
21	·816	·579	1·63	1·16	2·45	1·74	3·26	2·31	4·08	2·89	4·89	3·47	5·71	4·05	6·53	4·63	7·34	5·21	8·16	5·79	39
24	·815	·579	1·63	1·16	2·45	1·74	3·26	2·32	4·08	2·90	4·89	3·48	5·71	4·05	6·52	4·63	7·34	5·21	8·15	5·79	36
27	·815	·580	1·63	1·16	2·44	1·74	3·26	2·32	4·07	2·90	4·89	3·48	5·70	4·06	6·52	4·64	7·33	5·22	8·15	5·80	33
30	·814	·581	1·63	1·16	2·44	1·74	3·26	2·32	4·07	2·90	4·88	3·48	5·70	4·06	6·51	1·65	7·33	5·23	8·14	5·81	30
33	·814	·581	1·63	1·16	2·44	1·74	3·25	2·33	4·07	2·91	4·88	3·49	5·70	4·07	6·51	4·65	7·33	5·23	8·14	5·81	27
36	·813	·582	1·63	1·16	2·44	1·75	3·25	2·33	4·07	2·91	4·88	3·49	5·69	4·07	6·50	4·66	7·32	5·24	8·13	5·82	24
39	·813	·583	1·63	1·17	2·44	1·75	3·25	2·33	4·06	2·91	4·88	3·50	5·69	4·08	6·50	4·66	7·31	5·25	8·13	5·83	21
42	·812	·584	1·62	1·17	2·44	1·75	3·25	2·33	4·06	2·92	4·87	3·50	5·68	4·08	6·50	4·67	7·31	5·25	8·12	5·84	18
45	·812	·584	1·62	1·17	2·43	1·75	3·25	2·34	4·06	2·92	4·87	3·51	5·68	4·09	6·49	1·67	7·30	5·26	8·12	5·84	15
48	·811	·585	1·62	1·17	2·43	1·75	3·24	2·34	4·06	2·92	4·87	3·51	5·68	4·10	6·49	4·68	7·30	5·26	8·11	5·85	12
51	·811	·586	1·62	1·17	2·43	1·76	3·24	2·34	4·05	2·93	4·86	3·51	5·67	4·10	6·48	4·69	7·29	5·27	8·11	5·86	9
54	·810	·586	1·62	1·17	2·43	1·76	3·24	2·35	4·05	2·93	4·86	3·52	5·67	4·10	6·48	4·69	7·29	5·28	8·10	5·86	6
57	·810	·587	1·62	1·17	2·43	1·76	3·24	2·35	4·05	2·94	4·86	3·52	5·67	4·11	6·48	1·70	7·29	5·28	8·10	5·87	3
Dist.	Dep	Lat.	Dep	Lat	Dep	Lat	Dep	Lat.	Dep	Lat.	Dep	Lat.	Dep	Lat.	Dep	L t.	Dep	Lat	Dep	Lat	Dist.

Cha.	1		2		3		4		5		6		7		8		9		10		Cha.
′	Lat.	Dep	Lat.	Dep	Lat.	Dep	Lat	Dep	Lat	Dep	Lat.	Dep	Lat.	Dep	Lat.	Dep	Lat.	Dep	Lat	Dep	′
36	·809	·588	1·62	1·18	2·43	1·76	3·24	2·35	4·05	2·94	4·85	3·53	5·66	4·11	6·47	4·70	7·28	5·29	8·09	5·88	54°
3	·809	·588	1·62	1·18	2·43	1·77	3·23	2·35	4·04	2·94	4·85	3·53	5·66	4·12	6·47	4·71	7·28	5·30	8·09	5·88	57
6	·808	·589	1·62	1·18	2·42	1·77	3·23	2·36	4·04	2·95	4·85	3·54	5·66	4·12	6·46	4·71	7·27	5·30	8·08	5·89	54
9	·807	·590	1·61	1·18	2·42	1·77	3·23	2·36	4·04	2·95	4·84	3·54	5·65	4·13	6·46	4·72	7·27	5·31	8·07	5·90	51
12	·807	·591	1·61	1·18	2·42	1·77	3·23	2·36	4·03	2·95	4·84	3·54	5·65	4·13	6·46	4·72	7·26	5·32	8·07	5·91	48
15	·806	·591	1·61	1·18	2·42	1·77	3·23	2·37	4·03	2·96	4·84	3·55	5·65	4·14	6·45	4·73	7·26	5·32	8·06	5·91	45
18	·806	·592	1·61	1·18	2·42	1·78	3·22	2·37	4·03	2·96	4·84	3·55	5·64	4·14	6·45	4·74	7·25	5·33	8·06	5·92	42
21	·805	·593	1·61	1·19	2·42	1·78	3·22	2·37	4·03	2·96	4·83	3·56	5·64	4·15	6·44	4·74	7·25	5·33	8·05	5·93	39
24	·805	·593	1·61	1·19	2·41	1·78	3·22	2·37	4·02	2·97	4·83	3·56	5·63	4·15	6·44	4·75	7·25	5·34	8·05	5·93	36
27	·804	·594	1·61	1·19	2·41	1·78	3·22	2·38	4·02	2·97	4·83	3·56	5·63	4·16	6·44	4·75	7·24	5·35	8·04	5·94	33
30	·804	·595	1·61	1·19	2·41	1·78	3·22	2·38	4·02	2·97	4·82	3·57	5·63	4·16	6·43	4·76	7·23	5·35	8·04	5·95	30
33	·803	·596	1·61	1·19	2·41	1·79	3·21	2·38	4·02	2·98	4·82	3·57	5·62	4·17	6·43	4·76	7·23	5·36	8·03	5·96	27
36	·803	·596	1·61	1·19	2·41	1·79	3·21	2·38	4·01	2·98	4·82	3·58	5·62	4·17	6·42	4·77	7·23	5·37	8·03	5·96	24
39	·802	·597	1·60	1·19	2·41	1·79	3·21	2·39	4·01	2·98	4·81	3·58	5·62	4·18	6·42	4·78	7·22	5·37	8·02	5·97	21
42	·802	·598	1·60	1·20	2·41	1·79	3·21	2·39	4·01	2·99	4·81	3·59	5·61	4·18	6·41	4·78	7·22	5·38	8·02	5·98	18
45	·801	·598	1·60	1·20	2·40	1·79	3·21	2·39	4·01	2·99	4·81	3·59	5·61	4·19	6·41	4·79	7·21	5·38	8·01	5·98	15
48	·801	·599	1·60	1·20	2·40	1·80	3·20	2·40	4·00	3·00	4·80	3·59	5·61	4·19	6·41	4·79	7·21	5·39	8·01	5·99	12
51	·800	·600	1·60	1·20	2·40	1·80	3·20	2·40	4·00	3·00	4·80	3·60	5·60	4·20	6·40	4·80	7·20	5·40	8·00	6·00	9
54	·800	·600	1·60	1·20	2·40	1·80	3·20	2·40	4·00	3·00	4·80	3·60	5·60	4·20	6·40	4·80	7·20	5·40	8·00	6·00	6
57	·799	·601	1·60	1·20	2·40	1·80	3·20	2·40	4·00	3·01	4·79	3·61	5·59	4·21	6·39	4·81	7·19	5·41	7·99	6·01	3
37	·799	·602	1·60	1·22	2·40	1·81	3·19	2·41	3·99	3·01	4·79	3·61	5·59	4·21	6·39	4·81	7·19	5·42	7·99	6·02	53°
3	·798	·603	1·60	1·22	2·39	1·81	3·19	2·41	3·99	3·01	4·79	3·62	5·59	4·22	6·38	4·82	7·18	5·42	7·98	6·03	57
6	·798	·603	1·60	1·22	2·39	1·81	3·19	2·41	3·99	3·02	4·79	3·62	5·58	4·22	6·38	4·83	7·18	5·43	7·98	6·03	54
9	·797	·604	1·59	1·22	2·39	1·81	3·19	2·42	3·99	3·02	4·78	3·62	5·58	4·23	6·38	4·83	7·17	5·44	7·97	6·04	51
12	·797	·605	1·59	1·22	2·39	1·81	3·19	2·42	3·98	3·02	4·78	3·63	5·58	4·23	6·37	4·84	7·17	5·44	7·97	6·05	48
15	·796	·605	1·59	1·22	2·39	1·82	3·19	2·42	3·98	3·03	4·78	3·63	5·57	4·24	6·37	4·84	7·16	5·45	7·96	6·05	45
18	·795	·606	1·59	1·22	2·39	1·82	3·19	2·43	3·98	3·03	4·77	3·64	5·57	4·24	6·36	4·85	7·16	5·45	7·95	6·06	42
21	·795	·607	1·59	1·22	2·38	1·82	3·19	2·43	3·97	3·03	4·77	3·64	5·56	4·25	6·36	4·85	7·15	5·46	7·95	6·07	39
24	·794	·607	1·59	1·22	2·38	1·82	3·19	2·43	3·97	3·04	4·77	3·64	5·56	4·25	6·36	4·86	7·15	5·47	7·94	6·07	36
27	·794	·608	1·59	1·22	2·38	1·82	3·19	2·43	3·97	3·04	4·76	3·65	5·56	4·26	6·35	4·86	7·14	5·47	7·94	6·08	33
30	·793	·609	1·59	1·22	2·38	1·83	3·19	2·44	3·97	3·04	4·76	3·65	5·55	4·26	6·35	4·87	7·14	5·48	7·93	6·09	30
33	·793	·609	1·59	1·22	2·38	1·83	3·19	2·44	3·96	3·05	4·76	3·66	5·55	4·27	6·34	4·88	7·14	5·49	7·93	6·09	27
36	·792	·610	1·58	1·22	2·38	1·83	3·19	2·44	3·96	3·05	4·75	3·66	5·55	4·27	6·34	4·88	7·13	5·49	7·92	6·10	24
39	·792	·611	1·58	1·22	2·38	1·83	3·19	2·44	3·96	3·05	4·75	3·67	5·54	4·28	6·33	4·89	7·13	5·50	7·92	6·11	21
42	·791	·612	1·58	1·22	2·37	1·83	3·19	2·45	3·96	3·06	4·75	3·67	5·54	4·28	6·33	4·89	7·12	5·50	7·92	6·12	18
45	·791	·612	1·58	1·22	2·37	1·84	3·19	2·45	3·95	3·06	4·74	3·67	5·53	4·29	6·33	4·90	7·12	5·51	7·92	6·12	15
48	·790	·613	1·58	1·23	2·37	1·84	3·19	2·45	3·95	3·06	4·74	3·68	5·53	4·29	6·32	4·90	7·11	5·52	7·90	6·13	12
51	·790	·614	1·58	1·23	2·37	1·84	3·19	2·45	3·95	3·07	4·74	3·68	5·53	4·30	6·32	4·91	7·11	5·52	7·90	6·14	9
54	·789	·614	1·58	1·23	2·37	1·84	3·19	2·46	3·95	3·07	4·73	3·69	5·52	4·30	6·31	4·91	7·10	5·53	7·89	6·14	6
57	·789	·615	1·58	1·23	2·37	1·84	3·19	2·46	3·94	3·07	4·73	3·69	5·52	4·30	6·31	4·92	7·10	5·53	7·89	6·15	3
38°	·788	·616	1·58	1·23	2·36	1·85	3·15	2·46	3·94	3·08	4·73	3·69	5·52	4·31	6·30	4·93	7·09	5·54	7·88	6·16	52°
3	·787	·616	1·57	1·23	2·36	1·85	3·15	2·47	3·94	3·08	4·72	3·70	5·51	4·31	6·30	4·93	7·09	5·55	7·87	6·16	57
6	·787	·617	1·57	1·23	2·36	1·85	3·15	2·47	3·93	3·09	4·72	3·70	5·51	4·32	6·30	4·94	7·08	5·55	7·87	6·17	54
9	·786	·618	1·57	1·24	2·36	1·85	3·15	2·47	3·93	3·09	4·72	3·71	5·50	4·32	6·29	4·94	7·08	5·56	7·86	6·18	51
12	·786	·618	1·57	1·24	2·36	1·86	3·14	2·47	3·93	3·09	4·72	3·71	5·50	4·33	6·29	4·95	7·07	5·57	7·86	6·18	48
15	·785	·619	1·57	1·24	2·36	1·86	3·14	2·48	3·93	3·10	4·71	3·71	5·50	4·33	6·28	4·95	7·07	5·57	7·85	6·19	45
18	·785	·620	1·57	1·24	2·35	1·86	3·14	2·48	3·92	3·10	4·71	3·72	5·49	4·34	6·28	4·96	7·06	5·58	7·85	·20	42
21	·784	·620	1·57	1·24	2·35	1·86	3·14	2·48	3·92	3·10	4·71	3·72	5·49	4·34	6·27	4·96	7·06	5·58	7·84	6·20	39
24	·784	·621	1·57	1·24	2·35	1·86	3·13	2·48	3·92	3·11	4·70	3·73	5·49	4·35	6·27	4·97	7·05	5·59	7·84	6·21	36
27	·783	·622	1·57	1·24	2·35	1·87	3·13	2·49	3·92	3·11	4·70	3·73	5·48	4·35	6·27	4·97	7·05	5·60	7·83	6·22	33
30	·783	·623	1·57	1·25	2·35	1·87	3·13	2·49	3·91	3·11	4·70	3·74	5·48	4·36	6·26	4·98	7·04	5·60	7·83	6·23	30
33	·782	·623	1·56	1·25	2·35	1·87	3·13	2·49	3·91	3·12	4·69	3·74	5·47	4·36	6·26	4·99	7·04	5·61	7·82	6·23	27
36	·782	·624	1·56	1·25	2·34	1·87	3·13	2·50	3·91	3·12	4·69	3·74	5·47	4·37	6·25	4·99	7·03	5·61	7·82	6·24	24
39	·781	·625	1·56	1·25	2·34	1·87	3·12	2·50	3·90	3·12	4·69	3·75	5·47	4·37	6·25	5·00	7·03	5·62	7·81	6·25	21
42	·780	·625	1·56	1·25	2·34	1·88	3·12	2·50	3·90	3·13	4·68	3·75	5·46	4·38	6·24	5·00	7·02	5·63	7·80	6·25	18
45	·780	·626	1·56	1·25	2·34	1·88	3·12	2·50	3·90	3·13	4·68	3·76	5·46	4·38	6·24	5·01	7·02	5·63	7·80	6·26	15
48	·779	·627	1·56	1·25	2·34	1·88	3·12	2·51	3·90	3·13	4·68	3·76	5·46	4·39	6·23	5·01	7·01	5·64	7·79	6·27	12
51	·779	·627	1·56	1·25	2·34	1·88	3·12	2·51	3·89	3·14	4·67	3·76	5·45	4·39	6·23	5·02	7·01	5·65	7·79	6·27	9
54	·778	·628	1·56	1·26	2·33	1·88	3·11	2·51	3·89	3·14	4·67	3·77	5·45	4·40	6·23	5·02	7·00	5·65	7·78	6·28	6
57	·778	·629	1·56	1·26	2·33	1·89	3·11	2·51	3·89	3·14	4·67	3·77	5·44	4·40	6·22	5·03	7·00	5·66	7·78	6·29	3
Dist.	Dep	Lat.	Dep	Lat.	Dep	Lat.	Dep	Lat.	Dep	Lat.	Dep	Lat.	Dep	Lat.	Dep	Lat.	Dep	Lat	Dep	Lat.	Dis.

Chs.	1		2		3		4		5		6		7		8		9		10		Chs.
′	Lat	Dep	Lat.	Dep	Lat.	Dep	Lat	Dep	Lat.	Dep	Lat.	Dep	Lat.	Dep	Lat.	Dep	Lat.	Dep	Lat.	Dep	′
39′	·777	·629	1·55	1·26	2·33	1·89	3·11	2·52	3·89	3·15	4·66	3·78	5·44	4·41	6·22	5·03	6·99	5·66	7·77	6·29	51°
3	·777	·630	1·55	1·26	2·33	1·89	3·11	2·52	3·88	3·15	4·66	3·78	5·44	4·41	6·21	5·04	6·99	5·67	7·77	6·30	57
6	·776	·631	1·55	1·26	2·33	1·89	3·10	2·52	3·88	3·15	4·66	3·78	5·43	4·41	6·21	5·05	6·98	5·68	7·76	6·31	54
9	·775	·631	1·55	1·26	2·33	1·89	3·10	2·53	3·88	3·16	4·65	3·79	5·43	4·42	6·20	5·05	6·98	5·68	7·75	6·31	51
12	·775	·632	1·55	1·26	3·32	1·90	3·10	2·53	3·87	3·16	4·65	3·79	5·42	4·42	6·20	5·06	6·97	5·69	7·75	6·32	48
15	·774	·633	1·55	1·27	2·32	1·90	3·10	2·53	3·87	3·16	4·65	3·80	5·42	4·43	6·20	5·06	6·97	5·69	7·74	6·33	45
18	·774	·633	1·55	1·27	2·32	1·90	3·10	2·53	3·87	3·17	4·64	3·80	5·42	4·43	6·20	5·07	6·96	5·70	7·74	6·33	42
21	·773	·634	1·55	1·27	2·32	1·90	3·09	2·54	3·87	3·17	4·64	3·80	5·41	4·44	6·19	5·07	6·96	5·71	7·73	6·34	39
24	·773	·635	1·55	1·27	2·32	1·90	3·09	2·54	3·86	3·17	4·64	3·81	5·41	4·44	6·18	5·08	6·95	5·71	7·73	6·35	36
27	·772	·635	1·54	1·27	2·32	1·91	3·09	2·54	3·86	3·18	4·63	3·81	5·41	4·45	6·18	5·08	6·95	5·72	7·72	6·35	33
30	·772	·636	1·54	1·27	2·31	1·91	3·09	2·54	3·86	3·18	4·63	3·82	5·40	4·45	6·17	5·09	6·94	5·72	7·72	6·36	30
33	·771	·637	1·54	1·27	2·31	1·91	3·08	2·55	3·86	3·18	4·63	3·82	5·40	4·46	6·17	5·09	6·94	5·73	7·71	6·37	27
36	·771	·637	1·54	1·27	2·31	1·91	3·08	2·55	3·85	3·19	4·62	3·82	5·39	4·46	6·16	5·10	6·93	5·74	7·71	6·37	24
39	·770	·638	1·54	1·28	2·31	1·91	3·08	2·55	3·85	3·19	4·62	3·82	5·39	4·47	6·16	5·10	6·93	5·74	7·70	6·38	21
42	·769	·639	1·54	1·28	2·31	1·92	3·08	2·56	3·85	3·19	4·62	3·83	5·39	4·47	6·16	5·11	6·92	5·75	7·69	6·39	18
45	·759	·639	1·54	1·28	2·31	1·92	3·08	2·56	3·84	3·20	4·61	3·84	5·38	4·48	6·15	5·12	6·92	5·75	7·69	6·39	15
48	·768	·640	1·54	1·28	2·30	1·92	3·07	2·56	3·84	3·20	4·61	3·84	5·38	4·48	6·15	5·12	6·91	5·76	7·68	6·40	12
51	·768	·641	1·54	1·28	2·30	1·92	3·07	2·56	3·84	3·20	4·61	3·84	5·37	4·49	6·14	5·13	6·91	5·77	7·68	6·41	9
54	·767	·641	1·53	1·28	2·30	1·92	3·07	2·57	3·84	3·21	4·60	3·85	5·37	4·49	6·14	5.13	6·90	5·77	7·67	6·41	6
57	·767	·642	1·53	1·28	2·30	1·93	3·07	2·57	3·83	3·21	4·60	3·85	5·37	4·49	6·13	5·14	6·90	5·78	7·67	6·42	3
40°	·766	·643	1·53	1·29	2·30	1·93	3·06	2·57	3·83	3·21	4·60	3·86	5·36	4·50	6·13	5·14	6·89	5·79	7·66	6·43	50°
3	·765	·643	1·53	1·29	2·30	1·93	3·06	2·57	3·83	3·22	4·59	3·86	5·36	4·50	6·12	5·15	6·89	5·79	7·65	6·43	57
6	·765	·644	1·53	1·29	2·29	1·93	3·06	2·58	3·82	3·22	4·59	3·86	5·35	4·51	6·12	5·15	6·88	5·80	7·65	6·44	54
9	·764	·645	1·53	1·29	2·29	1·93	3·06	2·58	3·82	3·22	4·59	3·87	5·35	4·51	6·11	5·16	6·88	5·80	7·64	6·45	51
12	·764	·645	1·53	1·29	2·29	1·94	3·06	2·58	3·82	3·23	4·58	3·87	5·35	4·52	6·11	5·16	6·87	5·81	7·64	6·46	48
15	·763	·646	1·53	1·29	2·29	1·94	3·05	2·58	3·82	3·23	4·58	3·88	5·34	4·52	6·11	5·17	6·87	5·82	7·63	6·46	45
18	·763	·647	1·53	1·29	2·29	1·94	3·05	2·59	3·81	3·23	4·58	3·88	5·34	4·53	6·10	5·17	6·86	5·82	7·63	6·47	42
21	·762	·647	1·52	1·29	2·29	1·94	3·05	2·59	3·81	3·21	4·57	3·88	5·33	4·53	6·10	5·18	6·86	5·83	7·62	6·47	39
24	·762	·648	1·52	1·30	2·28	1·94	3·05	2·59	3·81	3·24	4·57	3·89	5·33	4·54	6·09	5·18	6·85	5·83	7·62	6·48	36
27	·761	·649	1·52	1·30	2·28	1·95	3·04	2·60	3·80	3·24	4·57	3·89	5·33	4·54	6·09	5·19	6·85	5·84	7·61	6·49	33
30	·760	·649	1·52	1·30	2·28	1·95	3·04	2·60	3·80	3·25	4·56	3·90	5·32	4·55	6·08	5·20	6·84	5·85	7·60	6·49	30
33	·760	·650	1·52	1·30	2·28	1·95	3·04	2·60	3·80	3·25	4·56	3·90	5·32	4·55	6·08	5·20	6·84	5·85	7·60	6·50	27
36	·759	·651	1·52	1·30	2·28	1·95	3·04	2·60	3·80	3·25	4·56	3·90	5·31	4·56	6·07	5·20	6·83	5·86	7·59	6·51	24
39	·759	·651	1·52	1·30	2·28	1·95	3·03	2·61	3·79	3·26	4·55	3·91	5·31	4·56	6·07	5·21	6·83	5·86	7·59	6·51	21
42	·758	·652	1·52	1·30	2·27	1·96	3·03	2·61	3·79	3·26	4·55	3·91	5·31	4·57	6·07	5·22	6·82	5·87	7·58	6·52	18
45	·758	·653	1·52	1·31	2·27	1·96	3·03	2·61	3·79	3·26	4·55	3·92	5·30	4·57	6·06	5·22	6·82	5·87	7·58	6·53	15
48	·757	·653	1·51	1·31	2·27	1·96	3·03	2·61	3·78	3·27	4·54	3·92	5·30	4·57	6·06	5·23	6·81	5·88	7·57	6·53	12
51	·756	·654	1·51	1·31	2·27	1·96	3·03	2·62	3·78	3·27	4·54	3·92	5·29	4·58	6·05	5·23	6·81	5·89	7·56	6·54	9
54	·756	·655	1·51	1·31	2·27	1·96	3·02	2·62	3·78	3·27	4·54	3·93	5·29	4·58	6·05	5·24	6·80	5·89	7·56	6·55	6
57	·755	·655	1·51	1·31	2·27	1·97	3·02	2·62	3·78	3·28	4·53	3·93	5·29	4·59	6·04	5·24	6·80	5·90	7·55	6·55	3
41°	·755	·656	1·51	1·31	2·26	1·97	3·02	2·62	3·77	3·28	4·53	3·94	5·28	4·59	6·04	5·25	6·79	5·90	7·55	6·56	49°
3	·754	·657	1·51	1·31	2·26	1·97	3·02	2·63	3·77	3·28	4·52	3·94	5·28	4·60	6·03	5·25	6·79	5·91	7·54	6·57	57
6	·754	·657	1·51	1·31	2·26	1·97	3·01	2·63	3·77	3·29	4·52	3·94	5·27	4·60	6·03	5·26	6·78	5·92	7·54	6·57	54
9	·753	·658	1·51	1·32	2·26	1·97	3·01	2·63	3·76	3·29	4·52	3·95	5·27	4·61	6·02	5·26	6·78	5·92	7·53	6·58	51
12	·752	·659	1·50	1·32	2·26	1·98	3·01	2·63	3·76	3·29	4·51	3·95	5·27	4·61	6·02	5·27	6·77	5·93	7·52	6·59	48
15	·752	·659	1·50	1·32	2·26	1·98	3·01	2·64	3·76	3·30	4·51	3·96	5·26	4·62	6·01	5·27	6·77	5·93	7·52	6·59	45
18	·751	·660	1·50	1·32	2·25	1·98	3·01	2·64	3·76	3·30	4·51	3·96	5·26	4·62	6·01	5·28	6·76	5·94	7·51	6·60	42
21	·751	·661	1·50	1·32	2·25	1·98	3·00	2·64	3·75	3·30	4·50	3·96	5·25	4·62	6·01	5·29	6·76	5·95	7·51	6·61	39
24	·750	·661	1·50	1·32	2·25	1·98	3·00	2·65	3·75	3·31	4·50	3·97	5·25	4·63	6·00	5·29	6·75	5·95	7·50	6·61	36
27	·750	·662	1·50	1·32	2·25	1·99	3·00	2·65	3·75	3·31	4·50	3·97	5·25	4·63	6·00	5·30	6·75	5·96	7·50	6·62	33
30	·749	·663	1·50	1·33	2·25	1·99	3·00	2·65	3·74	3·31	4·19	3·98	5·24	4·64	5.99	5·30	6·74	5·96	7·49	6·63	30
33	·748	·663	1·50	1·33	2·25	1·99	2·99	2·65	3·71	3·32	4·49	3·98	5·24	4·64	5·99	5·31	6·74	5·97	7·48	6·63	27
36	·748	·664	1·50	1·33	2·24	1·99	2·99	2·66	3·74	3·32	4·49	3·98	5·24	4·65	5·98	5·31	6·73	5·98	7·48	6·64	24
39	·747	·665	1·49	1·33	2·24	1·99	2·99	2·66	3·74	3·32	4·48	3·99	5·23	4·65	5·98	5·32	6·73	5·98	7·47	6·65	21
42	·747	·665	1·49	1·33	2·24	2·00	2·99	2·66	3·73	3·33	4·48	3·99	5·23	4·66	5·97	5·32	6·72	5·99	7·47	6·65	18
45	·746	·666	1·49	1·33	2·24	2·00	2·98	2·66	3·73	3·33	4·48	4·00	5·22	4·00	5·97	5·33	6·71	5·99	7·46	6·66	15
48	·745	·667	1·49	1·33	2·24	2·00	2·98	2·67	3·73	3·33	4·47	4·00	5·22	4·67	5·96	5·33	6·71	6·00	7·45	6·67	12
51	·745	·667	1·49	1·33	2·23	2·00	2·98	2·67	3·72	3·34	4·47	4·00	5·21	4·67	5·96	5·34	6·70	6·00	7·45	6·67	9
54	·744	·668	1·49	1·34	2·23	2·00	2·98	2·67	3·72	3·34	4·47	4·01	5·21	4·67	5·95	5·34	6·70	6·01	7·44	6·68	6
57	·744	·668	1·49	1·34	2·23	2·01	2·97	2·67	3·72	3·34	4·46	4·01	5·21	4·68	5·95	5·35	6·69	6·02	7·44	6·68	3
Dist.	Dep	Lat.	Dep	Lat.	Dep	Lat.	Dep	Lat.	Dep	Lat.	Dep	Lat.	Dep	Lat.	Dep	Lat.	Dep	Lat.	Dep	Lat.	Dist.

Chs.	1		2		3		4		5		6		7		8		9		10		Chs.
′	Lat	Dep	Lat.	Dep	Lat.	Dep	Lat.	Dep	Lat	Dep	Lat	Dep	Lat	Dep	Lat	Dep	Lat.	Dep	Lat.	Dep	′
48°	·743	·669	1·49	1·34	2·23	2·01	2·97	2·68	3·72	3·35	4·46	4·01	5·20	4·68	5·95	5·35	6·69	6·02	7·43	6·69	42°
3	·743	·670	1·49	1·34	2·23	2·01	2·97	2·68	3·71	3·35	4·46	4·02	5·20	4·69	5·94	5·36	6·68	6·03	7·43	6·70	57
6	·742	·670	1·48	1·34	2·23	2·01	2·97	2·68	3·71	3·35	4·45	4·02	5·19	4·69	5·94	5·36	6·68	6·03	7·42	6·70	54
9	·741	·671	1·48	1·34	2·22	2·01	2·97	2·68	3·71	3·36	4·45	4·03	5·19	4·70	5·93	5·37	6·67	6·04	7·41	6·71	51
12	·741	·672	1·48	1·34	2·22	2·02	2·96	2·69	3·70	3·36	4·44	4·03	5·19	4·70	5·93	5·37	6·67	6·05	7·41	6·72	48
15	·740	·672	1·48	1·34	2·22	2·02	2·96	2·69	3·70	3·36	4·44	4·03	5·18	4·71	5·92	5·38	6·66	6·05	7·40	6·72	45
18	·740	·673	1·48	1·35	2·22	2·02	2·96	2·69	3·70	3·37	4·44	4·04	5·18	4·71	5·92	5·38	6·66	6·06	7·40	6·73	42
21	·739	·674	1·48	1·35	2·22	2·02	2·96	2·69	3·70	3·37	4·43	4·04	5·17	4·72	5·91	5·39	6·65	6·06	7·39	6·74	39
24	·738	·674	1·48	1·35	2·22	2·02	2·95	2·70	3·69	3·37	4·43	4·05	5·17	4·72	5·91	5·39	6·65	6·07	7·38	6·74	36
27	·738	·675	1·48	1·35	2·21	2·02	2·95	2·70	3·69	3·37	4·43	4·05	5·17	4·72	5·90	5·40	6·64	6·07	7·38	6·75	33
30	·737	·676	1·47	1·35	2·21	2·03	2·95	2·70	3·69	3·38	4·42	4·05	5·16	4·73	5·90	5·40	6·64	6·08	7·37	6·76	30
33	·737	·676	1·47	1·35	2·21	2·03	2·95	2·70	3·68	3·38	4·42	4·06	5·16	4·73	5·89	5·41	6·63	6·09	7·37	6·76	27
36	·736	·677	1·47	1·35	2·21	2·03	2·94	2·70	3·68	3·38	4·42	4·06	5·15	4·74	5·89	5·42	6·62	6·09	7·36	6·77	24
39	·736	·678	1·47	1·36	2·21	2·03	2·94	2·71	3·68	3·39	4·41	4·07	5·15	4·74	5·88	5·42	6·62	6·10	7·36	6·78	21
42	·735	·678	1·47	1·36	2·20	2·03	2·94	2·71	3·67	3·39	4·41	4·07	5·14	4·75	5·88	5·43	6·61	6·10	7·35	6·78	18
45	·734	·679	1·47	1·36	2·20	2·04	2·94	2·72	3·67	3·39	4·41	4·07	5·14	4·75	5·87	5·43	6·61	6·11	7·34	6·79	15
48	·734	·679	1·47	1·36	2·20	2·04	2·93	2·72	3·67	3·40	4·40	4·08	5·14	4·76	5·87	5·44	6·60	6·11	7·34	6·79	12
51	·733	·680	1·47	1·36	2·20	2·04	2·93	2·72	3·67	3·40	4·40	4·08	5·13	4·76	5·87	5·44	6·60	6·12	7·33	6·80	9
54	·733	·681	1·47	1·36	2·20	2·04	2·93	2·72	3·66	3·40	4·40	4·08	5·13	4·77	5·86	5·45	6·59	6·13	7·33	6·81	6
57	·732	·681	1·46	1·36	2·20	2·04	2·93	2·73	3·66	3·41	4·39	4·09	5·12	4·77	5·86	5·45	6·59	6·13	7·32	6·81	3
47°	·731	·682	1·46	1·36	2·19	2·05	2·93	2·73	3·66	3·41	4·39	4·09	5·12	4·77	5·85	5·46	6·58	6·14	7·31	6·82	43°
3	·731	·683	1·46	1·37	2·19	2·05	2·93	2·73	3·65	3·41	4·38	4·10	5·12	4·78	5·85	5·46	6·58	6·14	7·31	6·83	57
6	·730	·683	1·46	1·37	2·19	2·05	2·92	2·73	3·65	3·42	4·38	4·10	5·11	4·78	5·84	5·47	6·57	6·15	7·30	6·83	54
9	·730	·684	1·46	1·37	2·19	2·05	2·92	2·74	3·65	3·42	4·38	4·10	5·11	4·79	5·84	5·47	6·57	6·16	7·30	6·84	51
12	·729	·685	1·46	1·37	2·19	2·05	2·92	2·74	3·64	3·42	4·37	4·11	5·10	4·79	5·83	5·48	6·56	6·16	7·29	6·85	48
15	·728	·685	1·46	1·37	2·19	2·06	2·91	2·74	3·64	3·43	4·37	4·11	5·10	4·80	5·83	5·48	6·56	6·17	7·28	6·85	45
18	·728	·686	1·46	1·37	2·18	2·06	2·91	2·74	3·64	3·43	4·37	4·11	5·10	4·80	5·82	5·49	6·55	6·17	7·28	6·86	42
21	·727	·686	1·46	1·37	2·18	2·06	2·91	2·75	3·64	3·43	4·36	4·12	5·09	4·81	5·82	5·49	6·54	6·18	7·27	6·86	39
24	·727	·687	1·45	1·37	2·18	2·06	2·91	2·75	3·63	3·44	4·36	4·12	5·09	4·81	5·81	5·50	6·54	6·18	7·27	6·87	36
27	·726	·688	1·45	1·38	2·18	2·06	2·90	2·75	3·63	3·44	4·36	4·13	5·08	4·81	5·81	5·50	6·53	6·19	7·26	6·88	33
30	·725	·688	1·45	1·38	2·18	2·07	2·90	2·75	3·63	3·44	4·35	4·13	5·08	4·82	5·80	5·51	6·53	6·20	7·25	6·88	30
33	·725	·689	1·45	1·38	2·17	2·07	2·90	2·76	3·62	3·44	4·35	4·13	5·07	4·82	5·80	5·51	6·52	6·20	7·25	6·89	27
36	·724	·690	1·45	1·38	2·17	2·07	2·90	2·76	3·62	3·45	4·35	4·14	5·07	4·83	5·79	5·52	6·52	6·21	7·24	6·90	24
39	·724	·690	1·45	1·38	2·17	2·07	2·89	2·76	3·62	3·45	4·34	4·14	5·07	4·83	5·79	5·52	6·51	6·21	7·24	6·90	21
42	·723	·691	1·45	1·38	2·17	2·07	2·89	2·76	3·61	3·45	4·34	4·15	5·06	4·84	5·78	5·53	6·51	6·22	7·23	6·91	18
45	·722	·692	1·44	1·38	2·17	2·07	2·89	2·77	3·61	3·46	4·33	4·15	5·06	4·84	5·78	5·53	6·50	6·22	7·22	6·92	15
48	·722	·692	1·44	1·38	2·17	2·08	2·89	2·77	3·61	3·46	4·33	4·15	5·05	4·85	5·77	5·54	6·50	6·23	7·22	6·92	12
51	·721	·693	1·44	1·39	2·16	2·08	2·88	2·77	3·61	3·46	4·33	4·16	5·05	4·85	5·77	5·54	6·49	6·23	7·21	6·93	9
54	·721	·693	1·44	1·39	2·16	2·08	2·88	2·77	3·60	3·47	4·32	4·16	5·04	4·85	5·76	5·55	6·48	6·24	7·21	6·93	6
57	·720	·694	1·44	1·39	2·16	2·08	2·88	2·78	3·60	3·47	4·32	4·16	5·04	4·86	5·76	5·55	6·48	6·25	7·20	6·94	3
46°	·719	·695	1·44	1·39	2·16	2·08	2·88	2·78	3·60	3·47	4·32	4·17	5·04	4·86	5·75	5·56	6·47	6·25	7·19	6·95	44°
3	·719	·695	1·44	1·39	2·16	2·09	2·87	2·78	3·59	3·48	4·31	4·17	5·03	4·87	5·75	5·56	6·47	6·26	7·19	6·95	57
6	·718	·696	1·44	1·39	2·15	2·09	2·87	2·78	3·59	3·48	4·31	4·18	5·03	4·87	5·75	5·57	6·46	6·26	7·18	6·96	84
9	·718	·697	1·44	1·39	2·15	2·09	2·87	2·79	3·59	3·48	4·31	4·18	5·02	4·88	5·74	5·57	6·46	6·27	7·18	6·97	51
12	·717	·697	1·43	1·39	2·15	2·09	2·87	2·79	3·58	3·49	4·30	4·18	5·02	4·88	5·74	5·58	6·45	6·27	7·17	6·97	48
15	·716	·698	1·43	1·40	2·15	2·09	2·87	2·79	3·58	3·49	4·30	4·19	5·01	4·88	5·73	5·58	6·45	6·28	7·16	6·98	45
18	·716	·698	1·43	1·40	2·15	2·10	2·86	2·79	3·58	3·49	4·29	4·19	5·01	4·89	5·73	5·59	6·44	6·29	7·16	6·98	42
21	·715	·699	1·43	1·40	2·15	2·10	2·86	2·80	3·58	3·50	4·29	4·19	5·01	4·89	5·72	5·59	6·44	6·29	7·15	6·99	39
24	·714	·700	1·43	1·40	2·14	2·10	2·86	2·80	3·57	3·50	4·29	4·20	5·00	4·90	5·72	5·60	6·43	6·30	7·14	7·00	36
27	·714	·700	1·43	1·40	2·14	2·10	2·86	2·80	3·57	3·50	4·28	4·20	5·00	4·90	5·71	5·60	6·42	6·30	7·14	7·00	33
30	·713	·701	1·43	1·40	2·14	2·10	2·85	2·80	3·57	3·50	4·28	4·21	4·99	4·91	5·71	5·61	6·42	6·31	7·13	7·01	30
33	·713	·702	1·43	1·40	2·14	2·10	2·85	2·81	3·56	3·51	4·28	4·21	4·99	4·91	5·70	5·61	6·41	6·31	7·13	7·02	27
36	·712	·702	1·42	1·40	2·14	2·11	2·85	2·81	3·56	3·51	4·27	4·21	4·98	4·92	5·70	5·62	6·41	6·32	7·12	7·02	24
39	·711	·703	1·42	1·41	2·13	2·11	2·85	2·81	3·56	3·51	4·27	4·22	4·98	4·92	5·69	5·62	6·40	6·32	7·11	7·03	21
42	·711	·703	1·42	1·41	2·13	2·11	2·84	2·81	3·55	3·52	4·26	4·22	4·98	4·92	5·69	5·63	6·40	6·33	7·11	7·03	18
45	·710	·704	1·42	1·41	2·13	2·11	2·84	2·82	3·55	3·52	4·26	4·22	4·97	4·93	5·68	5·63	6·39	6·34	7·10	7·04	15
48	·710	·705	1·42	1·41	2·13	2·11	2·84	2·82	3·55	3·52	4·26	4·23	4·97	4·93	5·68	5·64	6·39	6·34	7·10	7·05	12
51	·709	·705	1·42	1·41	2·13	2·12	2·84	2·82	3·54	3·53	4·25	4·23	4·96	4·94	5·67	5·64	6·38	6·35	7·09	7·05	9
54	·708	·706	1·42	1·41	2·13	2·12	2·83	2·82	3·54	3·53	4·25	4·24	4·96	4·94	5·67	5·65	6·38	6·35	7·08	7·06	6
57	·708	·706	1·42	1·41	2·12	2·12	2·83	2·83	3·54	3·53	4·25	4·24	4·95	4·95	5·66	5·65	6·37	6·36	7·08	7·06	3
45°	·707	·707	1·41	1·41	2·12	2·12	2·83	2·83	3·54	3·54	4·24	4·24	4·95	4·95	5·66	5·66	6·36	6·36	7·07	7·07	45°
Dist.	Dep	Lat.	Dep	Lat.	Dep	Lat.	Dep	Lat.	Dep	Lat.	Dep	Lat.	Dep	Lat.	Dep	Lat.	Dep	Lat.	Dep	Lat.	Dist